## **ADDENDUM**



## An Addendum to "Hypergroup deformations of semigroups"

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Unfortunately, in [2] we overlooked a fundamental class of examples studied by Michael Voit ([3], Sections 5 and 6), which are also generalizations of the class of Dunkl-Ramirez discrete hypergroups [1], and their duals are also almost discrete, i.e., one-point compactifications of discrete countably infinite spaces. These examples are clearly (hermitian) hypergroup deformations of the semigroup ( $\mathbb{Z}_+$ , <, max).

Voit studied his class of examples to illustrate factorization of probability measures on certain symmetric, i.e., hermitian hypergroups. In [2], we arrived at the class via necessary conditions for a hypergroup deformation (S,\*) of an infinite "max" semigroup  $(S,<,\cdot)$  with identity, the first necessary condition being that  $(S,<,\cdot)$  is isomorphic to  $(\mathbb{Z}_+,<,\max)$ . The sufficiency had a simple computational proof. Of course, it should have been attributed to Voit [3], had we been aware of this class of examples. As is clear from our Theorem 3.2, general discussion after Remark 3 and Corollary 3.3, the two classes, viz., Voit's class and the (hermitian) hypergroup deformations of  $(\mathbb{Z}_+,<,\max)$ , coincide. Because Voit discusses the dual  $\widehat{K}$  of any such hypergroup K, the detailed proofs in Sect. 3.3 should have been avoided.

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