Correction to: Flat coset decompositions of skew lattices

João Pita Costa¹ · Karin Cvetko-Vah²

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Abstract In this corrigendum, we correct the statement and the proof of Proposition 4.11, where the word “lower” is stated instead of “upper,” and vice versa.

Proposition 1 (Proposition 4.11) *Let S be a skew lattice such that $S/\mathcal{D}$ is a distributive lattice.*

- (i) *Let S be upper symmetric. Then, S is upper cancellative, if and only if $M \vee x \vee M = M \vee x' \vee M \Leftrightarrow B \vee x \vee B = B \vee x' \vee B$ holds for all skew diamonds $\{J > A, B > M\}$ in S and all $x, x' \in A$.*
- (ii) *Let S be lower symmetric. Then, S is lower cancellative, if and only if $B \wedge x \wedge B = B \wedge x' \wedge B \Leftrightarrow J \wedge x \wedge J = J \wedge x' \wedge J$ holds for all skew diamonds $\{J > A, B > M\}$ in S and all $x, x' \in A$.*

Proof (i). A skew lattice is upper cancellative if and only if it is upper symmetric and simply cancellative. Assume first that S is not upper cancellative, i.e., it is not simply cancellative. By a result of [5], it follows that S contains a subalgebra S' isomorphic to $\mathbf{NC}_5$, given by the diagram below:

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✉ Karin Cvetko-Vah
karin.cvetko@fmf.uni-lj.si
João Pita Costa
joao.pitacosta@ijs.si

¹ Institut Jožef Stefan, Jamova Cesta 39, 1000 Ljubljana, Slovenia

² Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia

In $\mathbf{NC}_5$ operations on $\{x_1, x_2\}$ are defined in one of the following ways: either $x_i \wedge x_j = x_j$ and $x_i \vee x_j = x_i$ which yields a right-handed structure; or $x_i \wedge x_j = x_i$ and $x_i \vee x_j = x_j$ yielding a left-handed structure. Let A, B, M and J denote the $\mathcal{D}$ -classes of elements x_1, y, u and v in S , respectively. Note that $x_2 \in A$. Since x_1 and x_2 are both contained in the image of u in A , they cannot lie in the same coset of M in A , i.e., we obtain $M \vee x_1 \vee M \neq M \vee x_2 \vee M$. On the other hand, the cosets $B \vee x_1 \vee B$ and $B \vee x_2 \vee B$ both contain v (because $y \vee x_1 \vee y = v = y \vee x_2 \vee y$) and hence coincide by Theorem 2.4. Therefore, the implication $B \vee x_1 \vee B = B \vee x_2 \vee B \Rightarrow M \vee x_1 \vee M = M \vee x_2 \vee M$ fails.

Assume next that S is upper cancellative and let $\{J > A, B > M\}$ be a skew diamond in S . Given any $x, x' \in A$, the direct implication $M \vee x \vee M = M \vee x' \vee M \Rightarrow B \vee x \vee B = B \vee x' \vee B$ holds by Remark 4.8. Assume that there exist $x_1, x_2 \in A$ such that $B \vee x_1 \vee B = B \vee x_2 \vee B$, but $M \vee x_1 \vee M \neq M \vee x_2 \vee M$. Take any $m \in M$ and consider $a_1 = m \vee x_1 \vee m$, $a_2 = m \vee x_2 \vee m$. Note that $a_1, a_2 \in A$, $m < a_1$ and $m < a_2$. Take $b \in B$ such that $b > m$. Since $m < a_1$ and $m < b$, it follows that $a_1 \wedge b = m = b \wedge a_1$, and likewise $a_2 \wedge b = m = b \wedge a_2$. By upper symmetry, $a_1 \vee b = b \vee a_1$ and $a_2 \vee b = b \vee a_2$ follow. Denote $j_1 = a_1 \vee b$ and $j_2 = a_2 \vee b$. The assumption $B \vee x_1 \vee B = B \vee x_2 \vee B$ together with $m < b$ implies $b \vee a_1 \vee b = b \vee m \vee x_1 \vee m \vee b = b \vee x_1 \vee b = b \vee x_2 \vee b = b \vee m \vee x_2 \vee m \vee b = b \vee a_2 \vee b$. It follows that $j_1 = j_2$, and the set $S' = \{m, a_1, a_2, b, j_1\}$ forms a subalgebra of S , given by the following diagram:

(Again, with the operations on $\{a_1, a_2\}$ defined either in a right-handed or in a left-handed fashion.) Subalgebra S' is isomorphic to $\mathbf{NC}_5$. By a result of [5], it follows that S is not simply cancellative and thus not upper cancellative, which is a contradiction.

The proof of (ii) is similar. $\square$