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CORRECTION

Correction to: Representation growth and rational singularities of the moduli space of local systems

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Abstract We explain and correct a mistake in Section 2.6 and Appendix C of the first and second author's paper "Representation Growth and Rational Singularities of the Moduli Space of Local Systems" [1].

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We use throughout the notation and conventions of [1]. The source of the mistake is the description of the set S_2 in page 272. The elements

$$(\{(d, d-1), (d-1, d-1)\}, (d, d-1)), \ \ (\{(d-1, d-1), (d-1, d)\}, (d-1, d)) \tag{1}$$

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are not considered. This spoils the proof of Lemma 2.40 (the blue and green subgraphs are no longer trees), which is used to prove Theorem 2.1.

1 A straightforward correction

The right description of S_2 is

$$S_2 = \left\{(\{(i,j),(j,l)\},(i,l)) \in I^{(2)} \times J \mid \{i,l\} = \{d-1,d\} \text{ and } j = \left\lfloor \frac{i+l}{2} \right\rfloor, \right.$$
$$\left. \text{or } \{i,l\} \neq \{d-1,d\} \text{ and } j = \left\lceil \frac{i+l}{2} \right\rceil + \delta_{i,l} \right\}.$$

Then, Γ_2 is the polygraph attached to the graph $\Gamma_3 = (I, E)$, with

$$E = \left\{\{(i,j),(j,l)\} \in I^{(2)} \mid \{i,l\} = \{d-1,d\} \text{ and } j = \left\lfloor \frac{i+l}{2} \right\rfloor, \right.$$
$$\left. \text{or } \{i,l\} \neq \{d-1,d\} \text{ and } j = \left\lceil \frac{i+l}{2} \right\rceil + \delta_{i,l} \right\}.$$

As for Fig. 1, 2 and 3 in [1], the two edges corresponding to (1) are missing.

Additionally, there are some mismatches concerning Fig. 2 and ω_3 in [1]. The simplest way to make the labels of Fig. 2 match is fist to multiply ω_3 in page 273 by 5, obtaining

$$w_3((i,j))(m) = \begin{cases} 5^{|i-j|+1} & \text{if } m \equiv i-j \pmod 3 \\ 3 \cdot 5^{|i-j|} & \text{if } m \equiv (i-j-sign(i-j+1/2)) \pmod 3 \\ 0 & \text{if } m \equiv (i-j+sign(i-j+1/2)) \pmod 3, \end{cases} \tag{2}$$

and then consider the colouring [red is $m=0$, green is $m=2$, blue is $m=1$], so that we just have to swap the values of the blue and green labels along the diagonal of Fig. 2 in page 314.

Finally, the key to prove now Lemma 2.40 in the closest way to that of [1] is redefining ω_3 at the nodes $(d-1,d-1)$, $(d-1,d)$ and $(d,d-1)$:

$$\begin{array}{lll} \omega_3(d-1,d-1)(0) = 3\cdot 5^1, & \omega_3(d-1,d)(0) = 5^2, & \omega_3(d,d-1)(0) = 3\cdot 5^0, \\ \omega_3(d-1,d-1)(2) = 3\cdot 5^1, & \omega_3(d-1,d)(2) = 4\cdot 5^1, & \omega_3(d,d-1)(2) = 0, \\ \omega_3(d-1,d-1)(1) = 0, & \omega_3(d-1,d)(1) = 0, & \omega_3(d,d-1)(1) = 5^2, \end{array}$$

so that the resulting diagram for $d = 6$ (Fig. 2 in [1]) is given by Image 1.

With the redefinition of ω_3 above, we only need to add to the proof of Lemma 2.40 in [1] an analysis of the edges around $(d-1,d-1)$. They look,

