Correction to: Symmetric quiver Hecke algebras and R-matrices of quantum affine algebras

Seok-Jin Kang¹ · Masaki Kashiwara^{2,3} ·
Myungho Kim⁴

Published online: 31 January 2019
© Springer-Verlag GmbH Germany, part of Springer Nature 2019

Correction to: *Invent. math.* (2018) 211:591–685
<https://doi.org/10.1007/s00222-017-0754-0>

Abstract We fix a mistake in [1] on the definition of commuting family of central objects.

Mathematics Subject Classification 81R50, 16G, 16T25, 17B37

We will freely make use of the notations from [1]. Let $\mathcal{T}$ be a tensor category, and let $\{P_i\}_{i \in I}$ be a family of objects in $\mathcal{T}$. In the beginning of Appendix A.6,

The original article can be found online at <https://doi.org/10.1007/s00222-017-0754-0>.

✉ Myungho Kim
mkim@khu.ac.kr

Seok-Jin Kang
sjkang@uaeu.ac.ae

Masaki Kashiwara
masaki@kurims.kyoto-u.ac.jp

¹ Department of Mathematical Sciences, United Arab Emirates University, P. O. Box 15551, Al Ain, Abu Dhabi, United Arab Emirates

² Research Institute for Mathematical Sciences, Kyoto University, Kyoto 606-8502, Japan

³ Korea Institute for Advanced Study, Seoul 02455, Korea

⁴ Department of Mathematics, Kyung Hee University, Seoul 02447, Korea

we say that $\{(P_i, R_{P_i})_{i \in I}\}$ is a *commuting family of central objects* if it satisfies two conditions

(a) for any $i \in I$, R_{P_i} is an isomorphism

$$R_{P_i}(X): P_i \otimes X \xrightarrow{\sim} X \otimes P_i$$

functorial in $X \in \mathcal{T}$ such that

$$P_i \otimes X \otimes Y \xrightarrow{R_{P_i}(X)} X \otimes P_i \otimes Y \xrightarrow{R_{P_i}(Y)} X \otimes Y \otimes P_i \xrightarrow{R_{P_i}(X \otimes Y)}$$

commutes for any $X, Y \in \mathcal{T}$,

(b) $R_{P_j}(P_i) \circ R_{P_i}(P_j) = \text{id}_{P_i \otimes P_j}$ for any $i, j \in I$ such that $i \neq j$.

If it further satisfies the condition

$$R_{P_i}(P_i) = \text{id}_{P_i \otimes P_i} \text{ for any } i \in I, \quad (\text{A.4})$$

then it was called a *strictly commuting family of central objects*.

In the next pages, we asserted that if $\{(P_i, R_{P_i})_{i \in I}\}$ is a commuting family of central objects, then the localization $\tilde{\mathcal{T}} = \mathcal{T}[P_i^{\otimes -1} \mid i \in I]$ of $\mathcal{T}$ becomes a tensor category with a unit object $(\mathbf{1}, \vec{0})$. This is incorrect since the following gives a counterexample: Let $\mathcal{T}$ be the category of vector spaces over a field $\mathbf{k}$ and let $P = \mathbf{k}^{\oplus 2}$. Let $R_P(X): P \otimes X \rightarrow X \otimes P$ be the map given by $v \otimes x \mapsto x \otimes v$ for $v \in P$ and $x \in X$. Then $\{(P, R_P)\}$ satisfy the conditions (a) and (b). Since the endomorphism ring

$$\begin{aligned} \text{End}_{\tilde{\mathcal{T}}}(\mathbf{1}, 0) &= \varinjlim_{\gamma \in \mathbb{Z}_{\geq 0}} \text{End}_{\mathcal{T}}(\mathbf{1} \otimes P^\gamma) \\ &\simeq \varinjlim_{\gamma \in \mathbb{Z}_{\geq 0}} \text{End}_{\mathbf{k}}(P^\gamma) \simeq \varinjlim_{\gamma \in \mathbb{Z}_{\geq 0}} \text{Mat}_{2^\gamma \times 2^\gamma}(\mathbf{k}) \end{aligned}$$

is not commutative, the object $(\mathbf{1}, 0)$ cannot be a unit object of a tensor category ([2, Exercise 4.11 (ii)]).

The problem occurs since we do not assume that the family satisfy condition (A.4). As the simplest remedy, we include condition (A.4) in the definition of commuting family of central objects from the beginning and remove the definition of strictly commuting family of central objects in the paper. Under the new definition of commuting family of central objects, we can construct the category $\mathcal{T}[P_i^{\otimes -1} \mid i \in I]$ as well as the category $\mathcal{T}[P_i \simeq \mathbf{1} \mid i \in I]$ in the graded case (Appendix A.7). All the arguments in the main body of our paper also work without any problem.

References

1. Kang, S.-J., Kashiwara, M., Kim, M.: Symmetric quiver Hecke algebras and R -matrices of quantum affine algebras. *Invent. Math.* **211**, 591–685 (2018)
2. Kashiwara, M., Schapira, P.: *Categories and Sheaves*, Grundlehren der mathematischen Wissenschaften 332. Springer, Berlin (2006)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.