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Thickness of zero-strength layer in timber beam exposed to fuel-controlled parametric fires

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Abstract

Fire resistance is one of the essential requirements to be fulfilled in the design of timber structures. For this purpose, a reduced cross-section method is given in the European standards, Eurocodes. The method is based on the assumption that an initial, rectangular timber cross-section exposed to fire conditions reduces to an effective cross-section, which has material properties as at a room temperature. The reduced part of the cross-section with no resistance is determined by a sum of two parameters, namely a charring depth and a thickness of zero-strength layer. Eurocodes give a value of the latter only for the standard fire exposure, which is only one of the fire curves proposed in the same standards. Therefore, the present paper examines the thickness of zero-strength layer in case of 46 different fuel-controlled parametric fire exposures applied to a timber beam from three sides. A four-phase numerical analysis is applied for this purpose that includes the use of a hygro-thermal model and a mechanical model to determine temperatures of timber over the cross-section and the mechanical resistance of timber beam in fire conditions, respectively. The results show that the thickness of zero-strength layer takes the values between 7.9 and 18.4 mm for the fuel-controlled parametric fire exposures. Since it is clearly dependent on the parameters describing the parametric fire curve, five equations are proposed that can be used for determination of the thickness of zero-strength layer in case of parametric fire exposures.

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Introduction

Nowadays, wood is an almost indispensable material in the design of new environmentally friendly and sustainable buildings and often represents the whole or a major part of the structural system. At the same time, the construction of tall wooden high-rise buildings has expanded in recent years, and with this the need for knowledge of the behavior of wooden structures in normal as well as in extreme conditions, including fire and the corresponding fire resistance of timber structures. In engineering practice, the fire resistance of timber structures is usually determined by applying a prescriptive approach that includes the use of simplified methods, which are given in standards such as Eurocodes. In the prescriptive approach, the fire resistance is usually determined for the standard fire exposure. For instance, the Eurocode standard EN 1995–1-2 (2005) provides two methods for this purpose. The reduced cross-section method is the most commonly used method to determine not only the fire resistance of timber members under bending, tension and compression (Schmid et al. 2014, 2015; Lange et al. 2015), but also the fire resistance of timber frame assemblies (Tiso et al. 2018, 2019). The method assumes that the fire resistance depends on the determination of the thickness of the charring layer, also called charring depth, and on the thickness of the zero-strength layer, which do not contribute to the determination of the loadbearing capacity of timber elements. Currently, EN 1995–1-2 (2005) prescribes only a value of the thickness of zero-strength layer for the standard fire exposure, i.e., $d_0 = 7$ mm. The same value is commonly used also in case of natural fire exposures as there is no specific recommendations for those types of fires. The development of the reduced cross-section method and the acquirement of the prescribed value of the thickness of zero-strength layer is described by Schmid et al. (2012) where it is pointed out that both might not be appropriate for use in current applications due to the incompatibility with the applications the method was developed for. Additionally, research in the last decade shows that the proposed value of the thickness of zero-strength layer is not appropriate neither for the standard fire exposure nor the natural fire exposures. For instance, based on an extensive review of the large-scale fire tests, Schmid et al. (2014) reports the values of the thickness zero-strength layer as high as 30 mm for timber structural elements in bending, tension and compression exposed to the standard fire. Additionally, a relatively large scatter of the magnitudes of the thickness of zero-strength layer is reported given to the reason of insufficient information on material properties at room temperature, besides their large natural scatter. Another study by Schmid et al. (2015) reports that the calculated thicknesses of zero-strength layer are in the range of 9.5 mm to 20.1 mm based on the experimental results of five tests of glulam beams in bending exposed to the standard fire from three sides. Lange et al. (2015) performed four-point bending tests of glulam beams exposed to the standard fire and two natural fires, namely a long-cool parametric fire and a short-hot parametric fire. The thickness of zero-strength layer calculated from the results of the performed tests by Lange et al. (2015) are about 15 mm, 16 mm and 8 mm for the standard fire, the long-cool parametric fire and

the short-hot parametric fire exposures, respectively. Clearly, the magnitude of thickness of zero-strength layer varies with the type of fire exposure and is in some cases much higher than the currently prescribed value of 7 mm. Further-more, Lange et al. (2015) reports that the thickness of zero-strength layer depends on parameter Γ defined in EN 1991–1-2 (2004) as it takes lower values in case of parametric fires with higher heating rates. Similar observations are also presented by Huč et al. (2021) where advanced hygro-thermal and mechanical models are used to study the magnitude of the thickness of zero-strength layer for a timber beam under bending exposed to numerous ventilation-controlled (VC) parametric fires. The obtained thicknesses of zero-strength layer take the values between 8.4 and 30.5 mm. A clear dependence of the thicknesses of zero-strength layer on the cooling rate, the maximum gas temperature and the duration of VC parametric fire is also observed. Brandon et al. (2017) also conducted similar study, where the expression of zero-strength layer is given as the function of parameter Γ. Obtained values of the zero-strength layer are smaller in comparison with pre-viously discussed studies and the present study, namely the zero-strength layer calculation in Brandon et al. (2017) is based on effective charring rate and also the relationship is determined at time when cooling starts. As shown in Huč et al. (2021) the thickness of zero-strength layer varies with time and if one wants that the timber beam survives the fire the thickness of zero-strength layer has to be determined at time when the thickness of zero-strength layer no longer changes with time. Based on the reports of the aforementioned studies, it is obvious that additional research is needed to get more accurate and thus safe values of the thickness of zero-strength layer in case of different fire exposures.

