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Erratum

Global uniqueness for an inverse boundary value problem arising in elasticity

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The proof of Theorem 0.7 in [NU1] is incorrect. Using the same method of proof in [NU1] we can show Theorem 1, below. Unfortunately, we have not been able to prove the global result stated in [NU1].

We now state the corrected result. Let Ω be a bounded domain in $\boldsymbol{R}^3$ with smooth boundary $\partial\Omega$ and let

$$\begin{aligned} Lu &:= (\lambda+\mu)\nabla(\nabla\cdot u) + \mu\Delta u + (\nabla\cdot u)\nabla\lambda + (\nabla u + {}^t(\nabla u))\nabla\mu \\ &= 0 \ \text{ in } \ \Omega \end{aligned} \tag{1}$$

be the isotropic elasticity system with Lamé moduli $\lambda,\ \mu \in C^\infty(\overline{\Omega})$ satisfying the strong convexity condition

$$\mu > 0, \quad 3\lambda + 2\mu > 0 \quad \text{on} \quad \overline{\Omega}. \tag{2}$$

Define the Dirichlet to Neumann map $\Lambda_{\lambda,\mu} : C^\infty(\overline{\Omega}) \longrightarrow C^\infty(\overline{\Omega})$ by

$$\Lambda_{\lambda,\mu} f := \sigma(u(f))\nu|_{\partial\Omega}, \tag{3}$$

where $u = u(f) \in C^\infty(\overline{\Omega})$ is the solution to

$$\begin{cases} Lu = 0 & \text{in} \quad \Omega \\ u|_{\partial\Omega} = f, \end{cases} \tag{4}$$

where ν is the outer unit normal vector of $\partial\Omega$ and $\sigma(u)$ is the stress tensor given by

$$\sigma(u) := \lambda(\text{trace}\nabla u)I + 2\mu\varepsilon(u), \tag{5}$$

with strain tensor

$$\varepsilon(u) := \frac{1}{2}(\nabla u + {}^t(\nabla u)) \tag{6}$$

Theorem 0.7 in [NU1] holds under additional assumption $\|\nabla\mu_i\|_{C^m(\overline{\Omega})} < \varepsilon$ $(i = 1, 2)$ with some $0 < \varepsilon \ll 1$ and $m \in \boldsymbol{N}$. That is we have the following theorem.

Theorem 1 *Let $\lambda_i,\ \mu_i \in C^\infty(\overline{\Omega})$ $(i = 1, 2)$ be the Lamé moduli satisfying the strong convexity condition. Then, there exist $\varepsilon > 0$ and $m \in \boldsymbol{N}$ such that if $\|\nabla\mu_i\|_{C^m(\overline{\Omega})} < \varepsilon$ $(i = 1, 2)$ and $\Lambda_{\lambda_1,\mu_1} = \Lambda_{\lambda_2,\mu_2}$, we have $\lambda_1 = \lambda_2$, $\mu_1 = \mu_2$ on $\overline{\Omega}$.*

The proof of Theorem 1 follows the general outline of the paper [NU1]. The full details are in [NU2]. Namely we first reduce the second order system of isotropic elasticity to a first order system perturbation of the Laplacian. It is more convenient, as already indicated in [U], to use the reduction of [C] and [An] rather than the one used in [NU1].

The key step in the construction of the exponentially growing solutions (also called complex geometrical optics solutions) is the intertwining property, Theorem 1.23 of [NU1]. The proof of this result goes through with some modifications. See [NU3] for the full details. The main problem in Lemma 1.35 in [NU1] is that we cannot solve in general the initial value problem for the first order system

$$H_{q_\zeta}\big(A^{(0)}_{\zeta,2}\big) + \psi_1\big(s^{-1}\xi_1\big)\psi_2(s^{-1}\xi')\sigma\big(N^{(0)}_\zeta\big)\big(A^{(0)}_{\zeta,2}\big) = 0. \tag{7}$$

We can just solve (1) with $(A^{(0)}_{\zeta,2})$ invertible for large ζ. We use throughout the notation of [NU1]. The method of proof proceeds as in [NU1] by reducing (7) to solve a system of the form

$$\overline{\partial}A + NA = 0 \text{ in } \boldsymbol{R}^2 \tag{8}$$

depending on parameters. This is straightforward to solve for scalar equations since this is a particular case of a pseudoanalytic equation and all the solutions of (8) can be written in the form of a product of a non-zero function and an holomorphic function. The case of systems is more complicated. Recently Eskin [E] proved that we can find solutions of (8) with A invertible for general systems. We gave an alternative proof of the existence of solutions of (8) in [NU3].

When replacing the exponentially growing solutions constructed in the identity (0.10) of [NU1] we get a pseudodifferential equation rather than a PDE acting on the difference of the Lamé parameters as claimed in [NU1]. We thank G. Eskin and J. Ralston for pointing this to us. We can conclude that we can uniquely identify the Lamé parameters if μ is a-priori close to a constant.

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