




Estimation of thermal transmittance based on temperature measurements with the application of perturbation numbers

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Abstract Fast estimation of thermal transmittance based on temperature measurements is uncertain, and the obtained results can be burdened with a large error. Nevertheless, such attempts should be undertaken merely due to the fact that a precise measurement by means of heat flux measurements is not always possible in field conditions (resentment of the residents during the measurements carried out inside their living quarters), and the calculation methods do not allow for the nonlinearity of thermal insulation, heat bridges or other fragments of building envelope of diversified thermal conductivity. The present paper offers the estimation of thermal transmittance and internal surface resistance with the use of temperature measurements (in particular with the use of thermovision). The proposed method has been verified through tests carried out on a laboratory test stand built in the open space, subjected to the influence of real meteorological conditions. The present elaboration involves the estimation of thermal transmittance by means of temperature measurements. Basing on the mentioned estimation, the authors present correction coefficients which have impact on the estimation accuracy. Furthermore, in the final part of the paper, various types of disturbance were allowed for using perturbation numbers, and the introduced by the authors “credibility area of thermal transmittance estimation” was determined.

1 Introduction

The estimated determination of thermal transmittance based on the measurements is burdened with substantial uncertainty when defining heat loss from a building envelope and consequently the energy efficiency in buildings. On the other hand, the calculation methods which consist in mathematical modeling resort to certain assumptions or simplifications, which also leads to some sort of uncertainty when estimating heat loss. The calculations do not allow for thermal diversity of walls, or, in other words, they do not take into consideration the fact that there are places of different thermal conductivity. When the thermal transmittance U is in reality higher than estimated, the conservation of energy is in fact lower than estimated, which in the macro scale (e.g. the whole country) can be significant. Many energy performance certificates can be

incorrectly compiled due to the incorrect estimation of thermal transmittance U . Such a situation was observed by Lowe who described it in his paper [1].

Over the recent years, many research works have been published in which in-situ measurements of thermal transmittance have been attempted, as for example in the work [2]. Another work covering the said issue was presented by Stevens [3], in which the consumption of energy measured in 93 apartments with respect to the measured value of thermal transmittance was presented. In the paper presented by Rye [4], the author declares that traditional buildings are energy-inefficient, in particular with respect to the heat loss through walls. Also Evangelisti [5] declares that an accurate assessment of the thermal transmittance U of walls is indispensable to calculate the annual consumption of energy. In his work Evangelisti analyzed three walls of the building having different thermal properties and he compared the obtained results with the calculation methods.

Basing on the quoted works as well as on other works [6–9], we know that the thermal properties of a building have direct impact on the annual consumption of energy. We also know that the thermal conductivity of a wall depends on the thermal conductivity of each layer of that wall, and therefore, the thermal resistance of the whole wall can be modeled in the way analogous to the resistance of resistors in electrical

Jerzy Skrzypczyk died before publication of this work was completed.

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current flow models. Such a thermal-electrical analogy has been extensively described in literature and applied for the calculation of heat exchange by the envelopes whereof thermal properties are known. Nevertheless, the real (measured) value of thermal transmittance U is frequently different from the calculated value, and it is definitely different when we take into consideration e.g. the nonlinearity of insulation layers or heat bridges. Therefore, in order to estimate the real heat loss through the outer envelope of the building, the measurements are essential [10, 11]. The generally acknowledged non-destructive measurement method of thermal resistance is the direct measurement of heat flux [12, 13]. The measurements of thermal resistance can fail to reflect the resistance of the whole envelope if the measurement points are selected at places having thermal conductivity different from the conductivity of the whole envelope. Therefore, it is worth while complementing this measurement with the thermographic analysis [14]. Measurement methods can be also applied for the analysis of historical buildings. Walker and Pavia [15] have been carrying out in-situ research investigation studies involving thermal insulation of seven envelopes with internal insulation on a historical wall. We must emphasize here that the analyses involving the measured values of thermal transmittance have not been carried out for the last two years. In 2009 Jimenez [16] was investigating the said issue, and Dudek [17] demonstrated in a series of in-situ measurements of masonry walls that the discrepancy between the measurements and the calculation methods reached 25%.

Naveros and Jiménez [18] investigated also the potentials and limitations of the regression method based on the average values applied for the thermal analysis of the performance of real-size building elements tested in the external dynamic atmospheric conditions.

Other researchers, e.g. Cesaratto [19], using FEM, have been simulating the impact of various external interference, or variable input data of temperature on the in-situ measurement results of thermal conductivity. They presented a general assessment of deviations involving the obtained results for various cases.

According to the authors [20], energy audits and the control of energy performance are very important in terms of energy conservation in buildings. Therefore, it is essential to be able to estimate thermal transmittance of building envelopes. Ficco as well as other researches mentioned in this Introduction base their measurements principally on the measurement of heat flux through the wall. Although the latter is investigating the impact of the meteorological conditions of the environment on the measurement results involving the estimation of thermal transmittance U and on the uncertainty of that estimation.

The latest research studies [21] present the design of a more straightforward and compact version of the traditional Hot-Box apparatus (it measures U -value) which instead determines the thermal resistance of the samples of building envelope.

The work [22] et al. presents the description of in-situ monitoring measurements, using the measurements of temperature and heat flux flows as a more precise measurement of the thermal performance of walls. However, the authors indicated some limitations, as for example the fact that with the use of standard methods, the measurements in winter must be carried out over the time period of two weeks. The work Krause [23] also presents thermovisual and numerical studies, but they are focused on the identification of thermal bridges. There are empirical models for the prediction of thermal conductivity estimation. To provide an example, we can mention the model for cement-based porous sensible heat storage materials and naturally occurring crystalline rock formations as a function of temperature [24].

And in the work [25], they applied the measurement data and Autoregressive models with exogenous (ARX) for the identification of physical parameters of the tested walls in real meteorological conditions.

One of the measurement methods of temperature is thermovision. In fact, we can state that the thermovisual method is burdened with the risk of incorrect interpretation of results. As we can read in the work of Fox et al. [26], the thermographic image of the building is reflecting the thermal condition at a given moment of time, and it does not reflect changes in time. They also noticed that the building was hardly ever in the state of thermal equilibrium, which might result in the incorrect interpretation of building defects in effect of the application of the standard thermographic method. For the same reason Taylor [27] is supporting the thermographic research studies with computer simulations, trying to find out how these two techniques can be used together. Numerical modeling and impact analysis of various factors on the final results necessitates that a lot of data is “memorized”. Is it particularly noticeable when creating the models of bigger, more complex layered systems. In such cases a considerable part of the operating memory must be engaged to ensure proper operation of respective programs. When we have to investigate a larger number of uncertain parameters, the task is becoming even more complicated. Hence, new algorithms are being created all the time to ensure better remembering capacity by the program of possibly the greatest amount of data with the minimum employment of memory. The analyses involving the calculation of temperature fields are most commonly carried out with the use of the finite elements method and boundary element method, as well as with the application of finite difference method and elementary balance method [28]. We can more and more frequently witness the application of neural networks, fractals theory or perturbation methods in different branches of engineering. Artificial neural networks have been applied among others to solve the issue of inverse heat conduction, with the assumption of functional dependence of heat

conduction on temperature, and to predict the effective heat conduction in porous materials [29, 30]. Perturbation methods are applied almost in all branches of science, including technology. For example, in the work [31] the material properties of thermoplastic materials were investigated. Perturbation methods are also used to analyze the stability of dynamic systems. Such investigation studies have been carried out for example by Nowoświat [32], and they involved the control and stability of Duffing oscillator. Perturbation methods have been applied to investigate the buckling of springy beams [33]. Very interesting modifications of this method and the explanation of perturbation numbers have been proposed by Skrzypczyk [34]. Perturbation methods are applied among others to obtain an approximate analytical solution involving the problems of one-dimensional thermal transmittance through porous materials. Such materials are characterized by a very low design thermal conductivity, which can play the role of a small parameter [28, 35].

