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Correction

Correction to: Exponentially Small Splitting of Separatrices Associated to 3D Whiskered Tori with Cubic Frequencies

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We correct an error in Sect. 2.1 of [DGG20]. Just before Proposition 4, it is claimed that one can choose a matrix $T = T(\omega) \in \mathrm{SL}(3, \mathbb{Z})$ whose real eigenvalue $\lambda = \lambda(\omega) > 1$ is minimal *or, equivalently, the norm* $|T|$ *is minimal*, and this matrix T is called "the principal Koch's matrix for ω" (we denote $|\cdot| = |\cdot|_2$ the matrix norm subordinate to the Euclidean norm for vectors). This equivalence is not true, and the matrix $T(\omega)$ should be defined simply as the one whose real eigenvalue $\lambda = \lambda(\omega) > 1$ is minimal.

The following example shows that the principal Koch's matrix is not necessarily the Koch's matrix whose norm is minimal. Let us consider the frequency vector $\omega = (1, \Omega, \Omega^2)$ where Ω is the cubic golden number: the real root of $\Omega^3 = 1 - \Omega$ (see Sect. 2.3). The principal Koch matrix for this vector is $T(\omega) = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ with the real eigenvalue $\lambda = 1 + \Omega^2 = 1/\Omega$ (see (68)). Now consider the vector $\widetilde{\omega} = (1,\ \Omega^2,\ -1 + \Omega + \Omega^2)$. We have the relation $\widetilde{\omega} = S\omega$, where the matrix $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ is unimodular. It is clear that the principal Koch's matrix for this new vector is $T(\widetilde{\omega}) = S\,T(\omega)\,S^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -2 & 1 \end{pmatrix}$, with the same eigenvalue $\lambda(\widetilde{\omega}) = \lambda(\omega)$. But its norm $|T(\widetilde{\omega})| \approx 2.978400$ is not the minimal one, since $T(\widetilde{\omega})^2$ is also a Koch's matrix, with smaller norm $\left|T(\widetilde{\omega})^2\right| \approx 2.457837$.

Hence, the paragraph previous to Proposition 4 should be rewritten as follows:

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Using this lemma, we next show the "uniqueness" of the matrix T satisfying Koch's result. More precisely, we can choose $T = T(\omega) \in \mathrm{SL}(3, \mathbb{Z})$ whose real eigenvalue $\lambda = \lambda(\omega) > 1$ is minimal, and we call this matrix "*the principal Koch's matrix for* ω". This matrix T is not necessarily the one of minimal norm among the Koch's matrices for ω.

Although the validity of the results of the paper is not affected by this mistake, the algorithm for determining the principal Koch's matrix in a concrete case, described between Lemma 5 and Remark 6, is no longer valid since it relies in finding the Koch's matrix of minimal norm $|T|$. Alternatively, to find the Koch's matrix with minimal real eigenvalue λ, we should reformulate this algorithm in the following way:

Now, in order to determine the principal Koch's matrix for ω we can carry out the following simple exploration. We consider the (integer) entries of the first row $T_{(1)}$ as successive data, say with increasing norm $\left|T_{(1)}\right|$, until the whole matrix T determined from Lemma 5 belongs to $\mathrm{SL}(3, \mathbb{Z})$ (i.e. it has integer entries and determinant 1) and has an eigenvalue $\lambda > 1$ in (30). By Koch's result, we know that such a matrix exists and will be reached after a finite exploration. It remains to check whether the matrix T^*, with eigenvalue λ^*, obtained in this way is the principal Koch's matrix for ω since, in principle, there could exist another Koch's matrix T with $\left|T_{(1)}\right| \geq \left|T^*_{(1)}\right|$ but with eigenvalue $\lambda < \lambda^*$. If this happens, such a new matrix T would satisfy $\left|T_{(1)}\right| < |T| \leq \kappa\lambda < \kappa\lambda^*$, where $\kappa = \kappa(\omega)$ is the condition number introduced in (33), and the inequality $|T| \leq \kappa\lambda$ appears in the proof of Lemma 3. Hence, after obtaining a first matrix T^*, it is enough to continue the exploration with increasing norms $\left|T_{(1)}\right|$ up to the value $\kappa\lambda^*$ and, if a new Koch's matrix T is obtained, check if its real eigenvalue λ is lower than λ^*, which would imply that the matrix T has to replace T^* as the principal one.

Declaration

Conflict of interest All authors declare that they have no conflicts of interest related to the work in this manuscript.

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Reference

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