Economic Theory
https://doi.org/10.1007/s00199-024-01566-9

RESEARCH ARTICLE

# Prediction algorithms in matching platforms

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Received: 1 May 2023 / Accepted: 24 February 2024


**Abstract**

We follow the future trajectory of more targeted wage formation in labor matching platforms, such as freelancing, crowd-sourcing, home-delivery, and ride-hailing, where local job search is coordinated by improving prediction algorithms. A labor matching platform is modelled as a directed search and matching market. We observe that targeted wage setting promotes efficient matching and longer employment spells. However, because a higher employment rate accentuates any disparities between available workers and vacancies, the effects of targeted wage setting on firm competition depend on prevailing market tightness. The impact of targeted wage formation on workers is positive when the vacancy-to-worker ratio is intermediate but turns negative at both extremes. Our results suggest that targeted wage setting may benefit occasional workers while potentially posing drawbacks for full-time platform workers.

**Keywords** Prediction algorithms · Matching platforms · Efficiency · Targeted wages · Employment · Competitive search equilibrium

**JEL Classification** D82 · D83 · E24 · J64

We are grateful to the editor and an anonymous reviewer whose outstanding comments played a pivotal role in helping our paper to reach its full potential. S. Hämäläinen acknowledges the financial support of the OP Group research foundation (Grants Nos. 201600053, 201500138, 20180119, and 20170107). V. Petrikaitė acknowledges the financial support of the Ministry of Economy and Competitiveness of Spain (Grants Nos. ECO2014-59959-P and ECO2015-74328-JIN (AEI/FEDER/UE)) and 2016 FBBVA grant “Innovación e información en la Economía Digital".

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Published online: 04 April 2024

## 1 Introduction

Labor matching platforms, such as *Upwork*, *PeoplePerHour*, *MTurk*, and *Designhill*, provide a centralized marketplace where workers can explore numerous job postings simultaneously.[1] Although information frictions are significantly diminished by public work offers, search frictions depend on the effectiveness of the platform's matching algorithm. As evidenced by the competitive search literature (Peters 1984, 1991; Montgomery 1991; Shimer 2005; Moen 1997; Burdett et al. 2001), inefficient coordination of matches can amplify frictions. Some firms might thus receive an overflow of job applications, while others might face difficulties in attracting any workers. Anticipating the availability of workers is, therefore, of substantial importance for firms operating on labor platforms.

Interestingly, artificial intelligence (AI) is demonstrating increasing adeptness at predicting and optimizing platform user behavior, such as forecasting excess labor demand, recommending suitable matches, and estimating related match values (Castillo et al. 2022; Afeche et al. 2022).[2] These advances show great promise in reducing matching frictions by enabling more targeted job searches (Pallais 2014; Horton 2017; Allon et al. 2023).

Concurrently, the use of AI to discern the most valuable matching patterns can also revolutionize wage formation on platforms. Typically, platforms in home-delivery and ride-hailing employ algorithms that set wages based on predicted demand and supply conditions. However, freelance and crowd-sourcing platforms afford greater wage flexibility, which allows wages to be influenced by information disseminated through the platform.

Because such platform-empowered wage discrimination can harm some workers while others may benefit (Adams and Yellen 1976; Thisse and Vives 1988; Chen et al. 2019, 2022), enriched platform capabilities and the monopsony power of labor platforms has evoked recent policy interest (OECD 2019, 2022; Naidu et al. 2018; Naidu and Posner 2022). The purpose of this paper is to inform this debate by looking into the future of platform work.

At present, the *DoorDash*, *Uber (Eats)*, *Wolt*, and *Deliveroo* apps gather extensive information about workers – including preferred routes, expected timing, and frequency of app use – that is applicable to determine local peak hours of expected labor shortage, during which average wages are higher. This is often denoted as 'surge pricing'. Similarly, at *PeoplePerHour*, firms observe the number of previous worker contacts to their job and, at *Upwork*, both workers and firms rate their partners after a job is completed. Wages can hence be conditioned on these crude measures of future labor supply. Options to 'edit' and 'change the job visibility' also make it possible to screen worker interest while an offer is active. New workers are usually welcomed with a 'start-up bonus'.[3]

[1] On some task platforms, such as *TaskRabbit* and *Fiverr*, workers offer services is different job categories and set their own wages.