All the methods offered by the researchers presented in this Introduction necessitate long-term measurements and very often interfere with the privacy of people of the involved apartments. Consequently, such measurements in real conditions in many cases might be impossible to implement. Therefore, the authors of the present work are putting forward a proposition enabling fast estimation of thermal transmittance. Such a fast diagnosis can be applied to estimate heat loss through building envelopes in real conditions when the measurement with a heat meter is not feasible. Furthermore, with the use of the proposed fast estimation, we can determine an equivalent thermal transmittance for envelopes having diversified thermal activity.

The application of approximate analytical methods, which encompass perturbation techniques, can be found useful to solve the problems of heat transport. The basic assumptions of the traditional perturbation theory necessitate that the model should be transformed into the dimensionless form in such a way so that the parameters and variables involving the behavior of the system could be determined. As the next step, we select a parameter (referred to as small parameter or perturbation value) which is small as compared to the other ones and labeled as ε . The present work is attempting to apply an alternative perturbation method which consists in the application of n -perturbation numbers defined by Skrzypczyk [34] and used by Winkler-Skalna [36]. The overview of methods instigated the authors to undertake works involving the estimation capability of thermal transmittance by means of temperature measurement. Furthermore, using the algebra of perturbation numbers, we determined in the work a so called “credibility area” which almost certainly contains the measurement result. The application of the algebra of perturbation numbers for the estimation of thermal transmittance U is an innovative approach in such applications.

2 Theoretical model

2.1 Estimation of thermal transmittance

Solving the Fourier’s heat conduction equation

$$q = -\lambda \frac{dT}{dx} \quad (1)$$

and using the definition of thermal resistance as the quotient of heat flow rate to the difference of the internal and external environment temperatures

$$U = \frac{q}{\theta_i - \theta_e} \quad (2)$$

we obtain: $U = \frac{\alpha_i(\theta_i - \theta_{si})}{\theta_i - \theta_e} = \alpha_i \left(\frac{(\theta_i - \theta_e)}{\theta_i - \theta_e} - \frac{(\theta_{si} - \theta_e)}{\theta_i - \theta_e} \right)$ which ultimately yields

$$U = \frac{1}{R_{si}} \left(1 - \frac{\theta_{si} - \theta_e}{\theta_i - \theta_e} \right) \quad (3)$$

where

q	heat flow rate, W/m ²
λ	design thermal conductivity, W/mK
θ_i	internal surrounding temperature, K
θ_e	external surrounding temperature, K
α_i	internal surface heat transfer coefficient, W/m ² K
R_{si}	internal surface resistance, with $R_{si} = \frac{1}{\alpha_i}$, m ² K/W
θ_{si}	internal surface temperature, K

When using the Eq. (3) for the estimation of thermal transmittance on the basis of temperature, we might find it difficult to determine, or estimate the internal surface resistance R_{si} . Thermal transmittance can be also determined with the use of external surface resistance (from the external side of building envelope) and external surface temperature, but it is burdened with a much higher error. The error in question is effected by high changeability of the surrounding temperature on the external side of the envelope, by the changing speed of wind influencing the external heat transfer coefficient, by surface temperature interference from the outside (sunlight, shading, wind, etc.).

The Eqs. (2) and (3) are used with the assumption of stationary heat flow. However, due to the inability to stabilize the external atmospheric conditions, it is not possible to stabilize the temperature of envelope surface. Therefore, the changes were defined as dynamic ones.

Taking into account the above factors, the estimation of thermal transmittance was carried out with the use of Eq. (3).

The determination of thermal transmittance is carried out on the basis of the criteria equation

$$Nu = f(\text{Re}, \text{Pr}, \text{Gr}, K_g) \quad (4)$$

where: Nu is Nusselt number, Re – Reynolds number, Pr – Prandtl number, Gr – Grashof number, K_g – geometric similarity.

In the conditions of free convection, the speed of air circulation in rooms is from several to a dozen or so centimeters per second, and the difference between air temperature and envelope surface is most commonly within the range of 2 ÷ 8 K. In such conditions, the surface heat transfer coefficient through/ by convection on the surface of walls can be determined from the approximate Equation [37]:

$$\alpha_i = 1.66\theta_i^{\frac{1}{4}} \tag{5}$$

The surface heat transfer coefficient can be also approximated with other relations. For example, with the temperature difference between the air and the surface not higher than 5 K the above equation can have the following form:

$$\alpha_i = 3.49 + 0.093(\theta_i - \theta_{si}) \tag{6}$$

For the differences higher than 5 K, but not only, instead of the Eq. (6) we can use

$$\alpha_i = 2.32(\theta_i - \theta_{si})^{0.25} \tag{7}$$

When we want to apply the calculation method of thermal transmittance by means of the Eq. (3), we take into account places in the envelope which have considerably different surface temperatures, as in Fig. 1.

$$\begin{aligned} \text{Let } U_1 &= \frac{1}{R_{1si}} \left(1 - \frac{\theta_{1si} - \theta_e}{\theta_{1i} - \theta_e} \right), & U_2 \\ &= \frac{1}{R_{2si}} \left(1 - \frac{\theta_{2si} - \theta_e}{\theta_{2i} - \theta_e} \right), & \dots, & U_n \\ &= \frac{1}{R_{nsi}} \left(1 - \frac{\theta_{nsi} - \theta_e}{\theta_{ni} - \theta_e} \right) \end{aligned}$$

then the equivalent thermal transmittance is:

$$U_{eqw} = \frac{U_1 + U_2 + \dots + U_n}{n} \tag{8}$$

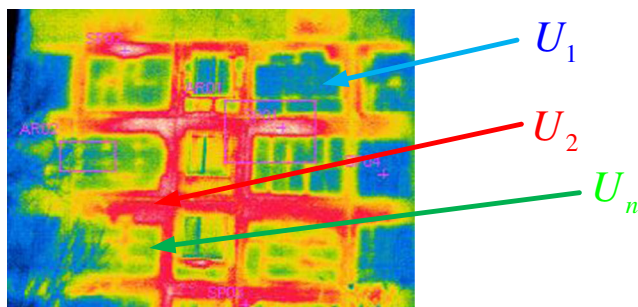


Fig. 1 Temperature distribution on the surface of envelope with the marked places of different surface temperatures

2.2 Perturbation numbers

Basing on the works of Skrzypczyk [34], we can write that the n-perturbation number is defined as an ordered set of real numbers $(x_0, x_1, x_2, \dots, x_n) \in \mathbb{R}^n$. The set of n-perturbation numbers is labeled as $R_{n\epsilon}$. If two numbers $x, y \in R_{n\epsilon}$ are n-perturbation numbers written as $x = (x_0, x_1, x_2, \dots, x_n)$, $y = (y_0, y_1, y_2, \dots, y_n)$, $x_i, y_i \in \mathbb{R}, i = 0, \dots, n$, we can say that $x \equiv y$ if and only if $x_i = y_i$ for each $i = 0, \dots, n$. In the set of n-perturbation numbers $R_{n\epsilon}$ we introduced the operations of adding ($+_{\epsilon}$) and multiplication (\bullet_{ϵ}), as well as neutral elements of adding $0_{n\epsilon} := \underbrace{(0, 0, \dots, 0)}_{n \text{ razy}}$ and multiplication $1_{n\epsilon} := \underbrace{(1, 0, \dots, 0)}_{n-1 \text{ razy}}$ were defined. In the set $R_{n\epsilon}$ defined in such a way, which was labeled as the body of n-perturbation numbers, we retain the commutativity and connectivity of adding, and also the commutativity and connectivity of multiplication, as well as the separation principle. The presented above body $R_{n\epsilon}$ does not contain the body of real numbers R . However, we can demonstrate that real numbers can be treated as certain elements of the body $R_{n\epsilon}$, with the algebraic operations and neutral elements of adding and multiplication being maintained [34].

