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Erratum to: The interior gradient estimate of prescribed Hessian quotient curvature equations

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In this note we correct a typo in the proof of the main result of [1], that is (3.10).

(3.10) should be

$$F^{1i}u_{1i} = \frac{\partial F}{\partial a_{1i}} \frac{1}{W^2} u_{1i} = \frac{\partial F}{\partial a_{1i}} a_{i1} \leq 0.$$

The last inequality in the above fomula holds from the concavity of $\left[\frac{\sigma_k}{\sigma_l}\right]^{\frac{1}{k-l}}$. We can give a detailed proof as follows. Following the proof in [1], we assume $A = \{a_{ij}\}_{n\times n}$ with $a_{ij} = a_{ji}$, and at x_0, we have $a_{ij} = 0$ for $2 \leq i \neq j \leq n$. For any $i_0 \geq 2$, consider

$$g(t) = \left[\frac{\sigma_k(B+tC)}{\sigma_l(B+tC)}\right]^{\frac{1}{k-l}}, \quad (-1 \leq t \leq 1),$$

where $B = \{b_{ij}\}_{n\times n}$ with $b_{1i_0} = b_{i_0 1} = 0$ and $b_{ij} = a_{ij}$ otherwise, and $C = \{c_{ij}\}_{n\times n}$ with $c_{1i_0} = c_{i_0 1} = a_{i_0 1}$ and $c_{ij} = 0$ otherwise. Then we have $B + C = A$.

Since $\sigma_1(B + tC) = \sigma_1(B)$ and

$$\sigma_m(B + tC) = \sigma_m(B) - t^2 a_{1i_0}^2 \sigma_{m-2}(B|1i_0)$$

for $m \geq 2$, we can get

$$\begin{aligned}\sigma_m(B + tC) =& \sigma_m(A) + (1 - t^2)a_{1i_0}^2 \sigma_{m-2}(B|1i_0) \\ =& \sigma_m(A) + (1 - t^2)a_{1i_0}^2 \sigma_{m-2}(A|1i_0)\end{aligned}$$

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$$\geq \sigma_m(A) \tag{0.1}$$

for $t \in [-1, 1]$ and $\sigma_m(B + tC) = \sigma_m(B - tC)$, which can also be obtained from

$$\begin{vmatrix} a_{11} & \cdots & ta_{1i_0} & \cdots \\ \vdots & \ddots & & \\ ta_{i_01} & & a_{i_0i_0} & \\ \vdots & & & \ddots \end{vmatrix}_{m\times m} = \begin{vmatrix} a_{11} & \cdots & -ta_{1i_0} & \cdots \\ \vdots & \ddots & & \\ -ta_{i_01} & & a_{i_0i_0} & \\ \vdots & & & \ddots \end{vmatrix}_{m\times m}.$$

Hence we know the eigenvalues of $B + tC$ are in the convex cone Γ_k for $t \in [-1, 1]$ and $g(-1) = g(1)$. From the concavity of $\left[\frac{\sigma_k}{\sigma_l}\right]^{\frac{1}{k-l}}$ in Γ_k, we have $g(t)$ is concave with respect to $t \in [-1, 1]$. Hence

$$0 \geq g'(1) = \frac{1}{k-l}\left[\frac{\sigma_k(A)}{\sigma_l(A)}\right]^{\frac{1}{k-l}-1}\left[\frac{\partial \frac{\sigma_k(A)}{\sigma_l(A)}}{\partial a_{1i_0}} a_{i_01} + \frac{\partial \frac{\sigma_k(A)}{\sigma_l(A)}}{\partial a_{i_01}} a_{1i_0}\right]. \tag{0.2}$$

Hence $\frac{\partial F}{\partial a_{1i_0}} a_{i_01} \leq 0$ for any $i_0 \geq 2$.

References

[1] Chen, C., Xu, L., Zhang, D.: The interior gradient estimate of prescribed Hessian quotient curvature equations. Manuscripta Math. **153**(1–2), 159–171 (2017). https://doi.org/10.1007/s00229-016-0877-4