manuscripta math. 171, 369–370 (2023)

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Erratum to: On homogeneous polynomials determined by their Jacobian ideal

Published online: 19 December 2022

Erratum to: manuscripta math. 146, 559–574 (2015)
https://doi.org/10.1007/s00229-014-0703-9

This is a corrigendum for the proof of [3, Corrollary 6.1]. We follow the notations in [3]. In the proof of [3, Corrollary 6.1], we claimed that the set $\mathscr{G}_1$, which is the set of homogeneous polynomials of ST type, is a Zariski closed subset of $\mathbb{P}(S_{n,d})$. This is incorrect, since, for instance, the polynomial $f = x^2y - y^2z = y(xy - yz) \in \mathbb{P}(S_{2,3})$ lies in $\overline{\mathscr{G}}_1 \setminus \mathscr{G}_1$, as is proved in [1, Example 1.4]. In fact, in our proof of [3, Corrollary 6.1], an error arises when we obtain the equality (6.3) by letting $i \to \infty$ which says

$$g_\infty(x_0, \cdots, x_n) = h_\infty(x'_0, \cdots, x'_l) + k_\infty(x'_{l+1}, \cdots, x'_n) \qquad \text{in } \mathbb{P}(S_{n,d});$$

however, it may happen that

$$h_\infty(x'_0, \cdots, x'_l) + k_\infty(x'_{l+1}, \cdots, x'_n) = 0;$$

thus the right-hand side does not make sense.

On the other hand, the result stated in [3, Corollary 6.1] is correct. Indeed, using [1, Theorem 4.5] and the remark following [2, Theorem 3.2], we obtain $\overline{\mathscr{G}}_1 \setminus \mathscr{G}_1 \subset \mathscr{G}_2$, the notation in [3] being used. Then [3, Corollary 6.1] immediately follows from this.

The author would like to thank Professor Hualin Huang and Professor Yu Ye. They asked him to give a talk about the Zariski closedness of $\mathscr{G}_1$ and after the talk, they told him they had a counterexample at hand. The author eventually found out the error in his original proof.

References

[1] Buczyńska, W., Buczyński, J., Kleppe, J., Teitler, Z.: Apolarity and direct sum decomposability of polynomials. Mich. Math. J. **64**, 675–719 (2015)

The original article can be found online at https://doi.org/10.1007/s00229-014-0703-9.
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https://doi.org/10.1007/s00229-022-01447-2

[2] Fedorchuk, M.: Direct sum decomposability of polynomials and factorization of associated forms. Proc. Lond. Math. Soc. (3) **120**(3), 305–327 (2020)
[3] Wang, Z.: On homogeneous polynomials determined by their Jacobian ideal. Manuscr. Math. **146**(3–4), 559–574 (2015)