Since the method has not been described in the world literature for the general audience, we will present here shortly the algebra of perturbation numbers defined by professor Skrzypczyk.

For three n-perturbation numbers $x = (x_0, x_1, x_2, \dots, x_n)$, $y = (y_0, y_1, y_2, \dots, y_n)$, $z = (z_0, z_1, z_2, \dots, z_n)$ and for the real number A we can write:

the sum of two n-perturbation numbers:

$$x +_{\epsilon} y := (x_0 + y_0, x_1 + y_1, x_2 + y_2, \dots, x_n + y_n)$$

the sum of the real number and the n-perturbation number:

$$A +_{\epsilon} y := (A + x_0, x_1, x_2, \dots, x_n)$$

the product of two n-perturbation numbers:

$$x \bullet_{\epsilon} y := (x_0 y_0, x_0 y_1 + x_1 y_0, x_0 y_2 + x_2 y_0, \dots, x_0 y_n + x_n y_0)$$

the inverse of n-perturbation number:

$$\begin{aligned} x^{-1} &= (x_0, x_1, x_2, \dots, x_n)^{-1} \\ &= \left(\frac{1}{x_0}, \frac{x_1}{x_0^2}, \frac{x_2}{x_0^2}, \dots, \frac{x_n}{x_0^2} \right), x_0 \neq 0 \end{aligned}$$

the division of two n-perturbation numbers:

$$x /_{\epsilon} y := \left(\frac{x_0}{y_0}, \frac{x_1}{y_0} - \frac{x_0 y_1}{x_0^2}, \frac{x_2}{y_0} - \frac{x_0 y_2}{x_0^2}, \dots, \frac{x_n}{y_0} - \frac{x_0 y_n}{x_0^2} \right), x_0 \neq 0, y_0 \neq 0$$

Let $D \subset R_{n\epsilon}$ be an arbitrary subset. We can say that we have a definite function $f_{n\epsilon}$, if to each number $z \in D$ is assigned precisely one element from the set $R_{n\epsilon}$. Then, we can say that $f_{n\epsilon}$ is the extension of the function defined on the subset D with the values contained in $R_{n\epsilon}$. The n -perturbation function can be written as follows: $f_{n\epsilon} : D \subset R_{n\epsilon}$ or,

let $f_{n\epsilon}(z) = u(z) + \epsilon_1 v(z) + \epsilon_2 g(z) + \dots + \epsilon_n h(z)$, with $u(\cdot)$, $v(\cdot)$, $g(\cdot)$, \dots , $h(\cdot)$ standing for the real functions of the n -perturbation variable $z = z_0 + z_1\epsilon_1 + z_2\epsilon_2 + \dots + z_n\epsilon_n$ or, in other words, $n + 1$ real variables $z_0, z_1, z_2, \dots, z_n$, which can be written as

$$\begin{aligned} f_{n\epsilon}(z) &= u(z) + \epsilon_1 v(z) + \epsilon_2 g(z) + \dots + \epsilon_n h(z) = \\ &= u(z_0, z_1, z_2, \dots, z_n) + \epsilon_1 v(z_0, z_1, z_2, \dots, z_n) \\ &+ \epsilon_2 g(z_0, z_1, z_2, \dots, z_n) + \dots + \epsilon_n h(z_0, z_1, z_2, \dots, z_n). \end{aligned}$$

Hence $u(\cdot)$, $v(\cdot)$, $g(\cdot)$, \dots , $h(\cdot)$ will be labeled as ordinary real functions without the index ϵ . The functions $u(\cdot)$, $v(\cdot)$, $g(\cdot)$, \dots , $h(\cdot)$ will be labeled respectively as main part, first perturbation, second perturbation and n -th perturbation of the function $f_{n\epsilon}(\cdot)$.

Powers of n -perturbation numbers:

$$z^2 = zz = (z_0^2, 2z_0z_1, 2z_0z_2, \dots, 2z_0z_n),$$

having in mind that: $z^2 \equiv (0, 0, 0, \dots, 0)$ if $z_0 = 0$. We obtain therefore:

$$\begin{aligned} z^3 &= zz^2 = (z_0, z_1, z_2, \dots, z_n)(z_0^2, 2z_0z_1, 2z_0z_2, \dots, 2z_0z_n) \\ &= (z_0^3, 3z_0^2z_1, 3z_0^2z_2, \dots, 3z_0^2z_n), \\ z^k &= zz^{k-1} = (z_0^k, kz_0^{k-1}z_1, kz_0^{k-1}z_2, \dots, kz_0^{k-1}z_n) \end{aligned}$$

Root square of perturbation numbers.

Let us note that $z^2 = (z_0^2, 2z_0z_1, 2z_0z_2, \dots, 2z_0z_n) = (a, b, c, \dots, p)$ if and only if when $a = \tilde{z}^2$ (that is when $a \geq 0$), $b = 2\tilde{z}\delta z_1$, $c = 2z_0z_2$ and $p = 2z_0z_n$.

The square root from the n -perturbation number can be therefore written as:

$$\begin{aligned} \sqrt{z} &= z^{1/2} = \sqrt{(a, b, c, \dots, p)} \\ &= \begin{cases} \pm \left(\sqrt{a}, \frac{b}{2\sqrt{a}}, \frac{c}{2\sqrt{a}}, \dots, \frac{p}{2\sqrt{a}} \right) & , \quad a > 0 \\ 0_\epsilon & , \quad a = 0. \end{cases} \end{aligned}$$

And for the determination of k -th algebraic root from the perturbation number $z = (z_0, z_1, z_2, \dots, z_n)$, with the appropriate assumptions involving a, b, c, \dots, p , we can write:

$$\begin{aligned} \sqrt[k]{z} &= \sqrt[k]{(z_0, z_1, z_2, \dots, z_n)} \\ &= \left(\sqrt[k]{a}, \frac{b}{k\sqrt[k]{a^{k-1}}}, \frac{c}{k\sqrt[k]{a^{k-1}}}, \dots, \frac{p}{k\sqrt[k]{a^{k-1}}} \right). \end{aligned}$$

3 Experiment

The test stand was made as a small test building, located at the place having the following geographical coordinates $50^\circ 17' 26.3''N + 18^\circ 40' 34.7''E$.

The external walls were made from light curtain walls filled with mineral wool. The wall on the southern side was divided into three parts. Each part separated by wooden elements was filled with a different type of material making up the envelopes. The discussed test stand is presented in Fig. 2.

The stand was furnished with three test envelopes marked in Fig. 3 as 1,2,3. All the test envelopes had the same width of 25 cm. The envelopes were selected, taking into account the materials most commonly applied in the general construction of multifamily residential buildings.

The test envelopes (described in Table 1) were selected in the way ensuring that they have different thermal conductivity and that they can reflect their common application in building practice.

Thermal performance of the envelopes was tested by means of the equipment presented below:

Main equipment – stationary:

- Multi-channel recorder type MA56902M09TG3,
- Heat flux transducer 250x250mm type FQA019C – 3 items (one item for each of the tested envelopes),
- Temperature sensor type Pt-100 - 20 items (6 items for each of the tested envelope) + spare ones.
- Stationary meteorological station – a meteorological multisensor with the heating type FMA510H.