[2] For operations research literature, see Yan et al. (2020); Möhlmann et al. (2021); Elmachtoub and Grigas (2022); Tao et al. (2023), etc.

[3] The Online Labor Index (Stephany et al. 2021) at www.onlinelabourobservatory.org/ indicates an increase in gig economy jobs.

To follow the potential trajectory of more targeted wage formation in matching platforms, pushing towards a still distant limit, this paper focuses on the effects of the strongest possible first-degree wage discrimination. We consider labor matching in a directed search and matching model, akin to Shi (2016). The focus is on labor platforms where (i) firms offer wages – or follow a profit maximizing wage algorithm of a platform – and (ii) workers contact firms. Wage discrimination could occur implicitly, by altering the algorithm that determines wages for platform workers (in ride-hailing and deliveries), or explicitly, by providing additional information to firms for optimal wage setting (in crowd-sourcing and freelance).

Our departure from the literature begins with the recognition that firms face changing prospects of matching with workers since their labor supply varies.[4] This is modelled by assuming that workers hold match value information about the firms they have previously worked for. If the match value is high, a worker prefers a repeat match, but otherwise seeks alternative job opportunities. Our analysis centers on a worker's match value, a key parameter of interest to a matched firm, as it serves as a sufficient statistic for efficiently estimating a firm's elasticity of labor supply. By observing the match value, a firm can better assess its prospects of rehiring a worker and offer different wages when it targets old versus new workers.

We compare two polar cases:

1. Under *uniform wage* setting, the match value remains private to a matched worker.
2. Under *targeted wage* setting, the match value is also observed by the previous firm.

These assumptions are adopted to establish bounds on the welfare effects of more informed wage setting, where wages reflect changing labor demand and supply conditions, without implying that firms actually observe worker match values. As discussed in the paper later, match values could be thought of as simply a representation of recurrent matching patterns.

Our stylized model delivers several substantial predictions. First, we demonstrate that targeted wage setting leads to *efficient matching and longer repeated spells of employment*. When vacancies outnumber workers, this could augment wages and labor surplus share. The underlying mechanism is potentially surprising: targeted wage setting permits firms to offer compensating differentials to their previous workers with high enough match values. This eliminates information rents in targeted wage setting. As a result, efficient hiring incentives increase repeat matching and the wage paid to a marginal rehired worker.

Second, we find that the effects of *wage discrimination can be either pro- or anti-competitive*. The impact on worker welfare is therefore equivocal. As it turns out, targeted wage setting alleviates competition among firms when it is already relaxed but exacerbates vigorous competition. Intuitively, this result arises because an increase in the employment rate means that the longer market side is looking for a new match relatively more frequently. This implies that, when there are more workers than vacancies, targeted wage setting reduces average wages: there are fewer firms hiring new workers relative to the number of such workers. On the other hand, when there are more vacancies than workers, targeted wage setting invigorates competition, hence

[4] For example, a restaurant might, unbeknownst to it, receive an order for delivery either during peak demand or during quiet times.

elevating wages. These positive wage effects are remarkable because the immediate benefit of first-degree wage discrimination will accrue solely to firms. As a key contribution to the literature, we thus demonstrate that the improved dynamism that comes with targeted wage setting can benefit workers to such an extent that not only all information rents lost to firms, but also a significant share of the gains from more efficient matching, will be distributed back to workers.

