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ORIGINAL

A meshless procedure for analysis of fluid flow and heat transfer in an internally finned square duct

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Abstract

Application of the method of fundamental solutions in combination with the global radial basis function collocation method for analysis of fluid flow and heat transfer in an internally finned square duct is presented in the paper. Fluid flow problem is solved using the modified method of fundamental solutions. After that, the average fluid velocity and product of friction factor and Reynolds number can be determined. Heat transfer problem in the fluid is governed by a nonlinear equation with linear boundary conditions. The Picard iteration method is employed in the paper in order to transform the nonlinear problem into a sequence of inhomogeneous problems. At each iteration step, the general solution is obtained using the modified method of fundamental solutions and the particular solution is obtained using the global radial basis function collocation method. When the iteration process is stopped, the Nusselt number can be determined.

Nomenclature

a	Half of internal width of channel [m]
b	Thickness of wall [m]
$\widehat{B}$	Dimensionless thickness of wall [−]
$\widehat{c}$	Shape parameter [−]
c_j	j-th unknown coefficient of the approximate solution for the fluid velocity [−]
c_p	Specific heat at a constant pressure [J/(kg·K)]
d	Half of thickness of fin [m]
$\widehat{D}$	Half of dimensionless thickness of fin [−]
e_j	j-th unknown coefficient of the particular solution for the fluid temperature [−]
f	friction factor [−]
g_j	j-th unknown coefficient of the general solution for the fluid temperature [−]
h_j	j-th unknown coefficient of the approximate solution for the wall temperature [−]
k_f	Thermal conductivity of the fluid [W/(m·K)]
k_w	Thermal conductivity of the wall [W/(m·K)]
K	Dimensionless thermal conductivity of the wall [−]
l	Length of fin [m]
$\widehat{L}$	Dimensionless length of fin [−]
$\dot{m}$	Mass flow rate [kg/s]
M_1	Number of harmonic functions in the approximate solution for the fluid velocity [−]
M_2	Number of harmonic functions in the general solution for the fluid temperature [−]
M_3	Number of harmonic functions in the approximate solution for the wall temperature [−]
n	Normal direction [−]
N_{int}	Number of internal points [−]
NS_f	Number of source points for fluid flow problem [−]
NS_{tf}	Number of source points for heat transfer problem in the fluid region [−]
NS_{tw}	Number of source points for heat transfer problem in the wall region [−]
Nu	Nusselt number [−]
p	Pressure [Pa]
$\widehat{P}$	Dimensionless wetted perimeter [−]
q_{av}	Average heat flux through external surface of the tube [W/m^2]
r_j	Distance between the point (X, Y) and the j-th internal point $(X_{\mathrm{int}j}, Y_{\mathrm{int}j})$ [−]
r_{Sj}	Distance between the point (X, Y) and the j-th source point (X_{Sj}, Y_{Sj}) [−]
Re	Reynolds number [−]
S	Distance between the pseudo-boundary and the considered region [−]
$\widehat{S}$	Dimensionless area of the repeated element [−]
T_f	Fluid temperature [K]

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T_o Constant temperature at the outer boundary of the tube [K]
T_w Wall temperature [K]
TOL Tolerance of the iteration process [−]
w Axial fluid velocity [m/s]
W Dimensionless fluid velocity [−]

Greek letters

Γ_i Dimensionless internal boundary of the duct in the repeated element [−]
$\widetilde{\Gamma}_i$ Internal boundary of the duct [m]
Γ_o Dimensionless outer boundary of the duct in the repeated element [−]
$\widetilde{\Gamma}_o$ Outer boundary of the duct [m]
θ_f Dimensionless temperature of the fluid [−]
θ_w Dimensionless temperature of the wall [−]
μ Dynamic viscosity of the fluid [Pa·s]
ρ Density of the fluid [kg/m^3]
Ω_1 Dimensionless fluid region [−]
$\widetilde{\Omega}_1$ Fluid region [m^2]
Ω_2 Dimensionless wall region [−]
$\widetilde{\Omega}_2$ Wall region [m^2]

Coordinate systems

(x, y, z) Cartesian coordinate system
(X, Y, Z) Dimensionless Cartesian coordinate system
(r_F, θ_F) Local polar coordinate system centered at F
(r_G, θ_G) Local polar coordinate system centered at G

Subscripts

av Average
f Fluid
g General
p Particular
w Wall

Superscripts

$[i]$ Iteration step

1 Introduction

Fluid flow and heat transfer in internally finned tubes is a very import problem from a practical point of view. In the literature one can find many different geometries of such tubes. Some of these [1–9] are depicted in Fig. 1. In order to analyze this problem the authors applied different numerical methods, e.g. the finite difference method [10], the finite element method [11] or the finite volume method [12]. Example of such a tube is also an internally finned square duct [13], which is considered in the paper, see Fig. 2.

In numerical analysis of boundary value problems meshless methods are more and more popular [14, 15]. The method of fundamental solutions (MFS) is one of such a method. It was proposed in 1964 by Kupradze and Aleksidze [16]. The method can be applied for these boundary value problems for which the fundamental solution of partial differential operator in the governing equation is known. In the MFS two types of points are used: the source points and the collocation points. The source points are located outside the considered domain on so called the pseudo-boundary (or the fictitious boundary) and the collocation points are put on the boundary of the considered region to satisfy the boundary conditions. Fundamental solution is a function of distance between a point inside the domain and the source point. The approximate solution in the MFS is a linear combination of fundamental solutions. The main problem in the MFS is how to distribute the source points. Distribution of the source points has been studied by some researchers in the literature [17–20]. Because of that, the method requires some experience in this matter. Nonetheless the method is quite easy to implement. Probably the first numerical implementation of the method was given by Mathon and Johnston [21]. Some authors considered also stability of the method [22, 23]. The MFS has been successfully applied for solving different boundary value problems, e.g., direct [24, 25] or inverse problems [26]. To the best knowledge of the author, the MFS has been applied for solving fluid flow and heat transfer only in one paper [27].