Therefore, the aim of the present paper is to examine the thickness of zero-strength layer of a timber beam under bending exposed to 46 different fuel-con-trolled (FC) parametric fires from three sides. A parametric fire is defined in EN 1991–1-2 (2002) as a temperature–time relationship of a gas in a fire compartment, depending on the compartment size, fuel load, boundary characteristics and ventila-tion. The parametric fire curve is a very simple model of natural fire and is for this reason the most adopted methodology to replicate natural fire exposures on struc-tural elements (Lucherini et al. 2021). The parametric fire curve consists of a heat-ing and a cooling phase, which are defined with a nonlinear and a linear equation, respectively. The cooling phase is absolutely important in the design of timber ele-ments in fire conditions, since the temperature of timber beam cross-section is still increasing in the cooling phase and the charring of timber continues for some time, although the gas temperature decreases (Gernay 2015; Wiesner et al. 2019). How-ever, the reader should be aware that, as reported in Lucherini et al. (2021), linear cooling rates are not the most appropriate to correctly characterize the parametric fire which should be improved with additional research. A distinction between the VC and the FC parametric fire curves is determined in EN 1991–1-2 (2002) by an empirical equation, which results in shorter duration and lower maximum gas tem-peratures reached for the FC fires as compared to the VC fires. A four-phase numeri-cal analysis is applied in the present paper to determine, firstly, a FC parametric fire scenario, secondly, temperatures' development over a timber beam cross-section, thirdly, the mechanical resistance of timber beam in fire conditions, and fourthly, the

charring depth and the thickness of zero-strength layer. In the second and the third phase, own-developed hygro-thermal (Pečenko et al. 2015, 2016) and mechanical (Pečenko et al. 2019; Pečenko 2016) models are applied, accounting for the natural phenomena of moisture transport and nonlinear behavior of timber simultaneously exposed to loading and fire conditions. Similar approaches to numerical modeling of fire resistance of timber can be found in the literature (Thi et al. 2017; Šulc et al. 2019; Di Blasi 1998; König 2006; Zhang et al. 2019), most of them highlighting the importance of considering temperature dependent material parameters. A posteriori analysis includes an extensive study of the obtained thicknesses of zero-strength layer in case of 46 FC parametric fires in regard to the prescribed value of 7 mm and in dependence on the so-called primary and secondary parameters required to determine a parametric fire curve. Finally, based on a comparison of the results of the present paper and the previously published results by Huč et al. (2021), several approaches and equations are proposed for a determination of the thickness of zero-strength layer in case of FC and VC parametric fire exposures.

Materials and methods

A simply supported timber beam of a rectangular cross-section loaded with two point loads F_i and exposed to fire from three sides is considered as depicted in Fig. 1. A span of a beam is $L = 3.6$ m, and the initial height and width are $h = 0.24$ m and $b = 0.2$ m, respectively. In the mechanical analysis, the bi-linear constitutive law of timber at elevated temperatures is considered. The material parameters describing the bi-linear constitutive law as presented in Pečenko et al. (2019) are: $f_{t,T0} = 22.5$ MPa, $f_{c,T0} = 25$ MPa, $D_{t,T0} = 1.6 \cdot 10^{-3}$, $D_{c,T0} = 1.9 \cdot 10^{-3}$, $D_{t,p} = 1.8 \cdot 10^{-2}$, $D_{c,p} = 3.5 \cdot 10^{-2}$, $E_{t,T0} = E_{c,T0} = 13,350$ MPa, and $E_{t,p,T0} = E_{c,p,T0} = 500$ MPa where the symbols f, D and E denote the strength, the limit elastic strain and the Young's modulus of timber, respectively. The subscripts t, c and $T0$ denote tension, compression and room temperature, respectively. In order to account for the nonlinear behavior of timber and redistribution of stresses that occurs in bending during fire, plastic hardening in compression and tension are considered in the stress–strain relationship of timber at elevated temperatures. The plastic hardening parameters were determined based on the validation cases presented in Pečenko (2016) and Pečenko et al. (2019). The temperature-dependent mechanical properties of timber at elevated temperatures are taken from EN 1995–1-2 (2005). The material is considered as transversely isotropic with the longitudinal material direction along the length of
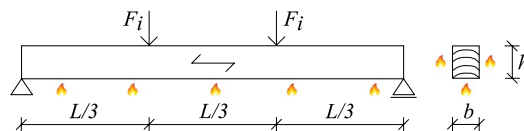

Fig. 1 Schematic of the analyzed simply supported timber beam of length L and the initial cross-section dimensions b/h, which is loaded with two point loads F_i and exposed to fire from three sides

Phase	Outcome of a phase
1) Determination of fire scenarios	Fire curve: gas temperature – time, $T_g - t$
2) Hygro-thermal analysis	Temperature over timber beam cross-section – time, $T - t$
3) Mechanical analysis	Mechanical resistance of timber beam in fire conditions, $M_{Rd,fi}$
4) Determination of the effective cross-section	Charring depth, d_{ch} Thickness of zero-strength layer, d_0