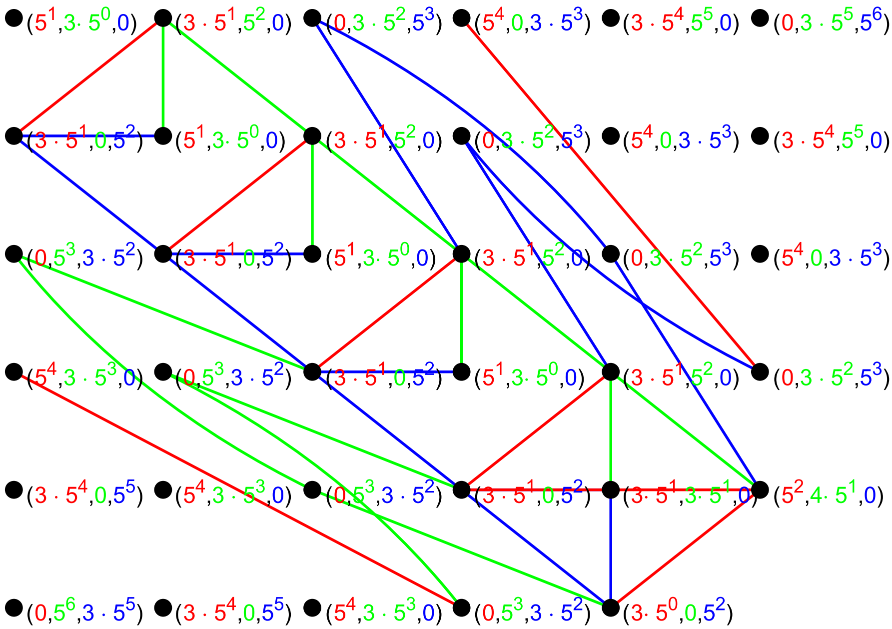

Image 1 Graph Γ_3 with the weights ω_3 for the case $d = 6$

for $d \geq 3$, like the ones in Image 1. We thus have forests with maximal degree ≤ 3, as we need. Finally, the cases $d = 1, 2$ are straigthforward.

2 An alternative solution

We indicate here how to get an alternative solution that, although requires more changes, would keep better the original intuition for the proof.

At the beginning of Section 2.6 in [1], recall that $L = \{1, \ldots, d\}$. Stay with $J = L \times L \backslash \{(d, d)\}$ and replace I by $I = L \times L \backslash \{(1, 1)\}$. This entails changes in S_j and Γ_j, which we omit here for the sake of brevity. With the definition of ω_3 as in (2) (without any redefinition) and the same conventions for the colours as described in Sect. 1, the corresponding Γ_3 for $d = 6$ is given by Image 2.

The general proof then follows along the same lines as the original one.

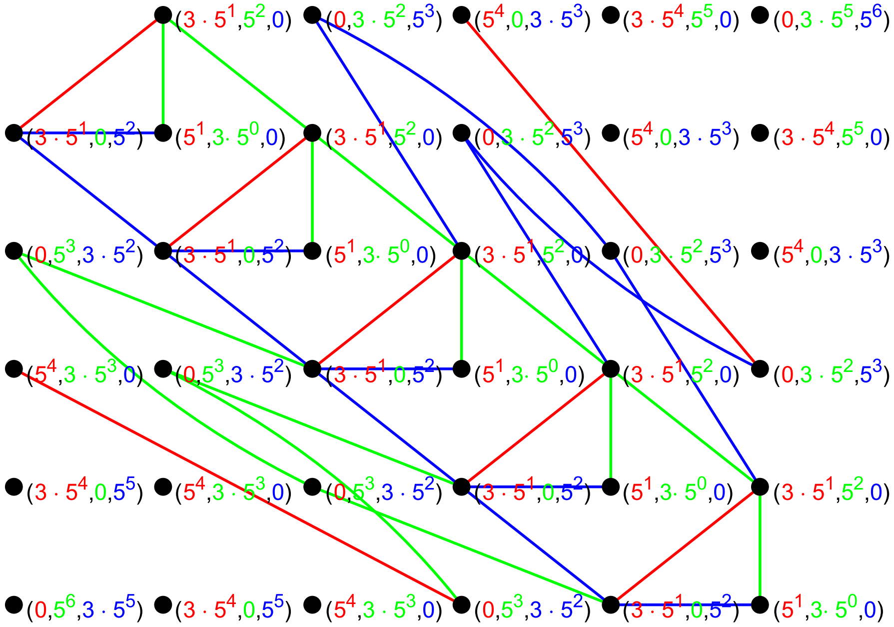

Image 2 Graph Γ_3 with $I \neq J$ for the case $d = 6$

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References

1. Aizenbud, A., Avni, N.: Representation growth and rational singularities of the moduli space of local systems. Invent. Math. **204**(1), 245–316 (2016)