In case of nonlinear problem, the MFS cannot be applied directly to solve it. In order to transform the nonlinear problem into a sequence of inhomogeneous equation, the Picard iteration method can be employed [28]. At the beginning in the nonlinear equation, the linear and nonlinear terms are distinguished. After that, at each iteration step to transform the problem in the nonlinear term, the solution from the previous iteration step is used and the problem to solve is inhomogeneous. Then, the method of particular solution can be applied. The solution of inhomogeneous equation consists of the general and particular solutions at each iteration step. The particular solution very often is obtained using the dual reciprocity method (DRM) [29]. The inhomogeneous term is interpolated, e.g., using the radial basis functions (RBFs), and simultaneously the particular solution is obtained if the particular solution of the RBF for the given linear operator is known. The DRM in combination with the MFS was compared with another method for obtaining the particular solution (based on the Newtonian potential [30]) by Golberg [31]. He showed that it gives more accurate results in case of the Poisson's equation. The numerical procedure with the DRM and the MFS is very common in the literature. It has been successfully applied in many engineering and scientific problems, e.g., flow in a wavy channel [32], non-Newtonian fluids flows [33, 34] or elastoplastic torsion of prismatic bars [35]. In the paper, the global radial basis function collocation method (GRBFCM) is applied to obtain the particular solution, which is not so typical in solving the inhomogeneous and nonlinear problems using the MFS. The GRBFCM is called also the Kansa method after the name of the author of the paper [36], who proposed the method for solving some boundary value problems.

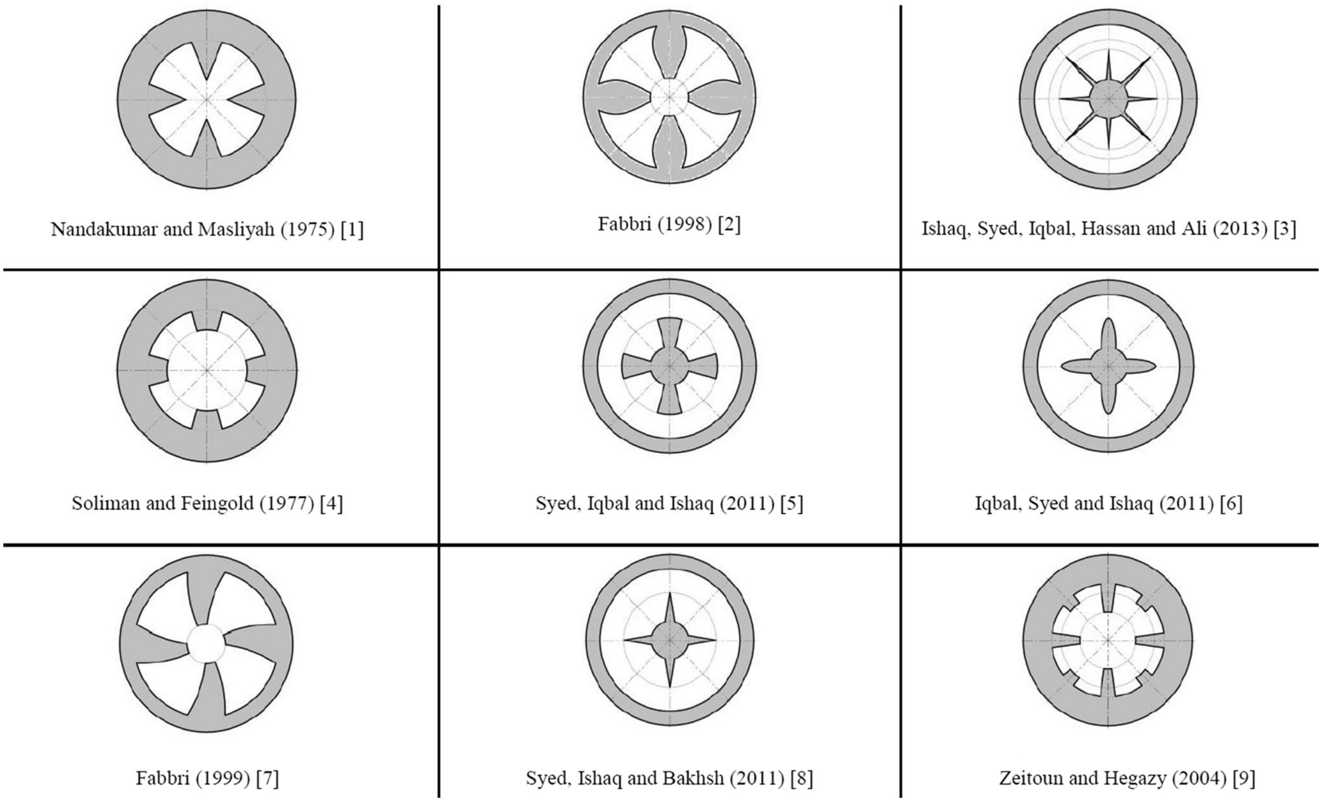

Fig. 1 Different geometries of internally finned tubes

For obtaining the general solution, the standard MFS can be applied in solving inhomogeneous problems. However, in this paper the problem to solve includes sharp corners and it generates additional singularity in solution. In the literature, one can find some modifications of the MFS for problems with boundary singularities. Antunes and Valychev applied the MFS for analysis of acoustic wave propagation problems in two-dimensional domains with corners and cracks [37]. Two-dimensional singular direct [38] and inverse [39] Helmholtz problems were analyzed using the MFS by Marin. Karageorghis proposed the modified MFS (MMFS) for solving harmonic and biharmonic problems with boundary singularities [40]. This modification of the MFS is employed in the paper for solving fluid flow and heat transfer in an internally finned square duct, which is a practical example of problems with sharp corners.