Third, when the vacancy-to-worker ratio surpasses four, we show that firms hold onto their work force so tenaciously that market dynamism stalls, leading to infrequent firm-to-firm transitions. Thus, the average wage offers will decline, although the expected wages of new and old workers will continue to increase. Labor share shrinks despite increasing average wages because, as the employment rate goes up in the matching platform, fewer workers receive the start-up bonus designated to compensate new workers for search. In statistics, the possibility that the average (total) effect can differ from the marginal (group) effects is referred to as Simpson's paradox. Contrary to intuition, we thus observe that even *short-term jobs can become so stable that labor surplus share is undermined*. Model extension shows that targeting can benefit workers if (i) entry into the platform is endogenous, (ii) the use of prediction algorithms is costly, or (iii) worker labor supply is persistent. However, in an extension where firms invest in future matches, we find that excessive repeat matching can engender negative wage externalities, harming workers in the long term, although the start-up bonus can increase markedly.

Our research extends the *competitive search literature with employee turnover*, starting from Burdett and Coles (2003); Shi (2009); Menzio and Shi (2011); Menzio et al. (2016). While we study task-matching platforms with short-term contracts, this previous work focuses mostly on long-term contracts. The closest papers in the literature introduce different job types: Faberman and Menzio (2018) explain lower turnover in higher-wage jobs by allowing for "regular" and "sensitive" jobs. In the labor matching platforms we study, most jobs can be classified as regular. Choi and Fernández-Blanco (2018) show that worker risk-aversion can encourage firms to offer excessive short-term jobs. We find that targeted wage setting decreases worker turnover.

Our work also contributes to the *literature that compares uniform wages and targeted wages*.[5] Earlier papers, such as Bulow and Levin (2006), Kojima (2007), and Niederle (2007), show that average wages are lower and the wage distribution more compressed when firms cannot offer targeted wages. Instead, our novel findings suggest that the average wage effects of targeting depend on market tightness and can be negative at both ends of the market tightness spectrum. Moreover, we show that targeting reduces turnover, on which these static models are silent. Empirical research generally supports the notion that targeted wage offers, like bonuses or benefits, decrease turnover (Frazis and Loewenstein 2013; Dale-Olsen 2006; Ekinci 2019).

Finally, our study contributes to research on price discrimination (Villas-Boas 1999; Armstrong and Vickers 2001; De Corniere 2016; Fabra and Reguant 2020; Hidir and Vellodi 2021; Bergemann and Bonatti 2022). Unlike these papers, which mostly concern product markets, we describe conditions under which workers benefit from

[5] For related models of directed or targeted search, see Lester (2011) and Yang (2020, 2013)

enhanced, potentially privacy-intrusive, platform matching algorithms.[6] This connects our work to the vast literature on *economics of privacy* (Acquisti et al. 2016; Acquisti 2024). Taylor (2004) and Acquisti and Varian (2005) observe that private information may decrease welfare by discouraging early transactions. Here *gains* from privacy suppression emanate from (i) reduced search frictions, (ii) improved match quality, and (iii) intensified firm competition; reduced search friction is similar to Shi (2016), who shows that firms have incentives to prioritize repeat matches.

The paper outline is as follows. Section 2 describes model assumptions. We characterize the equilibrium under uniform and targeted wages in Sects. 3 and 4 respectively. Section 5 contrasts these equilibria and Sect. 6 provides a closing discussion. Proofs are in Appendix, and extensive supplementary material in Online Appendix.

## 2 Model

We study a labor matching platform with infinite measures of firms and workers, where market tightness is captured by a finite vacancy-to-worker ratio $\nu$.[7]

### 2.1 Preferences

The workers in the platform have heterogeneous preferences for work. For concreteness, we assume that a worker's match value with a firm, $s$, is uniformly distributed over the unit interval $[0, 1]$,[8] and affects worker utility negatively – thus representing the disutility of work.

### 2.2 Technology

Firms employ workers in well-defined tasks where the output is easily observable through the platform. Because payment is contingent upon task completion, firms hold no preference for one worker over another. The value of a match for a firm can thus be normalized to unity.

