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ERRATUM

Erratum to: A Verification Theorem for Optimal Stopping Problems with Expectation Constraints

Stefan Ankirchner[1] · **Maike Klein**[1] · **Thomas Kruse**[2]

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We correct the statement of Lemma 2.2 in the original article. The solution of the SDE (2.2) is, in general, not a martingale but only a supermartingale. The set of controls is restricted to those processes such that the solution of Eq. (2.2) is a martingale. The remaining results and examples are valid for the new set of controls.

We first correct the statement of Lemma 2.2 in the original article. For $m \in \mathbb{R}_+$ the solution of the SDE

$$dM_t = \mathbb{1}_{\{M_t > H_t\}} \alpha_t \cdot dW_t, \quad M_0 = m \tag{2.2}$$

is a supermartingale but not necessarily a martingale (see Example 2.3 below for a counterexample). To show that M is a martingale we conclude in the original article that $\tau_n = \tau$ on $\{M_\tau \leq n\}$, which is not true in general. The corrected version of Lemma 2.2 reads as follows:

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✉ Stefan Ankirchner
s.ankirchner@uni-jena.de

Maike Klein
maike.klein@uni-jena.de

Thomas Kruse
thomas.kruse@uni-due.de

1 Institute for Mathematics, University of Jena, Ernst-Abbe-Platz 2, 07743 Jena, Germany

2 Faculty of Mathematics, University of Duisburg-Essen, Thea-Leymann-Str. 9, 45127 Essen, Germany

Lemma 2.2 *Let* $(\alpha_t)_{t\geq 0} = (\alpha_t^1, \ldots, \alpha_t^d)_{t\geq 0} \in L^2_{loc}(W)$ *and* $m \in \mathbb{R}_+$. *Then there exists a unique strong solution* M *of (2.2). This solution is a non-negative supermartingale.*

As a consequence, for the one-to-one-correspondence claimed in Proposition 2.3 to hold true, we need to require that the set of controls consists of processes $\alpha \in L^2_{loc}(W)$ such that the solution of (2.2) is a true martingale. More precisely, let

$$\mathcal{A} = \Big\{\alpha \in L^2_{loc}(W) \mid E[H_\tau] = M_0, \text{ where } M \text{ solves (2.2) for } \alpha \text{ and } \tau = \inf\{t \geq 0 \mid M_t \leq H_t\}\Big\}$$

and let $\mathcal{M}(m)$ be the set of all solutions M of (2.2) with $(\alpha_t)_{t\geq 0} \in \mathcal{A}$. Observe that Lemma 2.2 implies that for $\alpha \in \mathcal{A}$ the solution (M_t) of (2.2) is a true martingale with $M_t \to M_\infty$ in $L^1(\Omega)$ for $t \to \infty$. Moreover, $M_\infty = M_\tau = H_\tau$ by the definition of τ. On the other hand, if for $\alpha \in L^2_{loc}(W)$ the solution of (2.2) is a true martingale with $M_t \to M_\infty$ in $L^1(\Omega)$ for $t \to \infty$, then $E[H_\tau] = E[M_\tau] = M_0$. Notice that $\mathcal{A}$ is non-empty.

If $L^2_{loc}(W)$ is replaced by $\mathcal{A}$ in the subsequent statements, all results and arguments hold true. Moreover, observe that the processes α and α^* in Example 2.6, 2.7, 4.5, 4.6 and 4.7 are contained in $\mathcal{A}$. In the proof of the first part of Proposition 3.4 we now consider the control $\alpha_s = \mathbb{1}_{\{s\leq 1\}} a^\top$ with $a \in \mathbb{R}^d$. Then $\alpha \in \mathcal{A}$. For applying Itô's formula in (3.3) choose $t \in (0, 1)$. The remaining proof is unchanged.

The following example shows that $\mathcal{A} \neq L^2_{loc}(W)$.

Example 2.3 Let $d = 1$ and $h(y) = 1$ for all $y \in \mathbb{R}$. Let $\alpha_t = -\mathbb{1}_{\{t<1\}} W_t e^{-\frac{W_t^2}{2(1-t)}}/(1-t)^{3/2}$ and $m = 2$. Then $\tau_n = \inf\{t \geq 0 \mid |\alpha_t| \geq n\}$ is a localizing sequence for α and thus, $\alpha \in L^2_{loc}(W)$. Moreover, the solution M of (2.2) is given by

$$M_t = \begin{cases} 1 + \frac{1}{\sqrt{1-t}} e^{-\frac{W_t^2}{2(1-t)}}, & t < 1, \\ 1, & t \geq 1. \end{cases}$$

Then $M_t \geq 1$ for all $t \geq 0$ and $M_1 = 1 = H_1$. Thus, $\tau := \inf\{t \geq 0 \mid M_t \leq t\} = 1$, a.s. Moreover, (M_t) is a local martingale, but not a true martingale, because $M_0 = 2$ and $M_1 = 1$, a.s.

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