

Erratum

The inverse problem for the Hill operator, a direct approach

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The proofs of the main theorems were based on Theorem 5.1 about nonlinear functional analysis. In this abstract theorem we claim that the mapping $f : H \rightarrow H_1$ is an isomorphism. Unfortunately, this theorem is not correct and we have to add the following assumption

Condition v). There exists a linear isomorphism $J : H \rightarrow H_1$ such that $(f(V) - JV, e_n)_1 = O(1/n)$, as $n \rightarrow \infty$ uniformly on bounded subsets of H .

Additional part in the proof of Theorem 5.1. We must show that the topology on M_m , generated by the norm $\| \cdot \|$, coincides with the topology on M_m induced by the weak topology of H , that is, weak convergence implies strong convergence.

First, we prove the compactness of the mapping $F \equiv f - J$. Indeed, each component $F_n(\cdot) \equiv (F(\cdot), e_n)$ is compact since $(f(\cdot), e_n)$ and $(J\cdot, e_n)$ are compact. Then using Condition v) we deduce that $F : H \rightarrow H_1$ is compact. Second, assume that $V_p \rightarrow V_0$ weakly, as $p \rightarrow \infty$ and $V_p \in M_m$, $f(V_p) \in K_m$, $p \geq 1$. Then by Condition iv), $y_p \equiv f(V_p) \rightarrow y_0 = f(V_0)$ strongly. We have $y_p = JV_p + F(V_p)$ and hence $V_p = J^{-1}(y_p - F(V_p)) \rightarrow V_0$ strongly. \square

The main results remain true since, in fact, we checked Condition v) in our paper. Indeed, relations (3.38), (3.20), (3.30), (3.25), (3.10–11) show that the mappings h, l, μ, M, L satisfy Condition v).

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