In the paper. steady, fully-developed fluid flow and heat transfer in an internally finned square duct is considered using the MMFS and the GRBFCM. Fluid flow problem is described by a linear governing equation with linear boundary conditions. It can be solved using the MMFS. After that, the average fluid velocity and product of friction factor and Reynolds number can be determined. Heat transfer problem is governed by a nonlinear equation with linear boundary conditions. The nonlinear equation is transformed into a sequence of inhomogeneous equations using the Picard iteration method. Then in the nonlinear term at each iteration step, the solution from the previous iteration step is employed and solution of the inhomogeneous problem consists of the general and particular solutions. In order to obtain the particular solution the GRBFCM is employed and the inhomogeneous term is satisfied at a finite number of internal points. After that, the general solution is obtained using the MMFS and by satisfying the boundary conditions. In the end of each iteration step, the average fluid temperature is calculated. After stopping the iteration process, the Nusselt number can be determined. The paper presents quite new application of a meshless procedure for solving fluid flow and heat transfer in a internally finned square duct. In comparison to the previous paper on application of the MFS and RBFs for solving fluid flow and heat transfer in internally corrugated or finned ducts [27, 41–43], in this paper, the MMFS is applied in order to obtain stable solution because of boundary singularities. Furthermore, instead of the commonly used DRM, the GRBFCM is employed in the paper for obtaining the particular solution in the Picard iteration process, what is also not so typical for application of the MFS for nonlinear and inhomogeneous problems.

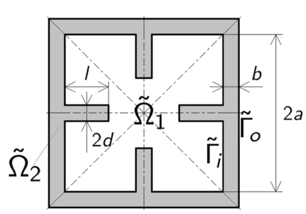

Fig. 2 Geometry of the considered problem

The paper is organized as follows. Statement of the considered problem is presented in Section 2. Mathematical description of the geometry, the governing equations and boundary conditions can be found in this part of the paper. Section 3 presents a meshless method applied in the paper. At the beginning of this section, the numerical algorithm is shown and then subsequent steps of the algorithm are described. The results of the conducted numerical experiments are shown in Section 4. Final conclusions are drawn in Section 5.

2 Mathematical formulation of the problem

Mathematical formulation (mathematical description of the geometry, governing equations and boundary conditions for fluid flow and heat transfer) of the considered problem is given in this section.

2.1 The considered geometry of the problem

Figure 2 shows an example of internally finned square duct cross-section. The characteristic dimensions, i.e., the internal width of the channel $2a$, the length of fins l, the thickness of fins $2d$ and the thickness of the wall b, are also depicted in the figure. Furthermore, one can notice that the internal region of the tube, where the fluid flows is denoted here by $\widetilde{\Omega}_1$ and the wall region by $\widetilde{\Omega}_2$. In a similar way, $\widetilde{\Gamma}_i$ is the internal boundary of the duct and $\widetilde{\Gamma}_o$ is the outer boundary.

Let us consider a repeated element of the considered region because of symmetry of the considered problem. The following dimensionless quantities can be introduced

$$\widehat{L}=\frac{l}{a},\widehat{D}=\frac{d}{a},\widehat{B}=\frac{b}{a}. \tag{1}$$

The repeated element with characteristic dimensionless quantities is depicted in Fig. 3. In this figure, Ω_1 and Ω_2 denote dimensionless fluid flow and wall regions, respectively.

In such a dimensionless repeated element, the area is given by

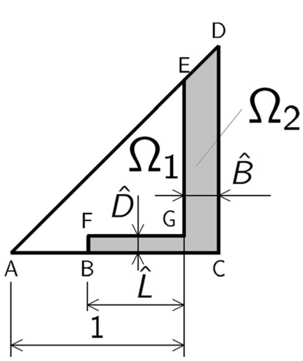

Fig. 3 The repeated element of the internally finned square duct with the characteristic dimensionless quantities

$$\widehat{S}=\frac{1}{2}-\widehat{D}\ \widehat{L} \tag{2}$$

and the wetted perimeter takes the form

$$\widehat{P}=1+\widehat{L}. \tag{3}$$

2.2 The momentum equation

Let us consider a steady, fully-developed flow of an incompressible Newtonian fluid in the axial direction. The problem can be mathematically described in the Cartesian coordinate system (x,y,z) by the following governing equation

$$\frac{\partial^2 w(x,y)}{\partial x^2}+\frac{\partial^2 w(x,y)}{\partial y^2}=\frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}z}\quad \text{in}\quad \widetilde{\Omega}_1, \tag{4}$$

where $w(x,y)$ is the axial velocity, $\mathrm{d}p/\mathrm{d}z$ is a constant pressure gradient in the z axis direction and μ is the dynamic viscosity. The non-slip boundary condition is formulated in this study and it is expressed by

$$w(x,y)=0\quad \text{on}\quad \widetilde{\Gamma}_i. \tag{5}$$

Introducing the dimensionless fluid velocity

$$W(X,Y)=\frac{w(x,y)}{-\dfrac{a^2}{\mu}\dfrac{\mathrm{d}p}{\mathrm{d}z}}, \tag{6}$$