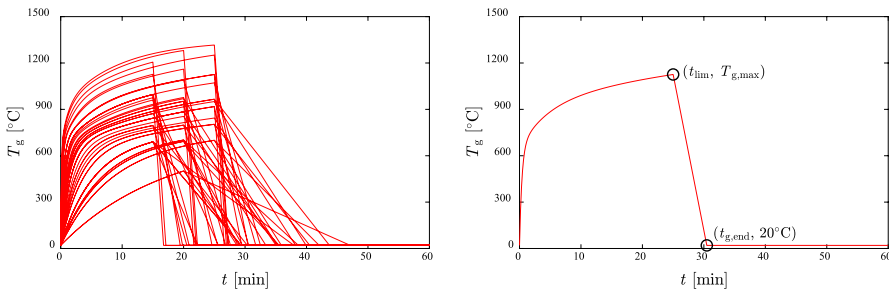**Fig. 2** Schematic of four phases of the analysis and their outcomes

Fig. 3 a Schematic of 46 fire scenarios, i.e., FC parametric fire curves; **b** Secondary parameters denoted on a single fire curve numbered 9 in Online Resource 1

the beam. In the present paper, the aforementioned timber beam is analyzed in four consecutive phases: (1) determination of fire scenarios, (2) hygro-thermal analysis, (3) mechanical analysis, and (4) determination of the effective cross-section. An outcome of each phase is considered as an input in the subsequent phase of the analysis as shown in Fig. 2.

In the first phase, 46 different fire scenarios are modelled. For this purpose, a FC parametric fire curve is chosen that is defined in EN 1991–1-2 (2002) by an analytical equation describing a relationship between gas temperature, T_g, and time, t. The input parameters, also referred as primary parameters in the present paper, that are required to obtain the FC parametric fire curve are: the thermal absorptivity for the total enclosure, b, the opening factor, O, the design value of the fire load density, q_{fd}, and the fire growth rate, t_{lim}. A fire compartment of dimensions 20 m × 15 m × 4 m is considered. 46 different FC parametric fire curves are obtained by varying the input parameters, i.e., b, O, q_{fd} and t_{lim} as shown tabularly in Online Resource 1 and plotted in Fig. 3a. In the present paper, each FC parametric fire curve is also characterized by so-called secondary parameters, i.e.: time at which the cooling phase ends, $t_{g,end}$, a linear cooling rate, v_{cool}, and maximum gas temperature, $T_{g,max}$, as seen in Fig. 3b. Additionally, a dimensionless ratio, $\Gamma = (O/b)^2 (0.04/1160)^{-2}$ according to

EN 1991–1-2 (2002), which accounts for the thermal properties of fire compartment boundaries is also considered in the results of the present paper.

Secondly, a hygro-thermal analysis is performed to determine a time development of a two-dimensional temperature field over the analyzed timber beam cross-section. For this purpose, an advanced hygro-thermal model for timber is used, hereafter denoted as HT model. The latter is an own developed heat and mass transport model, which is thoroughly presented and validated by Pečenko et al. (2015, 2016) and Pečenko (2016). In what follows, the main equations of the HT model are only presented. The HT model simultaneously solves a system of three continuity Eqs. (1)–(3) and the energy conservation Eq. (4), considering transports of air, bound water, water vapor and heat:

$$\frac{\partial\left(\varepsilon_g\left(\frac{P_g}{R_a T} - \frac{R_a}{R_v}\widetilde{\rho}_v\right)\right)}{\partial t} = -\nabla \cdot \mathbf{J}_a \tag{1}$$

$$\frac{\partial c_b}{\partial t} = -\nabla \cdot \mathbf{J}_b + \dot{c} \tag{2}$$

$$\frac{\partial\left(\varepsilon_g\widetilde{\rho}_v\right)}{\partial t} = -\nabla \cdot \mathbf{J}_v - \dot{c} \tag{3}$$

$$\rho C\frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T) - (\rho C\mathbf{v})\nabla T - \Delta H_s\dot{c} \tag{4}$$

The Eqs. (1)–(4) are solved for the main unknowns P_g, c_b, $\widetilde{\rho}_v$ and T, which are the air pressure, the bound water concentration, the water vapor concentration and temperature of timber, respectively. In Eqs. (1)–(4), ε_g denotes porosity of timber, k thermal conductivity, ρ density, C specific heat, ΔH_s latent heat of sorption, $\rho C\mathbf{v}$ heat convection and $\dot{c}$ the sorption rate. The symbol t denotes time, ∇ the nabla operator, and R_a and R_v gas constants for air and water vapor, respectively. To obtain a numerical solution of the Eqs. (1)–(4), constitutive relationships are needed for the mass fluxes of air, bound water, and water vapor, i.e., $\mathbf{J}_a$, $\mathbf{J}_b$ and $\mathbf{J}_v$, respectively. For a detailed description of the HT model formulation, please refer to Pečenko (2016). The initial conditions of the main unknowns at ambient temperature, $T_{g,0} = 20$ °C, are taken as follows: $c_{b,0} = 81.6$ kg/m³, $\widetilde{\rho}_{v,0} = 9$ g/m³, $P_{g,0} = 0.1$ MPa and $T_0 = 20.0$ °C. The initial density and the initial moisture content of timber are 420 kg/m³ and 12%, respectively. The rest of thermal properties of timber required for the HT model is taken from EN 1995–1-2 (2005). The necessary boundary conditions that resemble fire scenarios, which the analyzed timber beam is exposed to from three sides (the first phase of the analysis), are applied as the heat flux, pressure equilibrium and the water vapor flux through the surfaces exposed to fire. The pore pressure at the interface between the surroundings and the char layer is considered to be equal to ambient pressure of 0.1 MPa. The surface emissivity ε_m and the coefficient of heat transfer by convection α_c are

