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ERRATUM

Erratum to: Perturbation and interpolation theorems for the H^∞-calculus with applications to differential operators

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In our article [2], some of the general boundedness results in Sects. 7 and 8 for the H^∞-functional calculus may not be correct as stated since when applying [4, 1.2.4] our proofs use implicitly certain inclusions of interpolation spaces, which were not stated as assumptions and which do not hold in all generality. We would like to point out here that, with these assumptions added we obtain correct results. As a consequence, in an application to the Stokes operator in Sect. 9 we have to strengthen the regularity assumption on the underlying domain to ensure that our additional assumption is satisfied.

1 Abstract results

Theorem 7.9 in [2] should read:

Theorem 1.1 (cf. [2, Thm. 7.9]) *Let Y be a complemented subspace of a B-convex Banach space X. Let A have an H^∞-calculus on X and let B be almost R-sectorial on Y.*

If $P(\dot{X}_{\beta_j,A}) = \dot{Y}_{\beta_j,B}$ and $\dot{Y}_{\beta_j,B} \hookrightarrow \dot{X}_{\beta_j,A}$ for two different $\beta_1, \beta_2 \neq 0$ with $|\beta_j| \leq m$ then B has an H^∞-calculus on Y.

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As explained in [2] the equality $P(\dot{X}_{\gamma,A}) = \dot{Y}_{\gamma,B}$ is meant in the following sense: the projection $P : X \to Y$, restricted to $X \cap \dot{X}_{\gamma,A} = D(A^\gamma)$, has a continuous extension $\tilde{P} : \dot{X}_{\gamma,A} \to \dot{Y}_{\gamma,B}$ which is surjective. This also implies that P is compatible with the interpolation couples $(X, \dot{X}_{\gamma,A})$ and $(Y, \dot{Y}_{\gamma,A})$.

Similarly, the embedding $\dot{Y}_{\gamma,B} \hookrightarrow \dot{X}_{\gamma,A}$ is meant to mean that the inclusion $J : Y \to X$, restricted to $Y \cap \dot{Y}_{\gamma,B} = D(B^\gamma)$, has a continuous extension $\tilde{J} : \dot{Y}_{\gamma,B} \to \dot{X}_{\gamma,A}$. With this last assumption added, the proof given in [2] is correct.

One has to add the same assumption to Corollary 7.10 in [2].

Corollary 1.2 (cf. [2, Cor. 7.10]) *If, in the last theorem, B has even BIP, then $P(\dot{X}_{\alpha,A}) = \dot{Y}_{\alpha,B}$ and $\dot{Y}_{\alpha,B} \hookrightarrow \dot{X}_{\alpha,A}$ for one $\alpha \neq 0$ is sufficient for B to have an H^∞-calculus in Y.*

Theorem 8.2 in [2] should read:

Theorem 1.3 (cf. [2, Thm. 8.2]) *Let, in the situation described above, (X_0, X_1) be an interpolation couple of reflexive and B-convex spaces and assume that, for $j = 0, 1$, $P_j : X_j \to Y_j$ are compatible surjections with compatible right inverses $J_j : Y_j \to X_j$. Assume, for $j = 0, 1$, that A_j has an H^∞-calculus on X_j and that B_j is almost R-sectorial on Y_j. Assume moreover that there are $\alpha < 0 < \beta$ such that*

$$P_0((X_0)^{\cdot}_{\alpha,A_0}) = (Y_0)^{\cdot}_{\alpha,B_0}, \quad P_1((X_1)^{\cdot}_{\beta,A_1}) = (Y_1)^{\cdot}_{\beta,B_1} \tag{1}$$

and

$$J_0 : (Y_0)^{\cdot}_{\alpha,B_0} \to (X_0)^{\cdot}_{\alpha,A_0}, \quad J_1 : (Y_1)^{\cdot}_{\beta,B_1} \to (X_1)^{\cdot}_{\beta,A_1}. \tag{2}$$

Then, for $\theta \in (0, 1)$, the operator B_θ has an H^∞-calculus on the complex interpolation space $Y_\theta = [Y_0, Y_1]_\theta$.

Concerning the meaning of (1) and (2) see the remarks above. Assumption (2) is missing in [2, Thm. 8.2] where the P_j are assumed to be compatible projections. Assumption (2) does not follow from (1) in general.

However, the first part of the proof of [2, Thm. 8.2] for the case $Y_j = X_j$, $P_j = I$ is correct as it stands. Hence also [2, Cor. 8.3] and [2, Cor. 8.4] are correct, since they only used [2, Thm. 8.2] for the case $Y_j = X_j$, $P_j = I$.

2 Application to Stokes operators

Let Ω be a bounded domain in $\mathbb{R}^n$ and $\partial\Omega \in C^{1,1}$. We let $p \in (1, \infty)$ and use the notation of [2, Subsect. 9(d)]. Since [2, Thm. 8.2] is not correct, the proof of [2, Thm. 9.17] contains a gap, and it is not clear how this gap can be closed. We show that the corrected version Theorem 1.3 above yields a bounded H^∞-calculus for the Stokes operator if we assume a little more regularity on the boundary $\partial\Omega$.