Eq. (4) takes the following dimensional form

$$\frac{\partial^2 W(X,Y)}{\partial X^2}+\frac{\partial^2 W(X,Y)}{\partial Y^2}=-1\quad \text{in}\quad \Omega_1. \tag{7}$$

subject to the boundary conditions

$$W(X,Y)=0\quad \text{on}\quad \overline{\mathrm{BF}}\cup\overline{\mathrm{FG}}\cup\overline{\mathrm{GE}}\quad (\textit{non-slip boundary condition}), \tag{8}$$

$$\frac{\partial W(X,Y)}{\partial n}=0\quad \text{on}\quad \overline{\mathrm{AB}}\cup\overline{\mathrm{EA}}\quad (\textit{symmetry condition}), \tag{9}$$

where n is the normal direction.

The product of friction factor and Reynolds [10, 42] number takes the form

$$f\mathrm{Re}=\frac{8\widehat{S}^2}{W_{\mathrm{av}}\widehat{P}^2}, \tag{10}$$

where the dimensionless average velocity W_{av} is defined as follows

$$W_{\mathrm{av}} = \frac{\int_{\Omega_1} W(X,Y)\mathrm{d}\Omega_1}{\int_{\Omega_1} \mathrm{d}\Omega_1} = \frac{\int_{\Omega_1} W(X,Y)\mathrm{d}\Omega_1}{\widetilde{S}} \tag{11}$$

2.3 The energy equation

Let us consider heat transfer in such a tube. We put the following assumptions:

1) heat transfer in the fluid and wall regions is steady,
2) temperature profile is fully-developed,
3) heat transfer in the axial direction can be neglected,
4) temperature at the outer boundary of the tube is constant and known,
5) heat flux through the outer boundary of the tube on an unit length of the tube is constant.

Taking into account these assumptions, the governing equation in the fluid region takes the following form in the Cartesian coordinate system (x, y, z)

$$\frac{\partial^2 T_f(x,y)}{\partial x^2} + \frac{\partial^2 T_f(x,y)}{\partial y^2} = \frac{\rho c_p}{k_f} w(x,y) \frac{\partial T_f(x,y)}{\partial z} \quad \text{in} \quad \widetilde{\Omega}_1, \tag{12}$$

where $T_f(x, y)$ is the fluid temperature, k_f is the thermal conductivity of the fluid and c_p is the specific heat at a constant pressure.

Let us introduce the dimensionless fluid temperature

$$\theta_f(X,Y) = \frac{\left(T_f(x,y) - T_o\right)k_f}{q_{\mathrm{av}}a}. \tag{13}$$

Finally, the dimensionless governing equation for heat transfer problem in the fluid region Ω_1 after some mathematical operations [10, 42] takes the form

$$\frac{\partial^2 \theta_f(X,Y)}{\partial X^2} + \frac{\partial^2 \theta_f(X,Y)}{\partial Y^2} = f\mathrm{Re}W(R,\theta)\frac{\theta_f(X,Y)}{\theta_{f\mathrm{av}}} \quad \text{in} \quad \Omega_1, \tag{14}$$

where the dimensionless average fluid temperature $\theta_{f\mathrm{av}}$ is given by

$$\theta_{f\mathrm{av}} = \frac{\int_{\Omega_1} \theta_f(X,Y) W(X,Y)\mathrm{d}\Omega_1}{\int_{\Omega_1} W(X,Y)\mathrm{d}\Omega_1}. \tag{15}$$

The Nusselt number is formulated as

$$\mathrm{Nu} = -\frac{2}{\theta_{f\mathrm{av}}}. \tag{16}$$

The dimensionless temperature of the wall is expressed by

$$\theta_w(X,Y) = \frac{(T_w(x,y) - T_o)k_w}{q_{\mathrm{av}}a}, \tag{17}$$

$T_w(x, y)$ is the wall temperature and k_w is the thermal conductivity of the wall.

Using the above dimensionless temperature $\theta_w(X, Y)$, the energy equation in the wall region Ω_2 for steady, fully-developed heat transfer is given by

$$\frac{\partial^2 \theta_w(X,Y)}{\partial X^2} + \frac{\partial^2 \theta_w(X,Y)}{\partial Y^2} = 0 \quad \text{in} \quad \Omega_2. \tag{18}$$

Heat transfer problem is governed by the dimensionless Eqs. (14) in the fluid region Ω_1 and (18) in the wall region Ω_2. The boundary conditions are expressed as follows

$$\frac{\partial \theta_f(X,Y)}{\partial n} = 0 \quad \text{on} \quad \overline{\mathrm{AB}} \cup \overline{\mathrm{EA}} \quad (\mathit{symmetry\ condition}) \tag{19}$$

$$\theta_f(X,Y) = \theta_w(X,Y) \quad \text{on} \quad \overline{\mathrm{BF}} \cup \overline{\mathrm{FG}} \cup \overline{\mathrm{GE}} \quad (\mathit{continuity\ of\ the\ temperature}), \tag{20}$$

$$\frac{\partial \theta_f(X,Y)}{\partial n} = K\frac{\partial \theta_w(X,Y)}{\partial n} \quad \text{on} \quad \overline{\mathrm{BF}} \cup \overline{\mathrm{FG}} \cup \overline{\mathrm{GE}} \quad (\mathit{continuity\ of\ the\ heat\ flux}), \tag{21}$$

$$\frac{\partial \theta_w(X,Y)}{\partial n} = 0 \quad \text{on} \quad \overline{\mathrm{BC}} \cup \overline{\mathrm{ED}} \quad (\mathit{symmetry\ condition}), \tag{22}$$

$$\theta_w(X,Y) = 0 \quad \text{on} \quad \overline{\mathrm{CD}} \quad (\mathit{given\ value\ of\ the\ temperature}), \tag{23}$$

where $K = k_w/k_f$ is the dimensionless thermal conductivity of the wall.