considered according to EN 1991–1-2 (2002) and remain the same in the heating and in the decay phase of the parametric fire curve, although physically this is not entirely correct. However, to estimate temperatures and charring depth in timber members exposed to parametric fire exposure, the assumption of constant parameters for boundary conditions is a good approximation to real conditions as shown in Pečenko and Hozjan (2021). The mass transfer coefficient β is determined according to Cengel (1998). Due to symmetry only a half of the cross-section is modelled which is discretized to 2160 finite elements of dimensions 3.3×3.3 mm^2. Four node isoparametric finite elements and an implicit time integration scheme are used to obtain the solution.

In the third phase, the mechanical analysis is applied to the analyzed timber beam. National codes usually require that the structures in fire conditions have to be designed so that they survive the entire anticipated fire scenario. In the case of timber structures, it does not necessarily mean that the structure has only to survive the whole duration of a fire scenario, i.e., $[0, t_{g,end}]$, to fulfill the requirement of EN 1991–1-2 (2002), but it might need to survive even a longer period, i.e., $t > t_{g,end}$, as timber might continue to char even when fire conditions are no longer present, i.e., $T_g = 20$ °C. Therefore, for the analyzed timber beam of prescribed dimensions and properties, the magnitude of the loads F_i has to be determined in a way that the design bending moment, $M_{Ed,fi} = F_i L/3$, does not exceed the mechanical resistance in fire conditions, $M_{Rd,fi}$. In the present paper, the latter is determined by means of the mechanical model that is based on the Reissner's kinematically exact beam model (Reissner 1972). The mechanical model simultaneously numerically solves a system of equilibrium, kinematic and constitutive equations where the temperature fields over timber beam cross-section determined in the hygro-thermal analysis are taken as an input. The applied mechanical model has already been developed and presented in several publications over the years, therefore the reader is asked to see a detailed background on the derivation, solving, validation and application of the mechanical model to timber structures published by Bratina et al. (2004), Pečenko (2016) and Pečenko et al. (2019). In the mechanical model, the entire beam is discretized with six finite elements where five integration points are applied over each element. The input data to describe the bilinear material model as part of the advanced mechanical model are presented by Pečenko et al. (2019). Reductions of the strength and the elastic modulus of timber in compression and tension at elevated temperatures are considered in accordance with EN 1995–1-2 (2005). The loads F_i are applied in steps in a decreasing order until the mechanical resistance in fire conditions $M_{Rd,fi}$ is obtained with a difference of 1 N m between the current and the previous step. The mechanical resistance in fire conditions $M_{Rd,fi}$ determined as aforementioned is said to be the highest mechanical resistance for which the analyzed timber beam survives a prescribed fire exposure, i.e., prescribed FC fire curve. Time at which the mechanical resistance in fire conditions $M_{Rd,fi}$ is determined is denoted as t_{end}.

Finally, in the fourth phase, the thickness of zero-strength layer, d_0, is determined according to the reduced cross-section method given in EN 1995–1-2 (2005) and depicted in Fig. 4. The reduced cross-section method is based on the assumption that
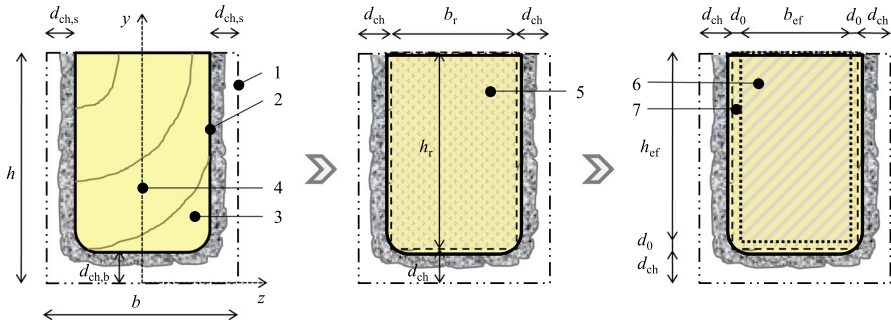
Fig. 4 Schematic of the reduced cross-section method. 1 Initial cross-section; 2 Charring line; 3 Residual cross-section with rounding; 4 Coordinate system; 5 Rectangular residual cross-section; 6 Effective cross-section; 7 Zero-strength layer