The measurement of heat flux (Fig. 4) was carried out with the use of a heat meter FQ90119 (Ahlborn) of the surface of 0.25×0.25 m. The sensor was placed on the internal surface of the wall in its geometrical centre in order to minimize the impact of possible thermal interference (e.g. heat bridges, air infiltration, etc.). The surface of the wall was appropriately prepared to ensure optimal adhesion between the tested wall and the heat meter. The temperature was measured in two



Fig. 2 Test stand

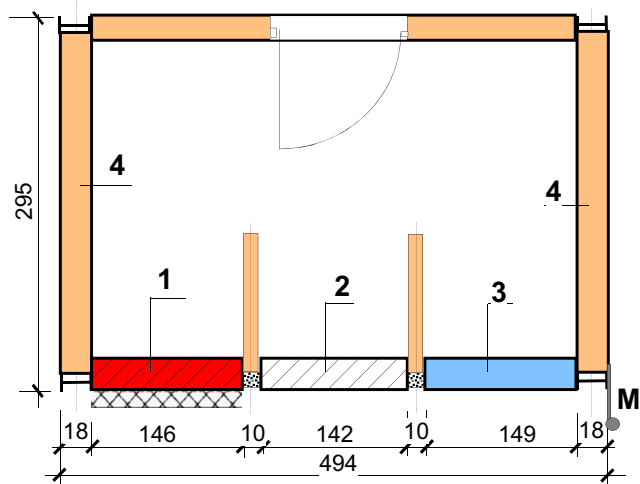


Fig. 3 Arrangement of test envelopes

ways: internal temperature in the tested room as well as on the internal and external surfaces of the envelope, using the sensors PT100. The temperature of the outside air was measured by means of the meteorological station described above.

The applied testing (Fig. 5) methodology consisted in observing the measurement procedure for stationary tests comprised in the Standard [38].

4 Results of laboratory tests

Assuming the internal surface resistance $R_{si} = \frac{1}{\alpha_i}$ and using the Eqs. (5), (6) and (7), we will define the best, out of the proposed ones, estimation method of the said resistance, verified by the measurement (Fig. 6). To define the measured value of surface resistance we will use the transformation of the Eq. (3), which yields

$$R_{si} = \frac{\theta_i - \theta_{si}}{q} \tag{9}$$

Knowing the value of surface resistance, we can estimate thermal transmittance from the Eq. (3). For that

Table 1 Structure and the calculated thermal performance of the test envelopes

Material type	λ^*	d	R	U
/ reference number/	[W/mK]	[m]	[m ² K/W]	[W/(m ² K)]
1 Porous ceramics+ Styrofoam	0.200 0.040	0.25 0.10	5.248	0.185
2 Plain concrete	0.850	0.25	0.294	2.155
3 Silicate	0.610	0.25	0.410	1.725
4 Light curtain walls	–	0.15	2.949	0.320

*properties of materials based on the manufacturer’s data confirmed by ETA

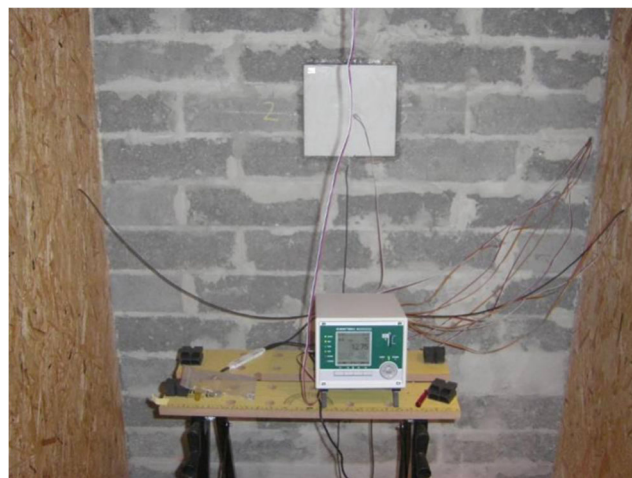


Fig. 4 View on the concrete test wall (2) with the multi-channel recorder

purpose, we apply the Eqs. (5)–(7). The thermal resistance estimated in that way was verified by the applied measurement method.

In order to define the measured value of thermal transmittance U, two approaches were applied. The first one (referred to as Measurement 1) consisted in the measurement of heat flow rate q and the temperature difference of the inside and outside air. The advantage of this method is that we can directly determine the U-value (with the internal and external heat transfer coefficients taken into consideration) from the obtained measurement results. The disadvantage is that the results are sensitive to fast changes of environmental conditions.

Depending on temperature fluctuations, the value of thermal transmittance U will be different, which leads to long-lasting measurements and it hampers the possibility to compare the results between the stands located in different places.

The second approach (referred to as Measurement 2), which employs the measurement procedure described here, requires the measurement of heat flow rate q and the difference in temperatures on the internal and external surfaces of the envelope. Additionally, when determining thermal resistance of the envelope, the values of surface resistance should

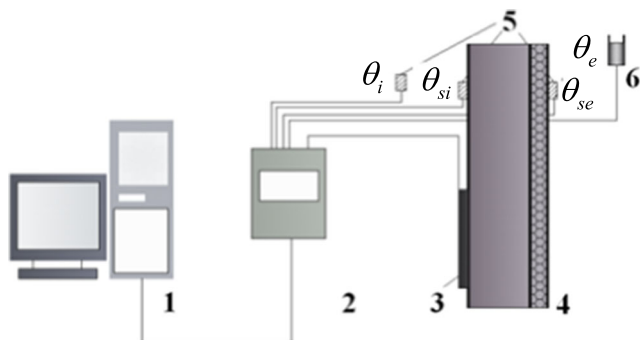
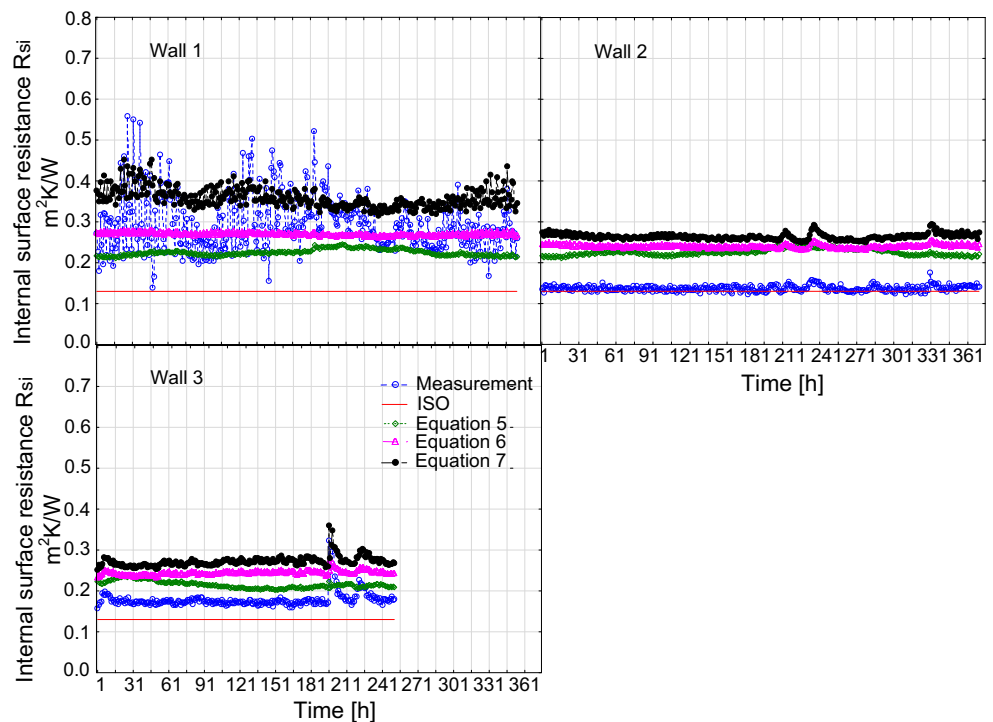


Fig. 5 Measurement stand: 1- computer with software, 2- multi-channel recorder, 3 – heat meter, 4- test envelope, 5- temperature sensors, 6- meteo station

Fig. 6 Surface resistance R_{si} determined in different ways



be added. This method provides higher repeatability of results and allows easier comparison of the results between test stands located in different places. Thermal resistance of the envelope can be determined by dividing the average value of temperature difference by the average value of heat flow rate:

$$R = \frac{\sum_{j=1}^n (\theta_{sij} - \theta_{sej})}{\sum_{j=1}^n q_j} \quad (10)$$

The studies were carried out at the stand as presented in Fig. 5.