3 Numerical procedure

Table 1 shows a general concept of the numerical procedure proposed in the paper.

3.1 Solution of Newtonian fluid flow problem using the modified method of fundamental solutions

The boundary value problem to solve is described by the governing Eq. (7) subject to the boundary conditions

Table 1 Numerical algorithm of the proposed method of solution

Step 1	Input parameters of the considered problem.
Step 2	Input parameters of the proposed method of solution.
Step 3	Solve fluid flow problem using the MMFS. Determine the unknown coefficients by satisfying the boundary conditions at a finite number of the collocation points (the boundary collocation technique).
Step 4	Calculate the average fluid velocity W_{av} (e.g. using the Gaussian quadrature) and product of friction factor and Reynolds number fRe.
Step 5	For the first approximation take $i = 1$.
Step 6	Determine the particular solution of the fluid temperature using the GRBFCM. Unknown coefficients can be determined by satisfying the inhomogeneous equation at a finite number of the internal points.
Step 7	Determine the general solutions of fluid and wall temperatures employing the MMFS. Unknown coefficients are calculated by satisfying the boundary conditions at a finite number of the collocation points (the boundary collocation technique).
Step 8	Calculate the dimensionless average temperature $\theta_{f\,\mathrm{av}}^{[i]}$ (using e.g. Gaussian quadrature).
Step 9	Check the condition for stopping the iteration process If $\left\lVert \theta_f^{[i]}(X,Y) - \theta_f^{[i-1]}(X,Y) \right\rVert < \mathrm{TOL}$ else TAKE $i = i + 1$ and GO TO Step 6, where TOL is tolerance of the iteration process.
Step 10	Calculate the Nusselt number Nu.

(8)–(9). The solution of this problem consists of two parts: the general solution and the particular solution

$$W(X,Y) = W_g(X,Y) + W_p(X,Y), \tag{24}$$

where $W_g(X, Y)$ and $W_p(X, Y)$ denote the general and particular solutions, respectively. The particular solution is given by

$$W_p(X,Y) = -\frac{X^2 + Y^2}{4}. \tag{25}$$

The general solution can be easily solved using the MMFS in which the approximate solution consists of a linear combination of fundamental solutions and harmonic functions. It can be written in the form

$$W_g(X,Y) = \sum_{j=1}^{NS_f} c_j \ln\left(r_{Sj}^2\right) + \sum_{m=1}^{M_1} \alpha_m r_{\mathrm{F}}^{\lambda_m} f_m(\theta_{\mathrm{F}}), \tag{26}$$

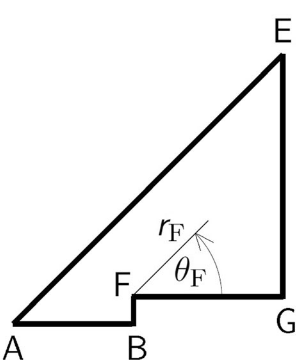

Fig. 4 Local polar coordinates centered at F

where NS_f is the number of the source points, M_1 is the number of the harmonic function in the solution, c_j ($j = 1, 2, \ldots, NS_f$) and α_m ($m = 1, 2, \ldots, M_1$) are unknown coefficients, (r_F, θ_F) are local polar coordinates centered at F (see Fig. 4) and r_{Sj} denotes distance between the point (X, Y) and the j-th source point (X_{Sj}, Y_{Sj})

$$r_{Sj} = \sqrt{\left(X - X_{Sj}\right)^2 + \left(Y - Y_{Sj}\right)^2}. \tag{27}$$

The additional term in the general solution (in comparison with the classical version of the MFS) approximates solution of the problem in the neighborhood of boundary singularity (for this case at the point F). For the Laplace equation and this form of singularity, the harmonic functions are expressed by

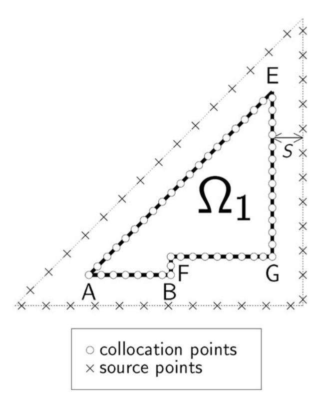

Fig. 5 Distribution of the source and collocation points for fluid flow problem

$$r_{\mathrm{F}}^{\lambda_m} f_m(\theta_{\mathrm{F}}) = r_F^{\frac{2}{3}m} \sin\left(\frac{2\theta_F m}{3}\right). \quad (28)$$

In order to obtain the unknown coefficients c_j ($j = 1, 2, \ldots, NS_f$) and α_m ($m = 1, 2, \ldots, M_1$) – **Step 3** of the numerical algorithm – the set of equations resulting from satisfying the boundary conditions (8)–(9) has to be solved. Distribution of the source and collocation points in the MMFS for the considered problem is depicted in Fig. 5. The source points are situated on a pseudo-boundary outside the considered region.

