Libor Polák (1950–2020)

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Received: 7 December 2020 / Accepted: 14 December 2020 / Published online: 2 March 2021

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Libor Polák passed away suddenly on July 10, 2020.

1 Prologue

To characterize Libor briefly, one could say he was a mathematician from the bottom of his soul. He also had quite a specific sense of humor. As he often applied his distinct way of thinking to everyday life, he could be considered as an eccentric or quirky person by some people from his social neighborhood. To illustrate these aspects of his character, we take the liberty of interpreting here his favorite

Communicated by Jorge Almeida.

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witticism, which he liked to tell students: “When you stand at a stop, and a tram comes, then there are two options: either you get on and leave, or the tram leaves without you. You cannot board partially or leave nearly.” Perhaps this narrative captures his two-valued logic, with which he approached both scientific and everyday life. Indeed, that approach found positive expression in his enthusiasm because he could not do things by halves. (One may see it as a little bit of irony that Libor dealt with semigroup theory for most of his scientific life.) On the other hand, the negative consequences of his approach included his inability to make compromises. That handicap then led to conflicts in his personal and professional life. Nevertheless, for his mathematical ideas and boundless enthusiasm, he gained respect from us, many students and colleagues, and researchers working in the same field. Especially for the latter, we attempt to summarize his most important contributions in our short article. Before that, let us start with elementary data.

2 Vita

Libor was born on June 4, 1950 in Brno, the second-largest city in the Czech Republic and the center of Southern Moravia. He spent his whole life in Brno, and he always perceived himself to be a proud Moravian. In high school, Libor was enchanted by mathematics so much that he represented Czechoslovakia at the International Mathematical Olympiad in 1968. He graduated in mathematics from J. E. Purkyně University (now Masaryk University) of Brno in 1973. Six years later, under the supervision of Milan Sekanina, he defended here the candidate thesis (an equivalent of a Ph.D. thesis at that time in Czechoslovakia). Then he worked at the same university as an assistant professor until 1987 and as an associate professor since then. He taught lectures on algebra, graph algorithms, game theory, and semigroup theory. For ordinary students, his subjects were demanding obstacles, but some interested students loved his inspiring lectures. Libor had stopped teaching in 2015 and continued doing research as a partial job until his definite retirement four years later. He supervised four doctoral students during his career, namely Martin Kuřil, Michal Kunc, and both authors. Libor was also quite active in the international community. During his scientific career, he regularly visited international conferences, and he spent more than three years altogether abroad at several universities.

Libor Polák is the author or coauthor of about fifty scientific papers. One could divide his research activity into three main periods. He started with applications of universal algebraic principles in the theory of completely regular semigroups. This contribution brought him renown in the semigroup community, and there is no doubt that this contribution was the most valuable part of his research ever. Then, in the second period, he obtained other remarkable results in areas where universal algebra intertwines with semigroup theory. These areas include concepts like hyperassociativity, locally inverse semigroups, and $*$ -regular semigroups. In the third period, he became interested in applications of algebra in theoretical computer science. The contribution from that period consists mainly in the introduction of some perspective concepts in the theory of regular languages. In what follows, we look at the essential contributions in more detail.

3 On varieties of completely regular semigroups

The early (joint) works of Libor Polák are concerned with various concepts connected with a specific view on universal algebra [13, 14, 28]. After that early period, Libor Polák moved his attention to semigroup theory. Naturally, he remained influenced by his previous experience with universal algebra, so he oriented himself mainly to varieties of semigroups and related issues. From this time comes his profound work on varieties of completely regular semigroups contained in his renowned trilogy of papers [16–18].

The approach of Libor Polák towards the study of varieties of completely regular semigroups was primarily syntactic. He studied varieties of completely regular semigroups by means of fully invariant congruences on the free completely regular semigroup on a countably infinite set of generators. Inspired by his previous case study [15], he introduced in [16] and [17] several operators on the lattice of all fully invariant congruences on the mentioned free completely regular semigroup and studied generic properties of these operators. This led him to several complete lattice congruences on the lattice of varieties of completely regular semigroups and to the respective partitions of this lattice into congruence classes, in fact, into intervals. These decompositions of the lattice of varieties of completely regular semigroups gave him novel insight into the structure of this lattice and brought him to a breakthrough result, nowadays commonly referred to as Polák's theorem, which is presented and fully proved in [17] and which is further discussed and applied in [18]. Polák's theorem subsequently found numerous applications in diverse works of other authors. From all these advancements, we choose the papers [27] by Reilly and [30] by Trotter, for instance, where various subdirect product decompositions of the lattice of varieties of completely regular semigroups are exhibited, generalizing many earlier results of this kind.

Another significant contribution of Libor Polák in the theory of completely regular semigroups is contained in [5]. There, together with Jiří Kad'ourek, they presented a transparent solution of the word problem for free completely regular semigroups. This result also became the source of inspiration for subsequent investigations of many researchers. We choose at random the paper [4] by Jones, for example, where the e-locality of the variety of all completely regular semigroups is established.