an inner, rectangular part of timber cross-section, also named the effective cross-section, is not affected by elevated temperatures and has a strength equal to that at room temperature. The outer part of the cross-section is assumed to be affected by elevated temperatures to such extent that timber has no strength. The thickness of the outer part of timber cross-section with no strength is constant around the perimeter of the initial, rectangular cross-section and is equal to the sum of the charring depth and the thickness of zero-strength layer, i.e., $d_{ch} + d_0$. The charring depth, d_{ch}, is analytically determined from the following equation

$$W_{y,rr} = \frac{(b - 2d_{ch})(h - d_{ch})^2}{6} \tag{5}$$

where $W_{y,rr}$, the section modulus of the residual cross-section with rounding (Fig. 4), is equal to the elastic section modulus of the rectangular, residual cross-section, i.e., right-hand side of the equation. The breadth of the rectangular residual cross-section is $b_r = b - 2d_{ch}$ and the height is $h_r = h - d_{ch}$ where b and h denote the dimensions of the initial rectangular cross-section. The residual cross-section with rounding is determined by means of the HT model where the isotherm 300 °C condition is applied to the numerically obtained temperature field over timber cross-section. It is worth emphasizing that the charring depth d_{ch} is an invented parameter that has no physical meaning and cannot be measured, since the charring depth is realistically not constant around the perimeter as it is greater at the bottom $d_{ch,b}$ than at the sides $d_{ch,s}$ of the cross-section (Frangi and König 2011) as illustrated in Fig. 4. The thickness of zero-strength layer, d_0, as a real, positive number is analytically determined from the following equation

$$M_{Rd,fi} = f_{m,k} \frac{(b_r - 2d_0)(h_r - d_0)^2}{6} \tag{6}$$

where $f_{m,k}$ denotes the characteristic bending strength of timber, and $b_r - 2d_0 = b_{ef}$ and $h_r - d_0 = h_{ef}$ are the breadth and the height of the rectangular effective

cross-section, respectively. The mechanical resistance $M_{\text{Rd,fi}}$ is determined by a mechanical model in the third phase of the analysis. Since $M_{\text{Rd,fi}}$ is determined at t_{end}, the thickness of zero-strength layer corresponds to the maximum value observed in each analysis. For further details see Huč et al. (2021).

Results and discussion

The main result of the present paper is the thickness of zero-strength layer d_0 for the analyzed simply supported timber beam exposed to 46 different FC parametric fires. The obtained thicknesses d_0 take the values between 7.9 and 18.4 mm, their mean value is 12.4 mm and median 12.0 mm. The findings are in agreement with the previously published studies by Lange et al. (2015) and Schmid el al. (2015). A histogram of the thicknesses d_0 is presented in Fig. 5a. It can be seen that most results (24) of the thickness d_0 are between 11 and 15 mm, 14 results are between 7 and 11 mm, and 8 results between 15 and 19 mm. Figure 5b shows a relationship between the thickness d_0 and the charring depth d_{ch} that is an additional thickness for which an initial timber beam cross-section is reduced due to fire exposure according to the reduced cross-section method (EN 1995–1-2 2005). The obtained charring depths d_{ch} take the values between 5.3 and 72.3 mm. The charring depth d_{ch} is in 76% of the analyzed FC parametric fires higher than the corresponding thickness d_0. It can be seen in Fig. 5b that the lower the charring depth d_{ch} the higher the thicknesses d_0. A comparison of the results of FC parametric fires that reach a maximum gas temperature at t_{lim} of 15 min (FC 15), 20 min (FC 20 min) and 25 min (FC 25) reveals that the mean values of thicknesses d_0 at those times are 12.3 mm, 13.0 mm and 12.6 mm, and the corresponding mean values of charring depths d_{ch} are 20.3 mm, 27.1 mm and 36.5 mm. While the mean values of charring depths d_{ch} are higher the longer the time t_{lim}, the same cannot be concluded for the mean values of thicknesses d_0 as they are almost alike and do not depend on time t_{lim}. The latter can be clearly seen in Fig. 5c.

Figure 6a, b and c present the thickness d_0 in relation to the primary parameters of FC parametric fire curve b, O and q_{fd}, respectively. In general, it is seen that the thickness d_0 increases with the increasing parameter b (Fig. 6a). The thicknesses d_0 are the least scattered with respect to the parameter b, while the scatter is more
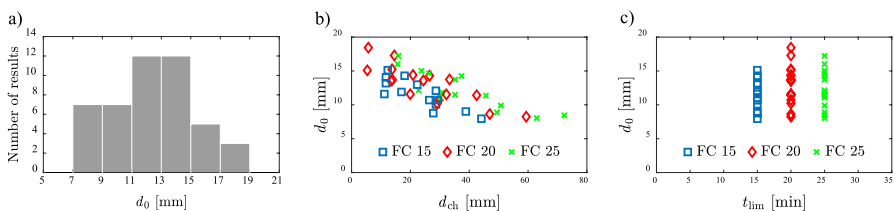