3.2 Solution of heat transfer problem using the modified method of fundamental solutions

Heat transfer problem is described by a nonlinear governing Eq. (14) in the fluid region Ω_1 with a linear governing Eq. (18) in the wall region Ω_2 subject to the linear boundary conditions (19)–(23). In this study, the nonlinear problem is transformed into a sequence of inhomogeneous problems using the Picard iteration method. In such a case, at the i-th iteration step, the inhomogeneous equation takes the form

$$\frac{\partial^2 \theta_f^{[i]}(X,Y)}{\partial X^2} + \frac{\partial^2 \theta_f^{[i]}(X,Y)}{\partial Y^2} = f\mathrm{Re}W(X,Y)\frac{\theta_f^{[i-1]}(X,Y)}{\theta_{f\mathrm{av}}^{[i-1]}} \quad \text{in} \quad \Omega_1. \quad (29)$$

In the first approximation (**Step 5** of the numerical algorithm), it is assumed that

$$\frac{\theta_f^{[0]}(X,Y)}{\theta_{f\mathrm{av}}^{[0]}} = 1. \quad (30)$$

It leads to the following governing equation for obtaining the first solution

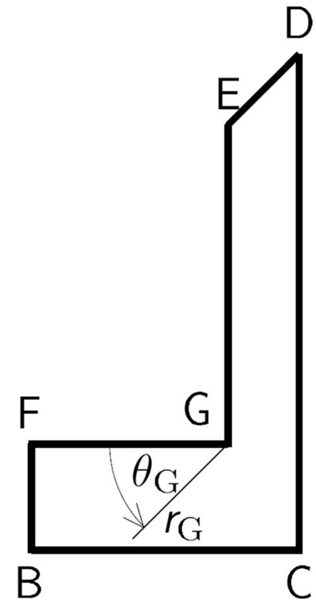

Fig. 7 Local polar coordinates centered at G

$$\frac{\partial^2 \theta_f^{[1]}(X,Y)}{\partial X^2} + \frac{\partial^2 \theta_f^{[1]}(X,Y)}{\partial Y^2} = f\mathrm{Re}W(X,Y) \quad \text{in} \quad \Omega_1. \quad (31)$$

At each iteration step, the approximate solution consists of the general and particular solutions

$$\theta_f^{[i]}(X,Y) = \theta_{fg}^{[i]}(X,Y) + \theta_{fp}^{[i]}(X,Y), \quad (32)$$

where $\theta_{fg}^{[i]}(X,Y)$ and $\theta_{fp}^{[i]}(X,Y)$ denote the general and particular solutions, respectively.

The GRBFCM is employed to obtain the particular solution which can be approximated by

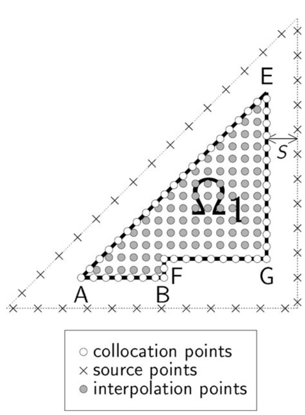

Fig. 6 Distribution of the internal, collocation and source points in the MMFS and GRBFCM for solving heat transfer problem in internally finned square duct (fluid region)

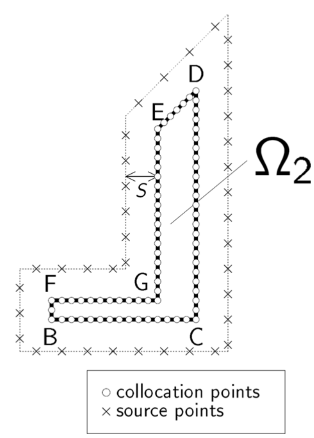

Fig. 8 Distribution of the collocation and source points in the MMFS and GRBFCM for solving heat transfer problem in internally finned square duct (wall region)

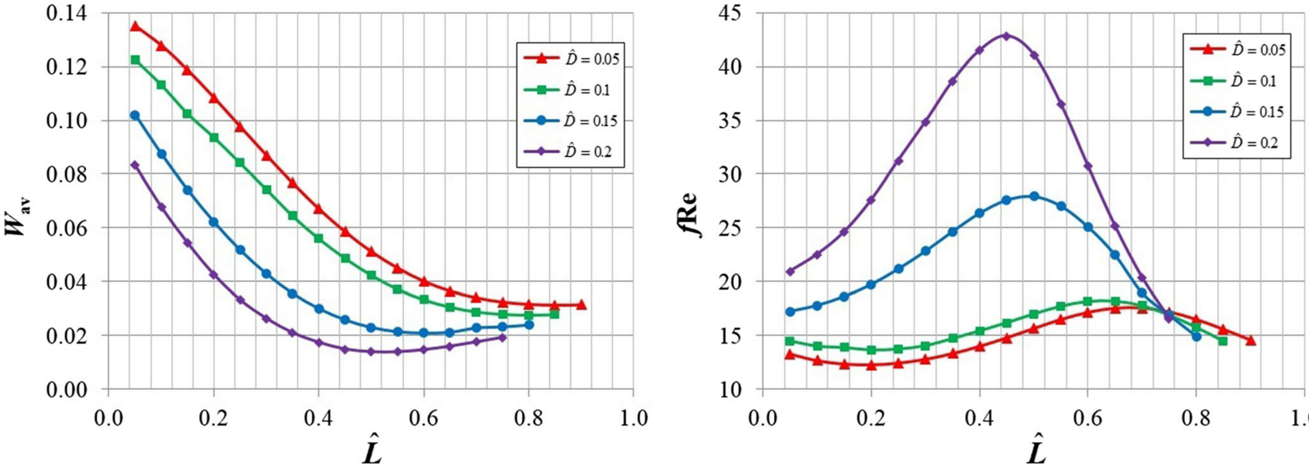

Fig. 9 Influence of the length and width of fins on the dimensionless average fluid velocity and product of friction factor and Reynolds number