4 On specific varieties of (regular) semigroups

Another theme of research pursued by Libor Polák was that of hyperassociativity and of solid varieties of semigroups. Recall that a given identity is hypersatisfied by a variety of algebras if every identity resulting from that identity by replacing every fundamental operation with a term operation of the same arity is also satisfied. Furthermore, a variety of algebras is said to be solid if it hypersatisfies all its identities. The contribution of Libor Polák to the study of solid varieties of

semigroups was indeed significant. In [19], he gave a simple equational basis for the largest solid variety of semigroups. He also provided a solution of the word problem for the completely regular part of the free semigroup (on any number of generators) in that variety. Then in [20], he crowned his laborious study of this subject with a detailed classification of all solid varieties of semigroups.

Nevertheless, Libor Polák remained interested in regular semigroups. His next topic was that of modifying the standard semidirect product of ordinary semigroups in order to obtain a kind of semidirect product on large classes of regular semigroups. Thus, in [1], jointly with Karl Auinger, they investigated from this viewpoint the existence variety of all locally inverse semigroups. This led them to a multiplication on the lattice of existence varieties of locally inverse semigroups extending the previously studied multiplication on the lattice of varieties of inverse semigroups. As an application of their results on bifree objects in such a product of two existence varieties of locally inverse semigroups, they have proved that the mentioned multiplication is associative. The ultimate generalization in this direction has eventually been exhibited later on by Billhardt in [2].

The next publications by Libor Polák already foreshadow his subsequent switch-over to theoretical computer science. His paper [21] still deals with regular semigroups, namely, it concerns the solvability of certain equations in relatively free completely simple semigroups. This matter of interest has to do with term unification, an essential concept in many areas of computer science. Of another character is his next paper [22], where a neat solution of the word problem for free $*$ -regular semigroups is obtained using a confluent reduction of words to canonical forms. Thus, his solution is a sort of term rewriting, another well-established concept in theoretical computer science.

5 On algebraic theory of regular languages

The first concept which Libor Polák invented in the theory of formal languages was the so-called syntactic semiring of a language. It is a classical construction that to any language one can assign its syntactic (ordered) monoid. According to Eilenberg's theorem, there is a one-to-one correspondence between the so-called varieties of regular languages and pseudovarieties of monoids. There is a refinement of this theorem due to Pin, according to which there is a one-to-one correspondence between the so-called positive varieties of regular languages and pseudovarieties of ordered monoids. In [25], Libor Polák established a further refinement of this theorem saying that there is a one-to-one correspondence between the so-called conjunctive varieties of regular languages and pseudovarieties of idempotent semirings. Subsequently, he found some applications of the notion of the syntactic semiring of a regular language in the paper [23], searching for maximal solutions of a certain type of language equations. We also mention the paper [24] containing his perspective of the notion of universal automaton, which led him to observations concerning the minimization of a non-deterministic finite automaton in [26].

In 2007, Libor Polák joined Ondřej Klíma and started a long-term cooperation with him, mostly related to Eilenberg-type correspondences. The first direction

of that research was the automaton counterpart to the syntactic semiring. The respective automata were equipped with an additional semilattice operation compatible with actions by letters and called meet automata. In [6], the correspondence between varieties of meet automata and conjunctive varieties of languages was established. The theory of meet automata recognizing a regular language goes along the same lines as the original algebraic theory of deterministic finite automata. In particular, the syntactic semiring is the transition semiring of the minimal meet automaton of a given language, which always exists. Using this relationship, alternative characterizations of natural subhierarchies of piecewise testable languages were given in [7].

Libor Polák and Ondřej Klíma also studied varieties of (ordered) automata. The considered automata are called actions in the paper [3] by Chaubard, Pin, and Straubing, who established the Eilenberg-type correspondence for varieties of actions. Ordered automata are automata equipped with a partial order on the set of states such that transitions by letters are isotone mappings. The paper [11] by Libor Polák and Ondřej Klíma was devoted to descriptions of varieties of (ordered) automata corresponding to well-known instances of (positive) varieties of regular languages. A continuation of that research is the paper [12], where they studied the phenomenon of forbidden patterns for automata in the frame of the theory of varieties of ordered automata. The above-mentioned papers actually deal with those notions in a more general setting of (positive) $\mathcal{C}$ -varieties of languages introduced by Straubing [29].

Motivated by Brzozowski's construction of the minimal automaton of a regular language via left quotients of the language, Libor Polák considered two-sided quotients, which led him to a new notion of biautomata [8], a kind of finite automata that arbitrarily alternate between reading the input word from the left and from the right. This notion was employed by Libor Polák and Ondřej Klíma in [9], where they characterized the class of all piecewise testable languages by a simple biautomata condition. In [10], they used techniques developed for proving this characterization to provide an alternative proof of Simon's original automata characterization of piecewise testable languages. As a consequence, they obtained a slight improvement of the best known polynomial-time algorithm for the membership problem for this class of languages.

6 Epilogue

Throughout his academic life Libor Polák worked on diverse topics within algebraic semigroup theory and theoretical computer science. His discoveries in these fields of research had a profound impact on further development in these areas of science. Libor Polák loved sharing his thoughts with collaborators and students. All of us who knew him and worked with him, have greatly benefited from encountering him and owe him much. His approach to doing mathematics and his insight into the problems under study were a source of inspiration for us.

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