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Erratum to: Kauffman bracket skein module of a connected sum of 3-manifolds

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Abstract. This note is an erratum pointing out an error in Theorem 7.1 in the paper [4] which was published in *Manuscripta Math.* in the year 2000.

The Kauffman bracket skein module of a 3-dimensional manifold was introduced by the author in April 1987 and published in [3]. The definition is as follows:

Definition. Let M be an oriented 3-manifold, R a commutative ring with identity and A its invertible element. Let $\mathcal{L}_{fr}$ be the set of unoriented framed links in M , including the empty link $\emptyset$. Let S be the submodule of $R\mathcal{L}_{fr}$ generated by skein expressions $L_+ - AL_0 - A^{-1}L_\infty$, where the triple L_+, L_0, L_∞ is shown in Fig. 1, and $L \sqcup T_1 + (A^2 + A^{-2})L$, where T_1 denotes the trivial framed knot. We define the Kauffman bracket skein module (KBSM), denoted by $S(M; R, A)$, as the quotient $S(M; R, A) = R\mathcal{L}_{fr}/S$. We write succinctly $S(M)$, as long as the ring is not unambiguous.

We proved in [2] that over the ring $R = \mathbb{Z}[A^{\pm 1}]$ the KBSM of $S^1 \times S^2$ has big torsion part (infinitely generated) and that torsion can appear in the case of connected sums of 3-manifolds. In [4] we show that over the field of rational functions over variable A there is a simple formula for the KBSM of connected sums. The main result proved in [4] is that if $(A^k - 1)$ is invertible in R for any $k > 0$, and M is the connected sum of M_1 and M_2 then $S(M) = S(M_1) \otimes S(M_2)$. This result is correct. However, at the end of the paper [4], in the section titled “Generalizations and Speculations” we stated, without a proof, the following theorem about the KBSM of the connected sum of handlebodies:

Theorem. Let H_n be a handlebody of genus n and P_n be a disk with n holes. Then,

$$S(H_n \# H_m) = S(H_{n+m})/\mathcal{I}$$

where $\mathcal{I}$ is the submodule generated by expressions $z_k - A^6 u(z_k)$, for any even $k \geq 2$, and $z_k \in B_k(P_{n+m})$, where $B_k(P_{n+m})$ is a subset of a basis $B(P_{n+m})$ of the

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Fig. 1. Kauffman bracket skein triple

Fig. 2. .

Fig. 3. .

KBSM of $P_{n+m} \times I$, composed of links with geometric intersection number with a disk D separating H_n and H_m equal to k . $u(z_k)$ is a modification of z_k in the neighborhood of D , as shown in Fig. 2. Our relation $z_k = A^6 u(z_k)$, is a result of the sliding relation $z_k = sl_{\partial D}(z_k)$ as illustrated in Fig. 3.

Unfortunately, this theorem does not hold for n or m greater than one. The paper [1] in this issue of *Manuscripta Math.* describes a counterexample for the theorem. The case of $m = n = 1$ is still open. I would like to thank Charlie Frohman who asked in March 2020 whether a proof of the theorem has been published, motivating the authors of [1] to think on the problem.

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