### 2.3 Information

The model concerns repeat matching in a setting where a fraction $\rho$ of workers have recently been matched and observed their match values $s$ with their previous firms. At the point of time we study, the matching platform is therefore populated by two types of workers and firms, determined by their match history. Recently matched and previously unemployed workers are classified as either *informed* ($i = 1$) or *uninformed* ($i = 0$) while the corresponding firms are called either *active* (type $j = 1$) or *inactive* (type

[6] Screening reveals information about worker types in Wolthoff (2018) and Feng et al. (2019).

[7] See Moen (1997); Acemoglu and Shimer (1999); Burdett et al. (2001) for a method review, and Galenianos and Kircher (2012) for our employed "market utility" approach.

[8] We have also considered alternative distributions of match values and have not found any significant deviation from our main results. More details are provided in Sect. 6.

$j = 0$), respectively. Heterogeneity among workers and firms captures the general idea, important in freelance, home-delivery, and ride-hailing platforms, that firms face different conditions of worker labor supply. Optimal wages hence differ for these firms. Informed workers who contact the same active firm are called *old* (to the firm). All other workers who contact unknown firms are referred to as *new* (to these firms).

*Payoffs*. The expected payoff of matching with a firm is $w - 1/2$ for a new worker without a recent history with this firm. The expected payoff for an old informed worker who has recently been matched with the firm is $w - s$.

*Actions*. Firms submit wages $w$ to the platform following a profit-maximizing wage setting algorithm. Workers observe these public wage offers $w$, and contact one firm each.[9] This setup lends itself to various interpretations[10]:

1. The wages are chosen by the firms, either autonomously or following the platform's "recommended wages", as in certain freelance and crowd-sourcing platforms such as *Upwork*, *PeoplePerHour*, and *MTurk*.[11]
2. The wages are given by the platform and cannot be changed, similar to delivery and ride-hailing platforms like *DoorDash*, *Uber* (Eats), and *Wolt*. In this case, the role of the firm is that of posting a vacancy.

We focus on matching platforms that seek to reduce search frictions by bringing together numerous workers and small firms, each with finite capacity to supply vacancies. For simplicity, we assume that a firm can only hire a single worker. If a firm receives contacts from multiple workers at the same time, we suppose that the platform matches one of the workers with the firm. The choice among workers can either be random (in Sect. 3) or favor the previous worker (in Sect. 4).[12]

We introduce matching frictions following the competitive search literature, by assuming that workers cannot coordinate their strategies with each other. For example, two workers cannot cooperatively increase their chances of being matched by agreeing that the first one targets one firm and the second one targets another.

This lack of coordination is captured by maintaining that workers must employ type-symmetric contact strategies, conditional on the same observable information, that is, public wage offers $w$ and private match values $s$. As we will see later in the paper, this leads to a mixed equilibrium where new workers trade off the wage against the corresponding matching probability by randomly choosing which unknown firms to contact. The following timeline summarizes the order of moves:

1. Informed workers observe match values with their last employers.
2. Firms offer jobs with wages optimized for the highest firm benefit.
3. Workers decide which firms to contact.

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[9] For multiple simultaneous contacts, see Galenianos and Kircher (2009); Moon (2023) and Lee and Wang (2023).

[10] While the platform's objective may differ from a firm's profit maximization problem, our characterization of optimal wages provides an important benchmark for all these cases. A platform may aim to maximize firms' profits in order to charge them higher fees.

[11] As discussed by Duch-Brown et al. (2022) and Dube et al. (2020), the modest labor supply elasticities in labor platforms imply that it is fair to view workers as wage-takers and firms as wage-setters.

[12] This assumption is primarily made for convenience. As we prove later, its significance is limited to instances where $\nu$ is less than one, and even in such cases, its impact on payoffs is negligible.

4. Matches are formed and payoffs realize.

In the following analysis, firms set wages themselves but, as the wording above suggests, we also want to incorporate the possibility that the platform defines the profit-maximizing wages for firms given the information transmitted by AI. We characterize the equilibrium with uniform wages in Sect. 3 and with targeted wages in Sect. 4. In the former case, match values remain private to workers. In the latter case, we assume that an active firm has access to an AI technology that enables it to predict the probability of a repeat match. This probability is determined by the match value $s_k$ of firm $k$'s previous matched worker. Under targeted wage setting, we assume that the firm observes $s_k$ together with its previous worker.