Procedure algorithm for the measurement no 1

- Mounting the heat meter on the representative fragment of the wall, Fig. 5, item 3,
- Mounting the sensors marked on Fig. 5 as θ_i (3 pcs.),
- Mounting the meteo station marked on Fig. 5 as θ_e , item 6,
- Mounting the multichannel recorder marked as item 2 on Fig. 5,
- Registering the readings of external and internal air temperature as well as heat meter readings in 1 h intervals,
- Selection of representative measurements; computer analysis of the obtained results.

Procedure algorithm for the measurement no 2

- Mounting the heat meter on the representative fragment of the wall, Fig. 5, item 3,

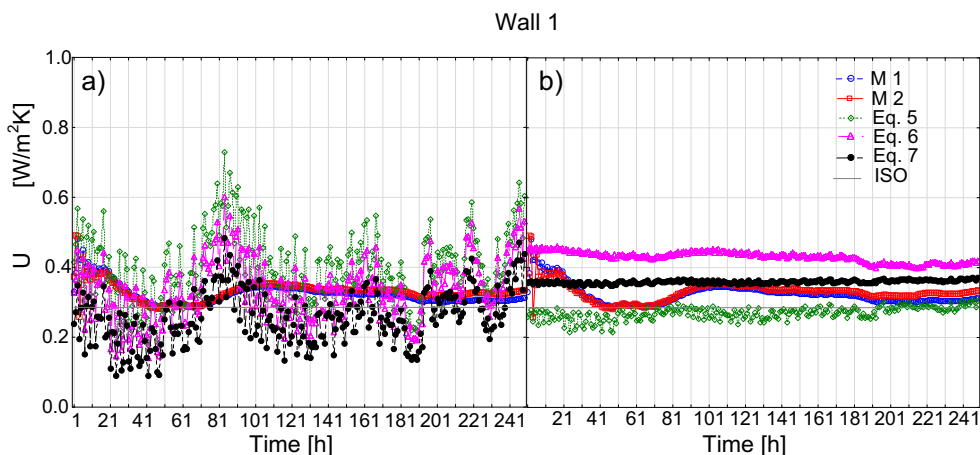
- Mounting the sensors on the internal surface, marked on Fig. 5 as θ_{si} (3 pcs.),
- Mounting the sensors on the external surface marked on Fig. 5 as θ_{se} (3 pcs.)
- Mounting the multichannel recorder marked as item 2 on Fig. 5,
- Registering the readings of surface temperature as well as heat meter readings in 1 h intervals,
- Selection of representative measurements; computer analysis of the obtained results.

The graphs below (Figs. 7, 8 and 9) present the results of the measurements M1 and M2 of thermal transmittance as well as the estimation of thermal transmittance basing on the Eq. (3), using the Eqs. (5)–(7). The graphs (a) presented on the left allow for dynamic changes of temperature in the Eqs. (5)–(7), i.e. the temperatures measured at each measurement time. And the graphs (b) on the right allow for average temperatures in the Eqs. (5)–(7), i.e. the average temperatures from the measurements at each measurement time.

The uncertainty of the measurements 1 and 2, as the measurements continuous in time, was calculated on the basis of the known error propagation methods obtained for the temperature measurements of: external environment, internal environment, external and internal surfaces of the wall as well as for heat flow rate.

Error estimation for the Eqs. 5, 6, 7 was based on the standard error determined for the particular temperatures and on the calculation of the standard uncertainty of composite number. The calculations were carried out in accordance with

Fig. 7 Thermal transmittance determined for the wall 1 with dynamic (a) and average (b) temperature changes of the surrounding and surface



the so called error propagation rule as the geometric total of partial differentials. The results are presented in Tables 2, 3 and 4.

The standard deviation of the averaged internal temperature of the surrounding is $\sigma_{\theta_i} = 1.93^\circ C$ and the surface temperature $\sigma_{\theta_{si}} = 2.12^\circ C$. The average temperature of the surrounding was $\bar{\theta}_i = 19.24^\circ C$ and the average temperature of the envelope surface was $\bar{\theta}_{si} = 17.06^\circ C$.

Using the error propagation rules, we can estimate the measurement error for the relation (3). The results of such an estimation are presented in Table 2.

Similar results are presented for the wall 2.

The standard deviation of the averaged internal temperature of the surrounding is $\sigma_{\theta_i} = 1.96^\circ C$ and the surface temperatures $\sigma_{\theta_{si}} = 2.3^\circ C, \sigma_{\theta_{si}} = 3.1^\circ C$. The average temperature of the surrounding was $\bar{\theta}_i = 19.25^\circ C$ and the average temperature of the envelope surface was $\bar{\theta}_{si} = 12.10^\circ C$.

Using the error propagation rules we can estimate the measurement error for the relation (3). The results of such an estimation are presented in Table 3.

The last measurement and estimation involved the wall 3.

The standard deviation of the averaged internal temperature of the surrounding is $\sigma_{\theta_i} = 2.6^\circ C$ and the surface temperature

$\sigma_{\theta_{si}} = 3.1^\circ C$. The average temperature of the surrounding was $\bar{\theta}_i = 22.15^\circ C$ and the average temperature of the envelope surface was $\bar{\theta}_{si} = 15.73^\circ C$.

Using the error propagation rules we can estimate the measurement error for the relation (3). The results of such an estimation are presented in Table 4.

We can see from the analysis of errors (Tables 2, 3 and 4) and from the presented graphs (Figs. 5, 6 and 7) that with the rise of thermal resistance of the investigated envelope, the Eqs. (5)–(7) yield the results closer to the measurements 1, 2. And together with the deterioration of thermal performance of the envelope, the results show lower agreement.

5 Analysis of results and discussion

As we already know, and which has also been confirmed in the present work, the thermal transmittance U determined with the use of Eq. (3) depends on the surface resistance. Furthermore, the estimation inaccuracy of thermal transmittance by means of the temperature method results from the sensitivity of the relation (3) to the slightest changes of this resistance. Therefore, the estimation of U on the basis of the

Fig. 8 Thermal transmittance determined for the wall 2 with dynamic (a) and average (b) temperature changes of the surrounding and surface