$$\theta_{fp}^{[i]}(X,Y)=\sum_{j=1}^{N_{\text{int}}}e_j\phi(r_j),\tag{33}$$

where N_{int} denotes the number of internal points inside the fluid region Ω_1 (see Fig. 6), e_j ($j = 1, 2, .., N_{\text{int}}$) are unknown coefficients, $\phi(r)$ is the form of the RBF, ($X_{\text{int}j}$, $Y_{\text{int}j}$) are coordinates of the j-th internal point and r_j is a distance between the point (X, Y) and the j-th internal point ($X_{\text{int}j}$, $Y_{\text{int}j}$)

$$r_j=\sqrt{\left(X-X_{\text{int}j}\right)^2+\left(Y-Y_{\text{int}j}\right)^2}.\tag{34}$$

The multiquadric function (MQ) is used in the paper as the form of the RBF in order to obtain the particular solution. The MQ is given by

$$\phi(r)=\sqrt{r^2+\widehat{c}^2},\tag{35}$$

where $\widehat{c}$ is the shape parameter.

In order to obtain (**Step 6** of the numerical algorithm), the unknown coefficients e_j ($j = 1, 2, .., N_{\text{int}}$) the inhomogeneous term

$$\frac{\partial^2\theta_{fp}^{[i]}(X,Y)}{\partial X^2}+\frac{\partial^2\theta_{fp}^{[i]}(X,Y)}{\partial Y^2}=f\text{Re}W(X,Y)\frac{\theta_f^{[i-1]}(X,Y)}{\theta_{f\text{av}}^{[i-1]}}\tag{36}$$

should be satisfied at a finite number of the internal points. Distribution of the internal points is depicted in Fig. 6.

The MMFS is also applied for obtaining the general solution of the fluid temperature and solution for the wall temperature. Thus the approximate general solution of the fluid temperature is given by

$$\theta_{fg}(X,Y)=\sum_{j=1}^{NS_{tf}}g_j\ln\left(r_{Sj}^2\right)+\sum_{m=1}^{M_2}\beta_m r_{\text{F}}^{\lambda_m}f_m(\theta_{\text{F}}),\tag{37}$$

where NS_{tf} is the number of the source points for heat transfer problem outside the fluid region (see Fig. 5), M_2 is the number of the harmonic function in the solution, g_j ($j = 1, 2, …, NS_{tf}$)

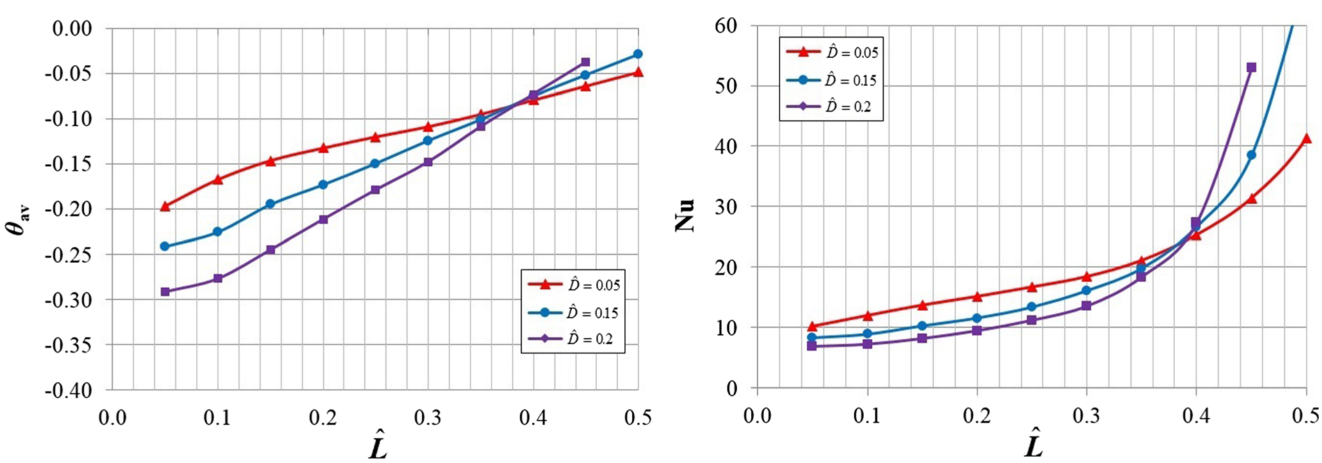

Fig. 10 Influence of the length and width of fins on the dimensionless average fluid temperature and Nusselt number

