Correction to: Some characterizations of the tensor product of complete lattices with applications to quantales

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For L a complete lattice, let $\text{Up}(L)$ be the complete lattice of all upclosed subsets of L ordered by $\subseteq^{op}$. An upclosed subset A is prime if $a \vee b \in A$ implies $a \in A$ or $b \in A$. Let $\text{PUp}(L)$ be the complete lattice of all prime members of $\text{Up}(L)$.

We apologize for the false statement on page 310 that $\text{Up}(L) \otimes M$ and $\text{PUp}(L) \otimes M$ are always order-isomorphic. Indeed, $M = \mathbf{2}$ is the unit of the tensor product. Thus, if that statement were true, this would mean that $\text{Up}(L)$ and $\text{PUp}(L)$ are order-isomorphic. But this is not always the case as simple counterexamples show. For instance, if L is the four-point diamond, then $\text{PUp}(L)$ has five elements, while $\text{Up}(L)$ has six elements.

That unfortunate statement played a role at the beginning of the proof of Corollary 2. Since this corollary is perfectly correct, it is the purpose of this note to start the proof of Corollary 2 appropriately. Let f be a tensor of $M \otimes L$. By complete distributivity of M and Lemma 3, f can uniquely be identified with the map determined by (see (4.4)):

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$$\text{Up}(L) \xrightarrow{\psi_f} M, \quad \psi_f(A) = \bigvee f^{-1}(A), \quad A \in \text{Up}(L).$$

Moreover, since $A = \bigcap_{a \notin A} L \setminus \downarrow a$, the following hold:

$$\psi_f(A) = \bigvee \left(\bigcap_{a \notin A} f^{-1}(L \setminus \downarrow a) \right) = \bigwedge_{a \notin A} \left(\bigvee f^{-1}(L \setminus \downarrow a) \right) = \bigwedge_{a \notin A} \psi_f(L \setminus \downarrow a),$$

i.e., ψ_f is uniquely determined by its values at all prime upclosed subsets of the form $L \setminus \downarrow a$. Hence ψ_f is completely determined by its restriction to $\text{PUp}(L)$. Thus, Lemma 3 implies that every tensor $f \in M \otimes L$ can uniquely be identified with a map $\text{PUp}(L) \xrightarrow{\varphi_f} M$ determined by (4.5).

The remaining part of the original proof of Corollary 2 remains unchanged.

None of the other results of the paper are affected.

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