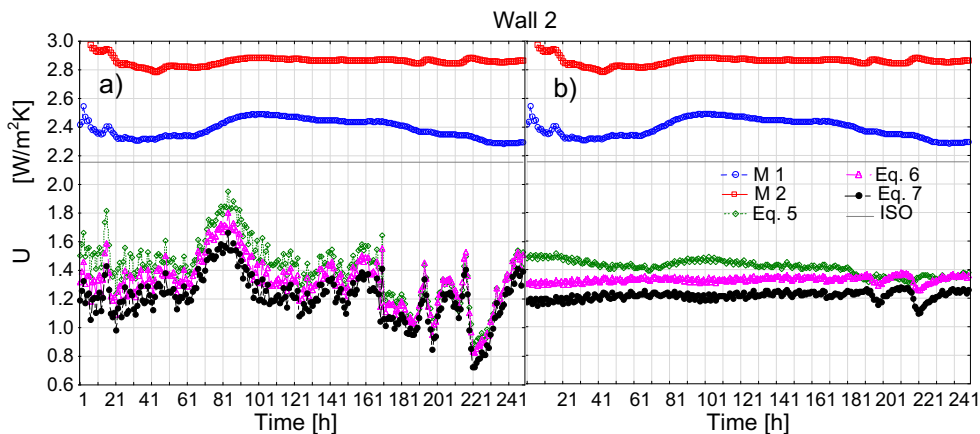
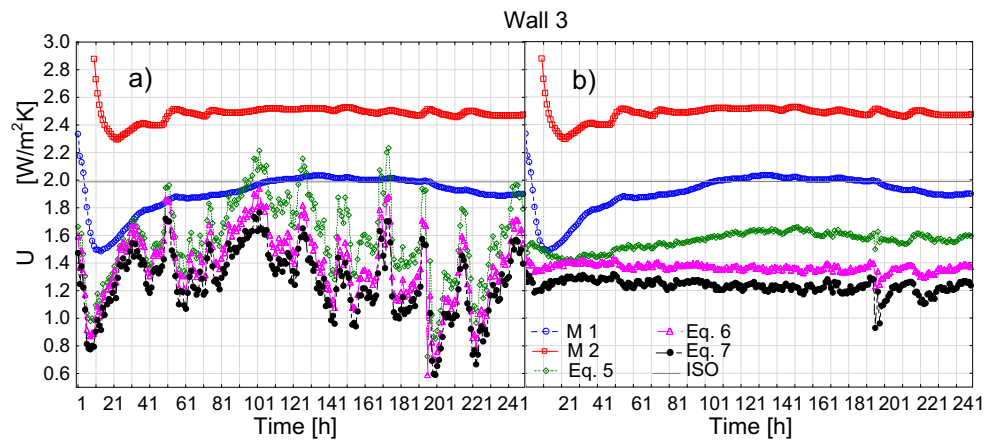


Fig. 9 Thermal transmittance determined for the wall 3 with dynamic (a) and average (b) temperature changes of the surrounding and surface



Eq. (3) is not always overlapping the measurement method. However, in view of the calculation methods acc. Standard [39], this estimation is satisfying. As we can see in Fig. 6 for the wall 1, the estimation of surface resistance with the Eqs. (5) or also (6) is closer to the tabular surface resistance [39] than the measurement. Admittedly, it does not bespeak of a wrong measurement method but rather of an inaccurate tabular data [39]. And looking at it the other way round, we can state that the estimation by means of Eqs. (5) or also (6) is closer to the measured value than the tabular value of resistance [39]. Therefore, although the estimation based on the Eqs. (5), (6) is worse than the measurement of heat flow rate, yet taking into account the measurement speed, the mentioned equations may be sufficient. It looks slightly worse on the wall 3 and the worst on the wall 2. We can observe that with the rise of temperature difference of the surrounding and the envelope, the estimation is becoming less precise.

Let us survey now the estimation results of the thermal transmittance U .

First of all, we can observe that for each of the investigated walls we have a different temperature difference of the surrounding and surface, which is demonstrated in the figure below (Fig. 10).

We can observe that the thermal transmittance determined with the Eq. (3) with the use of Eqs. (5)–(7) in all the investigated cases is different. And independently of the thermal performance and also independently of the temperature difference of the surrounding and wall surface, the thermal transmittance U (3) with the application of Eq. (5) is closest to the measurement, and with the use of the Eq. (7) it is the furthest. And the difference between the estimation (3) and the measurement is progressively higher with the rise of temperature difference of the surrounding and surface.

Another interesting observation involves the fact that the graphs (3) in the function of time with the application of Eqs. (5)–(7) respectively are almost parallel. It is particularly visible on graphs 7b, 8b and 9b at the average temperatures of the surrounding and surface. In the same way the measurement graph in the function of time is almost parallel to the remaining ones. And for the rooms having a small temperature difference of the surrounding and wall surface (walls with thermal insulation), the graphs are almost overlapping.

We can therefore state that for well insulated walls, the measurement and estimation based on (3), and in particular with the application of the Eq. (5), do not differ significantly. Whereas for the case of poorly insulated walls, where the differences of surrounding and wall surface are large, we can find a coefficient correcting the estimation with the Eq. (3).

Such an assumption is significant particularly when it involves the application of Eq. (8) which can assume now the following form:

$$U_{eqv} = \frac{k_1 U_1 + k_2 U_2 + \dots k_n U_n}{n} \tag{11}$$

where k_1, k_2, \dots, k_n are correcting coefficients.

And especially for places of the wall where the temperature difference of the surface and surrounding is small (good insulation, no discontinuities of insulation or heat bridge), we can assume $k_i = 1$.

For example, for the walls investigated in the present work.

$$k_1 = 1 \text{ for wall 1, } k_1 = \frac{U_{mea}}{U_{eq.(5)}} = 1.67 \text{ for wall 2, } k_2 = \frac{U_{mea}}{U_{eq.(5)}} = 1.23 \text{ for wall 3.}$$

The results of such a correction are presented in Fig. 11.

The determined correction coefficients quite well reflect the following cases:

Table 2 Estimation of measurement error U for all investigated cases

Method	Measurement 1	Measurement 2	Equation 5	Equation 6	Equation 7
Error δU	0.03	0.03	0.35	0.29	0.22

Table 3 Estimation of measurement error U for all investigated cases

Method	Measurement 1	Measurement 2	Equation 5	Equation 6	Equation 7
Error δU	0.06	0.04	0.27	0.25	0.25

k_1 for non-insulated envelopes $U \geq 2 \frac{W}{m^2K}$, and k_2 for poorly insulated envelopes $1.0 \frac{W}{m^2K} \leq U \leq 2.0 \frac{W}{m^2K}$,

We are tempted to claim here that when carrying out further research, it is possible to determine correction coefficients k for other cases, or to narrow them down and make them more specific for the cases that we suggested.

The results presented on the graphs 11 can be placed in the so called “credibility area” of results defined by the authors. For that purpose, perturbation methods will be applied.

Let us assume that in the Eq. (3) all parameters can be disrupted by various factors, with the assumption that each of the parameters will be dependent only on a part of the factors which may have impact on the final result. We assumed that the number of all factors may be n where of 11 were identified as follows:

- 1 - measurement errors,
- 2 - humidity of internal air,
- 3 - humidity of external air,
- 4 - temperature of internal air,
- 5 - temperature of external air,
- 6 - humidity content of the envelope,
- 7 - thermal conductivity index of the envelope,
- 8 - thickness of the envelope,

- 9 - surface of the envelope,
- 10 - impact of wind,
- 11 - insolation.

The Eq. (3), allowing for the expression (5), with the perturbation values of the variables assumes the following form:

$$U_{n\epsilon} = 1,66 \times \theta_{i,n\epsilon}^{1/3} \left(1_{n\epsilon} - \frac{\theta_{si,n\epsilon} - \theta_{e,n\epsilon}}{\theta_{i,n\epsilon} - \theta_{e,n\epsilon}} \right) \tag{12}$$

where the particular quantities will be dependent on the following disturbances:

- $\theta_{i,n\epsilon}$ temperature of the internal air (1,2)
- $\theta_{si,n\epsilon}$ temperature of the internal wall surface (1,2,4,6,7,8,9)
- $\theta_{se,n\epsilon}$ temperature of the external wall surface (1,3,5,6,7,8,9,10,11)