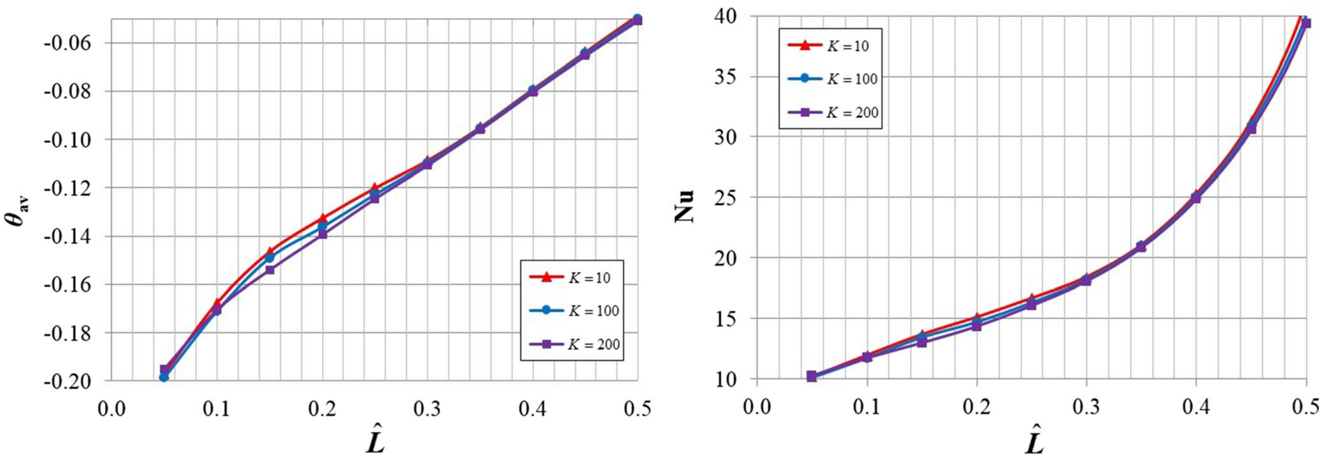

Fig. 11 Influence of the dimensionless thermal conductivity of the wall on the dimensionless average fluid temperature and Nusselt number

and β_m ($m = 1, 2, \ldots, M_2$) are unknown coefficients, (r_F, θ_F) are local polar coordinates centered at F (see Fig. 3) and r_{Sj} denotes distance between the point (X, Y) and the j-th source point (X_{Sj}, Y_{Sj}) defined in Eq. (27).

The wall temperature is also approximated using the MMFS. It takes the form

$$\theta_w(X,Y) = \sum_{j=1}^{NS_{tw}} h_j \ln\left(r_{Sj}^2\right) + \sum_{m=1}^{M_3} \gamma_m r_{\mathrm{G}}^{\lambda_m} f_m(\theta_{\mathrm{G}}), \tag{38}$$

where NS_{tw} is the number of the source points for heat transfer problem outside the wall region (see Fig. 7), M_3 is the number of the harmonic function in the solution, h_j ($j = 1, 2, \ldots, NS_{tw}$) and γ_m ($m = 1, 2, \ldots, M_3$) are unknown coefficients, (r_G, θ_G) are local polar coordinates centered at G (see Fig. 7) and r_{Sj} denotes distance between the point (X, Y) and the j-th source point (X_{Sj}, Y_{Sj}) defined in Eq. (27). The harmonic functions in the local polar coordinates (r_G, θ_G) in this solution are given by

$$r_{\mathrm{G}}^{\lambda_m} f_m(\theta_{\mathrm{G}}) = r_G^{\frac{2}{3}m} \sin\left(\frac{2\theta_G m}{3}\right). \tag{39}$$

In order to determine the unknown coefficients of the general solution of the fluid temperature g_j ($j = 1, 2, \ldots, NS_{tf}$) and β_m ($m = 1, 2, \ldots, M_2$) and unknown coefficients of the wall temperature h_j ($j = 1, 2, \ldots, NS_{tw}$) and γ_m ($m = 1, 2, \ldots, M_3$) – **Step 7** in the numerical algorithm – the boundary collocation method has to be applied. The boundary conditions (19)–(23) should be satisfied at a finite number of the collocation points. Distribution of the collocation and source points for the fluid region Ω_1 is depicted in Fig. 6 and for the wall region Ω_2 in Fig. 8.

4 Results

The results obtained using the presented meshless procedure are shown in this section. Influence of the geometry of the internally finned square duct on fluid flow and heat transfer in such a tube is here investigated.

Figure 9 presents influence of the length and width of fins on the dimensionless average fluid velocity and product of friction factor and Reynolds number. One can notice that the average fluid velocity decreases with increasing length of the fins. Only for $\hat{D} = 0.15$ and $\hat{D} = 0.2$ the average velocity slightly increases for higher values of the length of fins. The average fluid velocity is greater if the width of the fins is smaller for a given length of fins. The smallest product of friction factor and Reynolds number has been achieved for the thinnest fin ($\hat{D} = 0.05$).

Influence of the length and width of fins on the dimensionless average fluid temperature and Nusselt number is presented in Fig. 10. One can observe that the average fluid velocity and Nusselt number increase with increasing length of fin. For shorter fins ($\hat{L} < 0.4$) the average fluid velocity and Nusselt number are greater if the fin is thinner. For longer fins the dependency is reversed – the average fluid velocity and Nusselt number are greater for thicker fins.

Influence of the dimensionless thermal conductivity of the wall is not so significant. It is depicted in Fig. 11. The differences between the average fluid temperature and Nusselt number for different values of the thermal conductivity are very small. However the average fluid temperature and Nusselt number are greater for thinner fins.

5 Conclusions

In the paper, a meshless procedure has been proposed for analysis of fluid flow and heat transfer in an internally finned square duct. The procedure is based on the modified method of fundamental solutions and global radial basis function collocation method. The proposed numerical algorithm has been successfully applied for a practical problem with sharp corners, which introduce boundary singularities and makes the problem numerically more difficult to solve. The procedure gives stable solution and it can be easily extended in the future for another geometries of internally finned tubes.

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Compliance with ethical standards

Conflict of interest The author declare that he has no conflict of interest.

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