Fig. 5 **a** Histogram of the thicknesses of zero-strength layer d_0. A relationship between the thickness of zero-strength layer d_0 and **b** the charring depth d_{ch} at times t_{lim} 15 min (FC 15), 20 min (FC 20) and 25 min (FC 25), and **c** the primary parameter, time t_{lim}
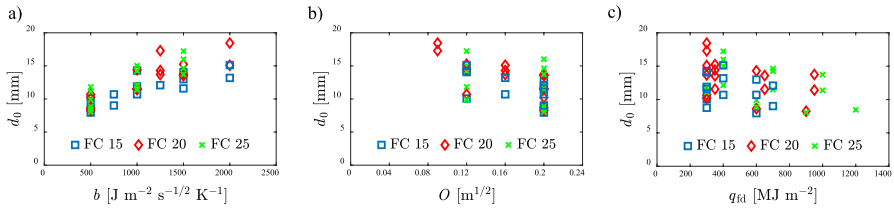
Fig. 6 The thicknesses d_0 plotted against primary parameters of FC parametric fire curve **a** b, **b** O, and **c** q_{fd}
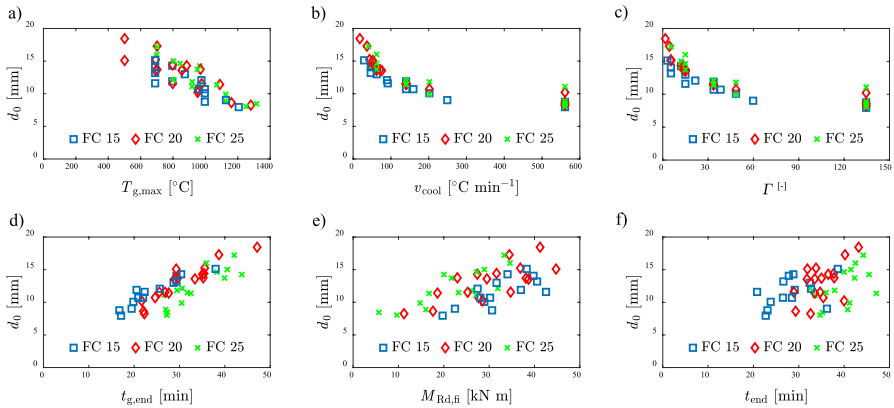

Fig. 7 The thicknesses d_0 plotted against secondary parameters of FC parametric fire curve, **a** $T_{g,max}$, **b** v_{cool}, **c** Γ and **d** $t_{g,end}$, and against results of the mechanical analysis, **e** $M_{Rd,fi}$ and **f** t_{end}

prominent at the parameters O (Fig. 6b) and q_{fd} (Fig. 6c), which implies there is no clear dependence between the thickness d_0 and the parameters O and q_{fd}. On contrary, a dependence of the thickness d_0 on the secondary parameters of the FC parametric fire curve is more evident as presented in Fig. 7. The thickness d_0 decreases with increasing maximum gas temperature $T_{g,max}$ (Fig. 7a), the cooling rate v_{cool} (Fig. 7b) and the increasing parameter Γ (Fig. 7c) that is determined by an empirical expression including the ratio between the parameters O and b. Figure 7d shows that the longer the duration of a FC parametric fire, $t_{g,end}$, the higher the thickness d_0. Apparently, an almost linear relation between the thickness d_0 and the parameters $T_{g,max}$ and $t_{g,end}$ is observed in Fig. 7a and d, respectively, while a more nonlinear relation between the thickness d_0 and the parameters v_{cool} and Γ is observed in Fig. 7b and c, respectively. The time at which a maximum gas temperature is reached, i.e., t_{lim} of 15 min (FC 15), 20 min (FC 20 min) and 25 min (FC 25), does not severely influence the maximum gas temperature itself as $T_{g,max}$ of about 700 °C or 1200 °C is reached in fire scenarios with time t_{lim} of 15 min, 20 min and 25 min (Fig. 7a). Expectedly, the fires with longer times t_{lim} also last longer (Fig. 7d), meaning that on average time $t_{g,end} = 35.0$ min at $t_{lim} = 25$ min is longer than $t_{g,end} = 31.1$ min at $t_{lim} = 20$ min and longer than $t_{g,end} = 26.0$ min

at $t_{\text{lim}} = 15$ min. Figure 7b and c show that 86% of the thicknesses d_0 obtained for FC parametric fires with time $t_{\text{lim}} = 15$ min are concentrated in the lower half of the parameters v_{cool} (< 300 °C min^{-1}) and Γ (< 75), while such thicknesses d_0 is 82% for $t_{\text{lim}} = 20$ min and 73% for $t_{\text{lim}} = 25$ min.

Figure 7e and f present the thicknesses d_0 in relation to the mechanical resistance of the beam in fire conditions, $M_{\text{Rd,fi}}$, and the corresponding time t_{end}, respectively, which are determined in the mechanical analysis. It is seen in Fig. 7e that the thickness d_0 tends to increase with increasing mechanical resistance in fire conditions, $M_{\text{Rd,fi}}$, while simultaneously relatively large scatter of the results is observed. The latter is even more clearly seen in Fig. 7f, therefore, it can be assumed that the thickness d_0 does not depend on time t_{end}, which is quite interesting as this conclusion is different to the conclusion drawn for time $t_{\text{g,end}}$ from Fig. 7d. As already mentioned, it is a requirement of EN 1991–1-2 (2002) that the timber structural elements exposed to fire conditions have to be designed to survive the entire anticipated fire scenario, which means that the analyzed timber beam has to survive not only the duration of prescribed FC fire scenarios, i.e., [0, $t_{\text{g,end}}$], but it should survive until time t_{end}. The performed analysis showed that time t_{end} is longer than time $t_{\text{g,end}}$ in 87% of the analyzed fire scenarios. It is on average 5 min longer. However, it can also be seen in Fig. 7f that the fires with longer times t_{lim} have also longer times t_{end}, which is similar to conclusions drawn from Fig. 7d. In general, time t_{lim} influences the parameter $M_{\text{Rd,fi}}$ (Fig. 7e), since the latter is on average approx. 6 kN m higher at each increase of time t_{lim} for 5 min.