Let us note that the numbers of factors can be arbitrary. In the provided example the first 11 factors were used to ensure better clarity of notation. We assume, therefore, that the remaining 89 perturbations are of zero value. Making use of the accepted numbering, we can write down the disturbed values of the particular parameters in the following form:

$$\begin{aligned} \theta_{si,n\epsilon} &= \theta_{si,0} + \epsilon_1 \theta_{si,1} + \epsilon_2 \theta_{si,2} + 0_{3\epsilon} + 0_{4\epsilon} + \dots + 0_{n\epsilon}, \\ \theta_{i,n\epsilon} &= \theta_{i,0} + \epsilon_1 \theta_{i,1} + \epsilon_2 \theta_{i,2} + 0_{3\epsilon} + \epsilon_4 \theta_{i,4} + 0_{5\epsilon} + \epsilon_6 \theta_{i,6} + \epsilon_7 \theta_{i,7} + \epsilon_8 \theta_{i,8} + \epsilon_9 \theta_{i,9} + 0_{10\epsilon} + \dots + 0_{n\epsilon}, \\ \theta_{e,n\epsilon} &= \theta_{e,0} + \epsilon_1 \theta_{e,1} + 0_{2\epsilon} + \epsilon_3 \theta_{e,3} + 0_{4\epsilon} + \epsilon_5 \theta_{e,5} + \epsilon_6 \theta_{e,6} + \epsilon_7 \theta_{e,7} + \epsilon_8 \theta_{e,8} + \epsilon_9 \theta_{e,9} + \epsilon_{10} \theta_{e,10} + \epsilon_{11} \theta_{e,11} + \dots + 0_{n\epsilon} \end{aligned}$$

The solution is obtained in the following form:

$$U_{n\epsilon} = \pm \left| U_0 + \sum_{k=1}^{11} \epsilon_k U_k + \theta_{12\epsilon} + \dots + \theta_{n\epsilon} \right| \tag{13}$$

The Fig. 12 present the results allowing for the Eq. (13).

The present paper confirms but also complements the research done by Li [40] demonstrating that the measurements of heat flux in inhabited estates are very difficult. Li is describing large sources of errors, such as: poor contact between the

heat flux sensor and the wall, difficulties to identify measurement points beyond thermal interference, etc.

The measurements described in the present paper were devoid of the described by Li error sources, and yet the estimation results could be accepted as satisfying. We can accept as true that with respect to an inhabited dwelling place the estimation uncertainty with (3) using the Eqs. (5)–(7) and in particular (5) will not be higher than the measurement of heat flux, or at least not higher than the uncertainty described by Li.

Table 4 Estimation of measurement error U for all investigated cases

Method	Measurement 1	Measurement 2	Equation 5	Equation 6	Equation 7
Error δU	0.13	0.19	0.31	0.27	0.26

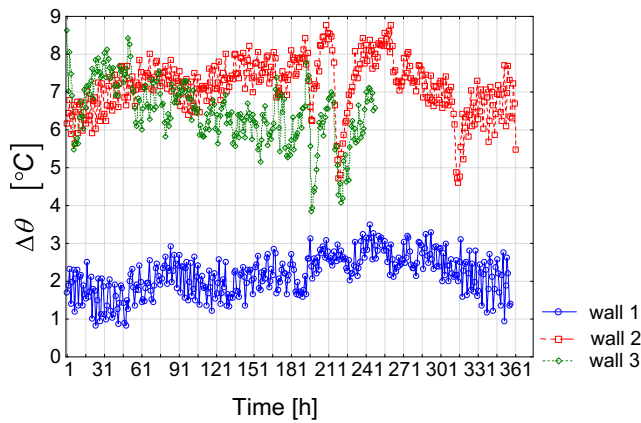


Fig. 10 Temperature differences of the surrounding and tested walls

The latest investigation studies [41] also show a considerable impact of the thermal conductivity envelope and temperature amplitudes on thermal resistance U . The research studies described in the present paper also demonstrate the influence of temperature changes on the estimation of thermal transmittance. However, the proposed estimation method is different.

Fig. 11 Graphs of thermal transmittance in the function of time; $U_{\text{measurement}}$ stands for the measured value, U_5 stands for the estimation with (3) using (5), $U_{k^*(5)}$ stands for the estimation with (3) using (5), taking into account the correction coefficient k

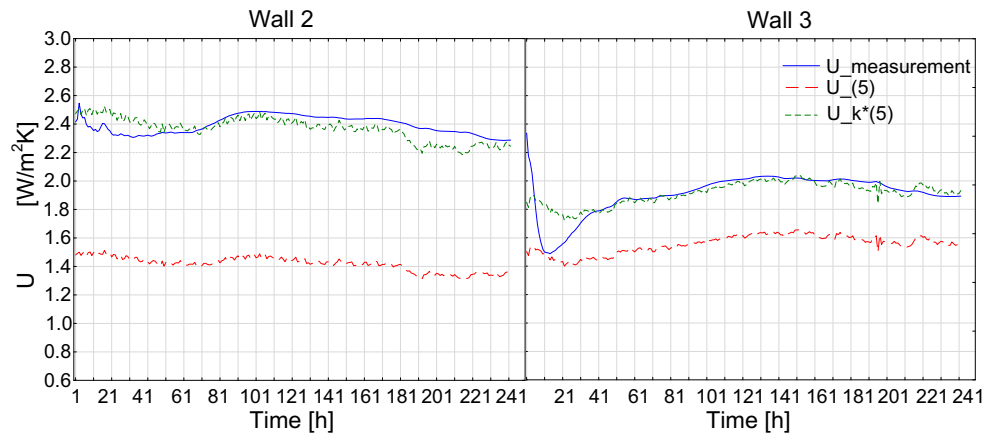
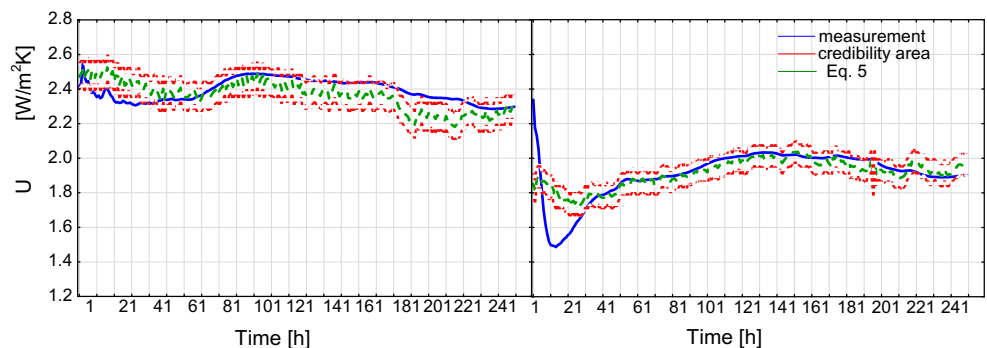


Fig. 12 Graphs of thermal transmittance as a function of time. Measurement stands for the measured value, Eq. 5 means the estimation by means of (3) using (5), credibility area was determined using the relation (13)



6 Conclusions

The present paper is attempting to estimate thermal transmittance U by means of temperature measurements. The research studies were carried out for three different wall constructions. The walls were selected in the way ensuring that they have a considerably different thermal resistance. Furthermore, the obtained estimation results were compared with the results based on the measurement of heat flow rate.

The results demonstrate that:

- the thermal transmittance U can be estimated by means of temperature measurement. Furthermore, with respect to the envelopes of low thermal resistance, we can find a correction coefficient k which makes it possible to estimate the result in a quite precise way,
- making use of the measurement of temperature and applying the Eq. (11), we can determine the equivalent thermal transmittance,
- using the algebra of perturbation numbers we can determine a so called “credibility area” which almost certainly comprises the measurement result.

We can also observe that the presented method can serve as an inspiration to use the thermovisual measurement for the estimation of thermal transmittance. Such an approach can replace the measurement by means of heat flow rate, which considerably facilitates the measurement without any substantial loss involving the results.

The elaboration of assumptions to apply thermovisual measurement as temperature measurement in the estimation of thermal transmittance U might be the subject of further research studies.

