Correction

Correction to: Decay of Determinantal and Pfaffian Correlation Functionals in One-Dimensional Lattices

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On the occasion of the extension of part of our results to complex times and higher dimensions [2], J.-B. Bru and W. de Siqueira Pedra made us aware of a missing assumption in Theorems 1.1 and 1.2 of [1]. We correct both statements below. Since neither Theorems 1.1 nor 1.2 have further application in [1], this does not affect other results concerning Pfaffian bounds and the time-dependent correlations of the XY spin chain.

The proof of the following statements is included in the original paper. What was missing there is the assumption on the norm bound of the involved matrices $M(s, t)$.

Theorem 1. Let $I \subseteq \mathbb{R}$, $s, t \in I$, and $\rho(s, t)$ be a family of operators on $\ell^2(\mathbb{Z})$. Let $n \in \mathbb{N}$, $x = (x_1, \dots, x_n)$, $y = (y_1, \dots, y_n) \in \mathbb{Z}^n$ be any pair of fermionic configurations, and $s, t \in I^n$. Suppose that matrix

$$M(s, t) := (\langle \delta_{x_j}, \rho(s_j, t_k) \delta_{y_k} \rangle)_{1 \leq j, k \leq n} \quad (1)$$

satisfies:

1. the norm bound $\|M(s, t)\| \leq 1$,
2. an exponential bound on its entries, i.e., for some $C \in (0, \infty)$, $\mu \in (\mu_0, \infty)$ and all $j, k \in \{1, \dots, n\}$:

$$|\langle \delta_{x_j}, \rho(s_j, t_k) \delta_{y_k} \rangle| \leq C e^{-\mu K(|x_j - y_k|)}. \quad (2)$$

Then:

$$|\det M(s, t)| \leq 8 \max\{CI(\mu_0), \sqrt{CI(\mu_0)}\} \exp\left(-\frac{\mu - \mu_0}{2} K\left(\frac{D(x, y)}{2}\right)\right). \quad (3)$$

Let us add that for a family of uniformly bounded operators $\rho(s, t)$ with $\sup_{s, t \in I} \|\rho(s, t)\| \leq 1$, the norm bound $\|M(s_0, t_0)\| \leq 1$ is true for all $n \in \mathbb{N}$ in the special case of constant time vectors $s_0 = (s, \dots, s)$ and $t_0 = (t, \dots, t)$ with $s, t \in I$ fixed. This covers the case of time-independent correlations functions of quasi-free fermions. However, this assumption fails to cover the case of general $s, t \in I^n$ when $\rho(s, t) = e^{isH} \varrho e^{-itH}$ with ϱ some state.

Similarly, the correct statement of Theorem 1.2 in [1], which pertains to random operators, is given below.

Theorem 2. *Let $I \subseteq \mathbb{R}$, $s, t \in I$, and consider a family of random operators $\rho(s, t)$ on $\ell^2(\mathbb{Z})$ which exhibit localization in the sense that for some $C, \mu \in (0, \infty)$ and for all $x, y \in \mathbb{Z}$:*

$$\mathbb{E} \left[\sup_{s, t \in I} |\langle \delta_x, \rho(s, t) \delta_y \rangle| \right] \leq C e^{-\mu|x-y|}. \quad (4)$$

Let $n \in \mathbb{N}$, $x = (x_1, \dots, x_n)$, $y = (y_1, \dots, y_n) \in \mathbb{Z}^n$ be any pair of fermionic configurations and suppose that the random matrix $M(s, t)$ defined in (1) satisfies the bound $\sup_{s, t \in I^n} \|M(s, t)\| \leq 1$ almost surely. Then,

$$\mathbb{E} \left[\sup_{s, t \in I^n} |\det M(s, t)| \right] \leq \frac{8 \max\{C, \sqrt{C}\}}{(1 - e^{-\mu})^2} \exp\left(-\frac{\mu}{4} D(x, y)\right). \quad (5)$$

References

1. Sims, R., Warzel, S.: Decay of determinantal and Pfaffian correlation functionals in one-dimensional lattices. *Commun. Math. Phys* **347**(3), 903–931 (2016)
2. Aza, N. J. B., Bru, J.-B., de Siqueira Pedra, W.: Decay of complex-time determinantal and Pfaffian correlation functionals in lattices. https://www.ma.utexas.edu/mp_arc/c/17/17-71

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