a. Proposal of *equations* for determination of thickness d_0 *in case of FC and VC parametric fire exposures*

It is obvious in Fig. 5 to Fig. 7 that the thicknesses d_0 for all analyzed FC parametric fires are higher than 7 mm, which is the value prescribed for the standard fire exposure in EN 1995–1-2 (2005). That implies that it is not appropriate to assume the thickness d_0 of 7 mm in case of FC parametric fire exposures. If the thickness d_0 is to be a single value, we suggest based on the presented results that a 95th percentile value is assumed, i.e., $d_0 = 17.2$ mm, to be on a safe side in the design of fire resistance of a timber beam exposed to a FC parametric fire from three sides. Similar suggestion can also be made for a timber beam exposed to a VC parametric fire where a 95th percentile value is equal to $d_0 = 29.9$ mm based on the previously published results (Huč et al. 2021). If the thickness d_0 is to be a function of one parameter, we suggest that it is expressed in dependence on one of the secondary parameters, i.e., $T_{\text{g,max}}$, v_{cool}, Γ or $t_{\text{g,end}}$, since these parameters the most clearly indicate dependence on the thickness d_0 among the results presented in Fig. 5 to Fig. 7. After combining the results of FC parametric fire exposures presented in Fig. 7a–d and the results of VC parametric fire exposures presented by Huč et al. (2021), the following equations for determination of the thickness d_0 are proposed

$$d_0 = -0.012 T_{\text{g,max}} + 25.15 \quad \text{for FC parametric fire exposure} \tag{7}$$

$$d_0 = -0.033 T_{g.max} + 56.10 \quad \text{for VC parametric fire exposure} \tag{8}$$

$$d_0 = 63.5 v_{cool}{}^{-0.5} + 7.2 \quad \text{for FC and VC parametric fire exposures} \tag{9}$$

$$d_0 = 27.2 \Gamma^{-0.28} + 3.0 \quad \text{for FC and VC parametric fire exposures} \tag{10}$$

$$d_0 = -0.00065 t_{g,end}{}^2 + 0.235 t_{g,end} + 9.0 \quad \text{for FC and VC parametric fire exposures} \tag{11}$$

Equations (7)–(11) are obtained in a way that at maximum 5% of the thicknesses d_0, i.e., less than 5 results for FC and VC parametric fire exposures are exceeding the proposed fitted curve on an unsafe side as can be seen in Fig. 8. It is seen in Fig. 8a that the thicknesses d_0 are substantially higher for VC than for FC parametric fires in the range of maximum gas temperatures between 800 and 1200 °C. The difference in magnitude of the thicknesses d_0 at VC and FC fire exposures decreases with increasing temperature $T_{g,max}$ until the thickness d_0 reaches the value of about 10 mm for both fire exposures at approx. $T_{g,max} = 1350$°C. Therefore, two separate, linear Eqs. (7) and (8) are proposed to determine the thickness d_0 in dependence on the parameter $T_{g,max}$ in case of FC and VC parametric fire exposures, respectively. Since the thicknesses d_0 show good alignment with respect to the parameters v_{cool},
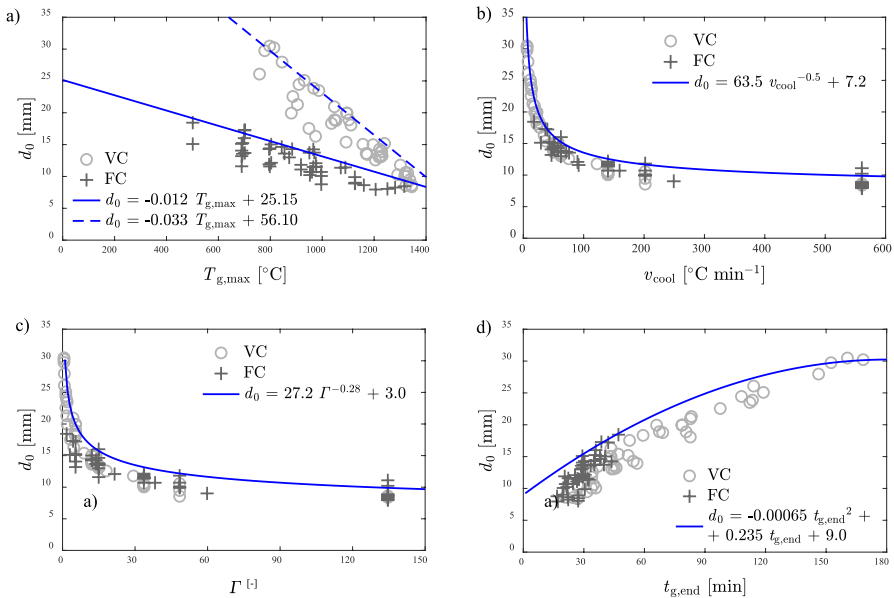