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References

1. Lowe RJ (2007) Technical options and strategies for decarbonizing UK housing. *Build Res Inf* 35(4):412–425
2. Baker P (2011) U-values and traditional buildings, in situ measurements and their comparisons to calculated values. *Historic Scotland*, Edinburgh
3. Stevens G, Bradford J (2013) Do u-value insulation? England’s field trial of solid wall insulation. *ECEEE Summer Study Proceedings*, 1269–1280
4. Rye C (2015) Are traditional buildings really carbon villains? Why it matters when insulating solid walls. *Journal of Building Survey, Appraisal & Valuation* 4(2):119–126
5. Evangelisti L, Guattari C, Gori P, de Lietto Vollaro R (2015) In situ thermal transmittance measurements for investigating differences between wall models and actual building performance. *Sustain For* 7:10388–10398
6. Friedman C, Becker N, Erell E (2014) Energy retrofit of residential building envelopes in Israel: a cost benefit analysis. *Energy* 77: 183–193
7. De Lieto Vollaro R, Guattari C, Evangelisti L, Battista G, Carnielo E, Gori P (2015) Building energy performance analysis: a case study. *Energy Build* 87:87–94
8. Pisello AL, Rossi F, Cotana F (2014) Summer and winter effect of innovative cool roof tiles on the dynamic thermal behavior of buildings. *Energies* 7:2343–2361
9. Baldinelli G, Bianchi F (2014) Windows thermal resistance: infrared thermography aided comparative analysis among finite volumes simulations and experimental methods. *Appl Energy* 136: 250–258
10. Baker P (2011) U-values and traditional buildings: in situ measurements and their comparisons to calculated values. (Historic Scotland Technical Paper 10) [PDF] Edinburgh: Historic Scotland. <https://webcache.googleusercontent.com/search?q=cache:WF-7bPYU8vgJ:https://pub-prod-sdk.azurewebsites.net/api/file/25a883fd-9a66-4cdd-9c5b-a67b01006f97+&cd=1&hl=en&ct=clnk&gl=pl> Accessed 24 Oct 2016
11. Ghazi Wakili K, Binder B, Zimmermann M, Tanner C (2014) Efficiency verification of a combination of high performance and conventional insulation layers in retrofitting a 130-year old building. *Energy Build* 82:237–242
12. Cabeza LF, Castell A, Medrano M, Martorell I, Pérez G, Fernández I (2010) Experimental study on the performance of insulation materials in Mediterranean construction. *Energy Build* 42:630–636
13. Hens H (2007). *Building physics – heat, air and moisture*. Ernst & Sohn, A Wiley Company, Berlin
14. Asdrubali F, D’Alessandro F, Baldinelli G, Bianchi F (2014) Evaluating in situ thermal transmittance of green buildings masonries—a case study. *Case stud. Constr. Dent Mater* 1:53–59
15. Walker R, Pavia S (2015) Thermal performance of a selection of insulation materials suitable for historic buildings. *Build Environ* 94(1):155–165
16. Jiménez MJ, Porcaar B, Heres MR (2009) Application of different dynamic analysis approaches to the estimation of the building component U value. *Build Environ* 44(2):361–367
17. Dudek SJ-M, Armstrong S, Stamatou G (1993) Thermal performance of insulated concrete blocks. *Build Res Inf* 21(1):36–41
18. Naveros I, Jiménez MJ, Haras MR (2012) Analysis of capabilities and limitations of the regression method based in averages, applied to the estimation of the U value of building component tested in Mediterranean weather. *Energy Buildings* 55:854–872
19. Cesaratto P, De Carli M, Marinetti S (2011) Effect of different parameters on the in situ thermal conductance evaluation. *Energy Buildings* 43(7):1792–1801
20. Ficco G, Iannetta F, Ianniello E, d’Ambrosio Alfano FR, Dell’Isola M (2015) U-value in situ measurement for energy diagnosis of existing buildings. *Energy Buildings* 104:108–121
21. Byrne A, Byrne G, Robinson A (2017) Compact facility for testing steady and transient thermal performance of building walls. *Energy Buildings* 152:602–614
22. Biddulph P, Gori V, Elwell VG, Scott C, Rye C, Lowe R, Oreszczyn T (2014) Inferring the thermal resistance and effective thermal mass of a wall using frequent temperature and heat flux measurements. *Energy Buildings* 78:10–16
23. Krause P (2017) The numeric calculation of selected thermal bridges in the walls of AAC. *Cem Wapno Beton* 22(5):371–380
24. Hailemariam H, Wuttke F (2017) Temperature dependency of the thermal conductivity of porous heat storage media. *Heat Mass Transf* doi: <https://doi.org/10.1007/s00231-017-2204-3>
25. Naveros I, Ghiaus C, Ruiz DP, Castano S (2015) Physical parameters identification of walls using ARX models obtained by deduction. *Energy Buildings* 108:317–329
26. Fox M, Coley D, Goodhew S, De Wilde P (2015) Time-lapse thermography for building defect detection. *Energy Buildings* 92:95–106
27. Taylor T, Counsell J, Gill S (2014) Combining thermography and computer simulation to identify and assess insulation defects in the construction of building facades. *Energy Buildings* 76:130–142
28. Aziz A, Na T.Y (1984) *Perturbation methods in heat transfer*, Hemisphere Publishing Corporation, USA
29. Czel B, Woodbury KA, Gróf G (2014) Simultaneous estimation of temperature-dependent volumetric heat capacity and thermal conductivity functions via neural networks. *Int J Heat Mass Transf* 68: 1–13
30. Singh R, Bhoopal RS, Kumar S (2011) Prediction of effective thermal conductivity of moist porous materials using artificial neural network approach. *Build Environ* 46:2603–2608
31. Abraham TN, George KE (2009) Short nylon fiber reinforced HDPE: melt rheology. *J Thermoplast Compos Mater* 22:293–304
32. Nowoświat A (2002) Optimal control of a duffing oscillator under parametric and external excitations. *J Theor Appl Mech* 40(2):435–447

33. Shan B, Pelegri A (2003) Approximate analysis of the buckling behavior of composites with delamination. *J Compos Mater* 37: 673–685
34. Skrzypczyk J (2008) II – order perturbation methods – new computational Metodology. *Architecture Civil Engineering Environment* 1(2):73–77
35. Villatoro F.R, Perez J, Santander J.L.G, Borovsky M.A, Ratisf Yu.L, Izzheurov E.A, Fernandez de Cordoba P (2011) Perturbation analysis of the heat transfer in porous media with small thermal conductivity. *J Math Anal Appl* 374: 57–70
36. Winkler-Skalna A (2016) Perturbation numbers – evaluation of the influence of the uncertainty of parameters on unidirectional temperature distribution in a plate. *Architecture Civil Engineering Environment* 9(1):95–100
37. Fokin KF (1973) *Thermotechnics of a building envelopes parts*. Stroizdat, Moskwa
38. ISO 9869–1: 2014 Thermal insulation – Building elements – In-situ measurement of thermal resistance and thermal transmittance – Part 1: Heat flow meter method
39. ISO 6946: 2007 Building components and building elements – Thermal resistance and thermal transmittance – Calculation method
40. Li FGN, Smith AZP, Biddulph P, Hamilton IG, Lowe R, Mavrogianni A, Oikonomou E, Raslan R, Stamp S, Stone A, Summerfield AJ, Veitch D, Gori V, Oreszczyn T (2015) Solid-wall U-values: heat flux measurements compared with standard assumptions. *Build Res Inf* 43(2):36–41
41. Ma P, Wang L, Guo N (2015) Maximum window-to-wall ratio of a thermally autonomous building as a function of envelope U-value and ambient temperature amplitude. *Appl Energy* 146:84–91