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CORRECTION

Correction to: Infinitesimal Lyapunov functions for singular flows

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Correction to: Math. Z. (2013) 275:863–897 https://doi.org/10.1007/s00209-013-1163-8

- On p. 874: In the statement of Lemma 2.2, which reads "Let V be a real finite dimensional vector space endowed with a *non-positive definite* and non-degenerate quadratic form", should read "Let V be a real finite dimensional vector space endowed with a *indefinite* non-degenerate quadratic form".
- On p. 875 at item (2) of Theorem 2.7, the last line should be replaced by:

In particular, we get $\partial_t \log |\mathcal{J}(A_t(x)v)| \geq \delta(X_t(x)), x \in V, t \geq 0, v \in E_x$ and $\mathcal{J}(v) > 0$; or $\partial_t \log |\mathcal{J}(A_t(x)v)| \leq \delta(X_t(x)), x \in V, t \geq 0, v \in E_x$ and $\mathcal{J}(v) < 0$.

- On p. 876: although not used anywhere in the article, item (5) of Theorem 2.7 is false. The proof presented in p. 878 has a wrong sign in the calculation. The corrected calculation gives only $\partial_t \left(\frac{|\mathcal{J}(A_t(x)w)|}{\mathcal{J}(A_t(x)v)}\right) |_{t=0} \geq 0$ for the $\mathcal{J}$-separated cocycle $A_t(x)$ and strictly positive if $A_t(x)$ is a strictly $\mathcal{J}$-separated cocycle.
- On p. 884: items (1) and (2) in the statement of Theorem 2.23 in [1] need to be corrected, since [1, Example 5 with index 2] shows that hyperbolic behavior of the subbundles cannot be expressed as necessary and sufficient conditions using the sign of the function δ. Item (3) of [1, Thm. 2.7] is correct. We reformulate the statement of items (1) and (2) as follows.

Theorem 0.1 [1, Theorem 2.23] *Let Γ be a compact invariant set for X_t admitting a dominated splitting $E_\Gamma = F_- \oplus F_+$ for a linear multiplicative cocycle $A_t(x)$ over Γ with values*

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in E. Let $\mathcal{J}$ be a C^1 field of indefinite quadratic forms such that $A_t(x)$ is strictly $\mathcal{J}$-separated admitting a function $\delta : \Gamma \to \mathbb{R}$ as given in Theorem 2.7. Then

(1) *If $\Delta_s^t(x) \xrightarrow[(t-s)\to+\infty]{} -\infty$ for all $x \in \Gamma$, then F_- is a uniformly contracted subbundle.*
(2) *If $\Delta_s^t(x) \xrightarrow[(t-s)\to+\infty]{} +\infty$ for all $x \in \Gamma$, then F_+ is a uniformly expanding subbundle.*

Since the short proof of these items uses the corrected expressions in item (2) of Theorem 2.7, we present it below.

Proof If $\Delta_0^t(x) \xrightarrow[t\to+\infty]{} -\infty$, then from item (2) of [1, Thm 2.7] we get $\frac{\mathcal{J}(A_t(x)v)}{\mathcal{J}(v)} \le e^{\Delta_0^t(x)} \xrightarrow[t\to+\infty]{} 0$ for all $x \in \Gamma$ and $v \in F_-(x)$. So F_- in uniformly contracted, by [1, Lemmas 2.18 & 2.24].

If $\Delta_s^t(x) \xrightarrow[(t-s)\to+\infty]{} +\infty$, then analogously $\frac{\mathcal{J}(A_t(x)v)}{\mathcal{J}(v)} \ge e^{\Delta_0^t(x)} \xrightarrow[t\to+\infty]{} +\infty$ for all $x \in \Gamma$ and $v \in F_+(x)$. So F_+ in uniformly contracted, again by [1, Lemmas 2.18 & 2.24]. □

Reference

1. Araujo, V., Salgado, L.: Infinitesimal Lyapunov functions for singular flows. Mathematische Zeitschrift **275**, 863–897 (2013)