Theorem 2.1 (cf. [2, Thm. 9.17]) *Let $\Omega \subset \mathbb{R}^n$ be bounded with $\partial\Omega \in C^{2+\mu}$ where $\mu \in (0, 1)$. Then, for $1 < p < \infty$, the operator B_p has an $H^\infty(\Sigma_\nu)$-calculus for all $\nu \in (0, \pi)$.*

Proof As in [2, Prop. 9.16], for $1 < p < \infty$, the operator B_p is R-sectorial in $\mathbb{L}_{p,\sigma}$ and $\omega_R(B_p) = 0$. By [2, Prop. 9.14], for $1 < p < \infty$, $X := \mathbb{L}_p$ and $Y := \mathbb{L}_{p,\sigma}$, the Helmholtz projection $\mathbb{P}_p : X \to Y$ has a continuous and surjective extension $\widetilde{\mathbb{P}_p} : X_{-1,A_p} \to Y_{-1,B_p}$. The right inverse is given by $J_p := A_p \iota_p B_p^{-1}$ where $\iota_p : \mathbb{L}_{p,\sigma} \to \mathbb{L}_p$ denotes the inclusion. Then $J_p : Y \to X$ has a continuous extension $Y_{-1,B} \to X_{-1,A}$.

By [2, Prop. 9.15] we also have that, for any $s \in (0, 1/4)$, $\mathbb{P}_2$ maps $D(A_2^s)$ onto $D(B_2^s)$. The only property that we have to check in addition to what had been done in [2] is contained in the following lemma. Then we apply Theorem 1.3. □

Lemma 2.2 *For small $s > 0$ we have $J_2 : D(B_2^s) \to D(A_2^s)$.*

Proof The assertion is clearly equivalent to $D(B_2^{1+s}) \hookrightarrow D(A_2^{1+s})$ for small $s > 0$. For $|s| < \min\{1/4, \mu/2\} =: \delta_0$ we have

$$D(A_2^{1+s}) = \mathbb{H}_2^{2(1+s)} \cap \mathbb{H}_{2,0}^1, \quad D(A_2^s) = \mathbb{H}_2^{2s},$$

so these are complex interpolation scales (the additional boundary regularity enters in the first equality). We set, for $|s| < \delta_0$, $\hat{\mathbb{H}}_2^{1+2s} := \{v \in \mathbb{H}_2^{1+2s} : \int_\Omega v\, dx = 0\}$. Also this is a complex interpolation scale.

The map $T : (u, p) \mapsto (-\Delta u + \nabla p, \operatorname{div} u)$ acts boundedly

$$(\mathbb{H}_2^{2(1+s)} \cap \mathbb{H}_{2,0}^1) \times \hat{\mathbb{H}}_2^{1+2s} \to \mathbb{H}_2^{2s} \times \hat{\mathbb{H}}_2^{1+2s}, \quad |s| < \delta_0.$$

By [1, Thm. 1.2] (with $\lambda = 0$, $q = 2$) this map is an isomorphism for $s = 0$. By [3, Thm. 2.7] this then holds also for $s \in (0, \delta)$ and some $0 < \delta \le \delta_0$.

Now we recall from the proof of [2, Prop. 9.15] that

$$D(B_2^s) = D(A_2^s) \cap \mathbb{L}_{2,\sigma} = \mathbb{H}_2^{2s} \cap \mathbb{L}_{2,\sigma} = \mathbb{P}(\mathbb{H}_2^{2s}) = \mathbb{P}(D(A_2^s)), \quad s \in (0, 1/4).$$

We claim that

$$B = -\mathbb{P}\Delta : \mathbb{H}_2^{2(1+s)} \cap \mathbb{H}_{2,0}^1 \cap \mathbb{L}_{2,\sigma} \to \mathbb{P}\mathbb{H}_2^{2s} = D(B_2^s), \quad s \in (0, \delta).$$

is an isomorphism for $s \in (0, \delta)$. The operator is clearly bounded and injective, but also surjective: For $f \in D(B_2^s) = \mathbb{H}_2^{2s} \cap \mathbb{L}_{2,\sigma}$ let $(u, p) := T^{-1}(f, 0) \in (\mathbb{H}_2^{2(1+s)} \cap \mathbb{H}_{2,0}^1) \times \hat{\mathbb{H}}_2^{1+2s}$. Then $u \in \mathbb{H}_2^{2(1+s)} \cap \mathbb{H}_{2,0}^1 \cap \mathbb{L}_{2,\sigma}$ and $-\Delta u + \nabla p = f$, so $-\mathbb{P}\Delta u = f$, i.e. $Bu = f$.

We conclude that $D(B_2^{1+s}) = \mathbb{H}_2^{2(1+s)} \cap \mathbb{H}_{2,0}^1 \cap \mathbb{L}_{2,\sigma}$ for $s \in (0, \delta)$, and this space embeds into $\mathbb{H}_2^{2(1+s)} \cap \mathbb{H}_{2,0}^1 = D(A_2^{1+s})$. □

References

1. Farwig, R., Sohr, H.: Generalized resolvent estimates for the Stokes system in bounded and unbounded domains. J. Math. Soc. Jpn. **46**(4), 607–643 (1994)

2. Kalton, N.J., Kunstmann, P.C., Weis, L.: Perturbation and interpolation theorems for the H^∞-calculus with applications to differential operators. Math. Ann. **336**, 747–801 (2006)
3. Kalton, N.J., Mitrea, M.: Stability results on interpolation scales of quasi-Banach spaces and applications. Trans. Am. Math. Soc. **350**(10), 3903–3922 (1998)
4. Triebel, H.: Interpolation, function spaces, differential operators, vol. 18. North-Holland Mathematical Library, Amsterdam (1978)