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Erratum

Localization Regions of Local Observables

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It was kindly pointed out to the author by S. J. Summers that the proof of the lemma presented in the appendix of [3] contains an error. The assumption that the bounded operator A used in the proof is a local observable is redundant, and the argument in the last three lines of the proof, which uses this assumption, does not complete the proof.

Instead, if A is taken to be any element of $\mathcal{C}'_{\mathcal{O},a}$, then the original argument proves correctly that $A\Omega = \alpha\Omega$ for some $\alpha \in \mathbb{C}$. As Ω is cyclic with respect to $\mathcal{C}_{\mathcal{O},a}$ by the Reeh–Schlieder property, it is separating with respect to $\mathcal{C}'_{\mathcal{O},a}$, so one obtains $A = \alpha$, which proves $\mathcal{C}'_{\mathcal{O},a} = \mathbb{C}\mathbb{1}$ and, hence, $\mathcal{C}''_{\mathcal{O},a} = \mathcal{B}(\mathcal{H})$, as stated.

While this is all to be rectified, we recall the lemma and its proof here in a self-contained fashion for the reader's convenience, as it appears to be less well known than expected.

As in [3], let $\mathfrak{A}$ be a 1+s-dimensional local net of local observables on a Hilbert space $\mathcal{H}$, and assume $\mathfrak{A}$ to be in an irreducible vacuum representation, which is expressed by spacetime translation covariance, the spectrum condition, and the existence of an (up to a phase) unique translation invariant cyclic unit vector Ω.

The *Reeh–Schlieder property* that $\mathfrak{A}(\mathcal{O})\Omega$ is dense in $\mathcal{H}$ is well known to hold for the vacuum state of a Wightman field [5]. For the present setting, a sufficient condition for the Reeh–Schlieder property is *weak additivity* [1]: if $\mathcal{O}$ is any bounded open region, then

$$\left(\bigcup_{a\in\mathbb{R}^{1+s}} \mathfrak{A}(\mathcal{O}+a)\right)'' = \mathfrak{A}''_{\mathrm{loc}}.$$

Conversely, weak additivity can be derived from the Reeh–Schlieder property as well, provided the net is, as assumed above, in an irreducible vacuum representation. This is not really new (cf. Thm. 4 in [4] and Lemma 2.6 in [6]), but as there existed no fully self-contained reference, the following lemma was included in [3] for the reader's convenience.

Lemma. *Under the above assumptions, let $\mathfrak{A}$ satisfy the Reeh–Schlieder property, let $\mathcal{O} \subset \mathbb{R}^{1+s}$ be a bounded open region, and let $a \in \mathbb{R}^{1+s}$ be some timelike vector. Then*

$$\mathcal{C}_{\mathcal{O},a} := \left(\bigcup_{t\in\mathbb{R}} \mathfrak{A}(\mathcal{O} + ta) \right)'' = \mathcal{B}(\mathcal{H}).$$

Proof. For any $A \in \mathcal{C}'_{\mathcal{O},a}$ and $B \in \mathfrak{A}(\mathcal{O})$, define $f_+(t) := \langle \Omega, A^* U(ta) B \Omega \rangle$ and $f_-(t) := \langle \Omega, BU(-ta)A^*\Omega \rangle$. By the spectral theorem and the spectrum condition, the Fourier transforms of these functions are (not necessarily positive, but bounded) measures one of which has its support in the closed positive half axis, while the other one has its support in the closed negative half axis. Since f_+ and f_- coincide by construction, it follows that the Fourier transform of f_+ is a measure with support $\{0\}$, i.e., some multiple of the Dirac measure, so that f_+ is a constant function. Using this, the spectral theorem, and the uniqueness of the vacuum, one concludes

$$\langle A\Omega, B\Omega \rangle = f_+(0) = f_+(t) = \langle \Omega, A\Omega \rangle \langle \Omega, B\Omega \rangle =: \overline{\alpha} \langle \Omega, B\Omega \rangle = \langle \alpha\Omega, B\Omega \rangle.$$

The Reeh-Schlieder property implies that $A\Omega = \alpha\Omega$ (since $B \in \mathfrak{A}(\mathcal{O})$ is arbitrary) and that Ω is cyclic with respect to $\mathcal{C}_{\mathcal{O},a}$ or, equivalently, separating with respect to $\mathcal{C}'_{\mathcal{O},a}$. As $A \in \mathcal{C}'_{\mathcal{O},a}$, it follows that $A = \alpha \mathbb{1}$, and as $A \in \mathcal{C}'_{\mathcal{O},a}$ is arbitrary, one finds $\mathcal{C}'_{\mathcal{O},a} = \mathbb{C}\mathbb{1}$, and $\mathcal{C}_{\mathcal{O},a} = \mathcal{C}''_{\mathcal{O},a} = \mathcal{B}(\mathcal{H})$, as stated. □

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