Fig. 8 Expressions for the thickness d_0 depending on parameters, **a** $T_{g,max}$, **b** v_{cool}, **c** Γ and **d** $t_{g,end}$ of FC and VC parametric fire exposures

Γ and $t_{g,end}$ for VC and FC parametric fires (Fig. 8b, c and d), a single equation is proposed to determine thickness d_0 in dependence on each of those parameters, i.e., Eqs. (9), (10) and (11). The goodness-of-fit of the proposed equations is assessed by the R^2 measure, which takes the following values: 0.147 for Eq. (7) (Fig. 8a), 0.744 for Eq. (8) (Fig. 8a), 0.886 for Eq. (9) (Fig. 8b), 0.672 for Eq. (10) (Fig. 8c) and 0.492 for Eq. (11) (Fig. 8d). These results indicate that the thickness d_0 is the most accurately determined by considering its dependence on the cooling rate v_{cool} (Eq. (9)). Although, Eqs. (7), (8), (10) and (11) give worse fits, their application can still be safe if one of the parameters $T_{g,max}$, Γ or $t_{g,end}$ better suits the fire engineer to determine the thickness d_0.

It is also worth mentioning that a simplified approach for determination of the effective cross-section of timber beam exposed to VC parametric fire, which is proposed by Huč et al. (2021), can be used in case of FC parametric fire exposure as well. The simplified approach suggests that the effective cross-section is determined by reading combined thickness $d_{ch} + d_0$ at temperature of a timber beam cross-section of $T_{0.05} = 89.5°C$. In case of FC parametric fire exposure combined thickness $d_{ch} + d_0$ should be determined at temperature $T_{0.05} = 86.6°C$ and time t_{end}. The same linear equation as given by Huč et al. (2021) can be used to determine time t_{end} depending on time $t_{g,end}$ for VC and FC parametric fire exposures.

Conclusion

The paper presents the results of comprehensive numerical analyses performed to determine the thickness of zero-strength layer d_0 of a timber beam exposed to 46 different FC parametric fires from three sides. The thickness d_0 is an essential parameter of the reduced cross-section method given in the European standards Eurocodes. Since the latter only proposes a value of $d_0 = 7mm$ for standard fire exposure, there is a need to determine if the same value is suitable for use also for parametric fire exposures. It is found out in the present paper that the thickness d_0 for FC parametric fire exposures takes the values between 7.9 and 18.4 mm. If a constant value of the thickness d_0 is to be applied, it is suggested to consider a 95th percentile value that is $d_0 = 17.2$ mm to be on a safe side in the design of fire resistance of a timber beam exposed to FC parametric fire from three sides. Additionally, it is found out that the results of the performed analyses show no clear dependence of the thickness d_0 on the primary parameters describing FC parametric fire curve, i.e., b, O, and q_{fd}. The same can also be concluded for the outcome of mechanical analysis, i.e., parameters $M_{Rd,fi}$ and t_{end}. Since a more apparent dependence of thickness d_0 on the secondary parameters describing FC parametric fire curve, i.e., $T_{g,max}$, v_{cool}, Γ and $t_{g,end}$ is observed, five equations for determination of thickness d_0 with respect to the aforementioned parameters are proposed for a timber beam exposed to parametric fires from three sides. The proposed equations consider the results of the present paper for FC parametric fire exposures as well as the results for VC parametric fire exposures. A non-linear equation is found out to be the most suitable for determining the thickness d_0 in dependence on the cooling rate v_{cool}. Two different linear equations

are proposed to determine the thickness d_0 in dependence on the maximum gas temperature $T_{g,max}$, since obtained thicknesses d_0 are substantially higher for VC parametric fires than for FC parametric fires. To determine thicknesses d_0 in dependence on parameters Γ and $t_{g,end}$ two non-linear equations are also given. The present paper thus gives two options that may be considered when determining the thickness d_0 by applying the reduced cross-section method in case of parametric fire exposures. The first option is to take a constant value of thickness d_0 of 17.2 mm for FC and of 29.9 mm for FC parametric fire exposures. The second option is to consider more exact value of thickness d_0 by applying one of the five proposed equations depending on one of the parameters $T_{g,max}$, v_{cool}, Γ or $t_{g,end}$. A designer of fire resistance of timber beam exposed to fire from three sides can make his own decision of which equation to use based on the available data. At this point, we would like to point out that the above-mentioned findings are valid only for pure bending applications. To extend the applicability of the presented results in the future, additional analyses should be made by studying the influence of cross-section dimensions, timber strength, density and loading modes on the thickness d_0. It would also be interesting to find out if the proposed equations are suitable for determining the thickness d_0 in case of timber beam exposed to natural fire curves that are either different parametric curves or fire curves obtained by advanced numerical models, such as zone or CFD models.

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Declarations

Conflict of interest The authors declare no competing interests.

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