Our simple model yields a dynamic behavior pattern, wherein workers reconnect with the same firms, not because of stronger preferences but rather due to superior information on previously matched firms – a plausible scenario. Worker preferences for accepting tasks can change rapidly and unpredictably in freelancing and deliveries, which might be secondary occupations for students or parents whose primary occupation takes precedence. Still, current matching may depend on matching history because a worker may find it more convenient to deal with the same known firm or rely on information transmitted by the firm's previous workers.[13]

Our model also approximates some environments where labor supply has a *persistent component*, represented by informed workers, and a *transient component*, captured by uninformed workers. For example, in delivery platforms, a firm's labor supply often depends on the location of the firm with respect to workers and clients – which could be different, say, at 11 AM on Monday and at 7 PM on Sunday, or during major events like concerts or festivals. Our scenario of targeted (uniform) wage setting corresponds with the equilibrium where platform wages are (not) conditional on the location-specific patterns of labor supply and labor demand.

## 3 Uniform wages

In an equilibrium with uniform wage setting, only second-degree wage discrimination is feasible because information about labor supply is lacking. Thus, inactive firms offer $w_0$, and active firms $w_1$. Because a worker's *disutility* of re-matching with a firm is increasing in the match value, an informed worker contacts the same firm again if $s$ remains below a cutoff denoted by $c$. Otherwise, an informed worker contacts a random unknown firm, thus facing the same problem as uninformed workers.

### 3.1 Preliminaries

#### 3.1.1 Queue lengths

Workers employ mixed strategies when selecting which unfamiliar firms with the same wages to apply to. In a finite market, the expected number of contacts that a firm

[13] To introduce network effects and word-of-mouth, we could think that an exogenous proportion of workers exit upon matching, and transfer their match value information to another entering worker.

receives from new workers would hence be distributed according to the binomial distribution. Here, we consider a continuum market obtained as the limit of a finite market, which must be taken into account when deriving the expected numbers of workers. In a continuum market, the contacts by new workers follow Poisson distributions, $P(q)$, as recognized by the competitive search literature (Burdett et al. 2001). The Poisson parameters $q$ are called 'queue lengths'. A queue length $q_j$ can be thought of as the number of new workers who apply to firms of type $j$ divided by the number of firms of type $j$ receiving these approaches, which gives the average number of contacts from new workers per firm.[14]

The following adding-up condition (1) is derived in Appendix to ensure that the probabilities with which workers contact different types of firms add up to one.

$$(\nu - \rho)q_0 + \rho q_1 = 1 - \rho c \tag{1}$$

It requires that the fraction of workers in queue $q_0$ to inactive firms or in queue $q_1$ to active firms cannot exceed the fraction of workers $1 - \rho c$ seeking new matches.

#### 3.1.2 Match probabilities for firms

Because the number of contacts from new workers follows $P(q_j)$, there is a positive probability defined by $e^{-q_j}$ that a firm fails to attract any new workers. Therefore, the match probability of an inactive firm is $1 - e^{-q_0}$. Similarly, the match probability of an active firm is $1 - e^{-q_1}(1 - c)$, where $e^{-q_1}$ denotes the probability that no new worker contacts the firm, and $(1 - c)$ captures the probability that its previous worker does not contact it.

#### 3.1.3 Match probabilities for workers

Matching is random with uniform wage setting. This implies that, if the number of workers who contact a firm is $n$, the match probability of any individual worker is $1/n$. Therefore, if an uninformed worker contacts an inactive firm, his probability of matching is $\left(1 - e^{-q_0}\right)/q_0$. Similarly, if an informed worker contacts his previous active firm, his match probability is $\left(1 - e^{-q_1}\right)/q_1$. In each case, $\left(1 - e^{-q_j}\right)/q_j$ is the sum of probabilities over $k$ that a worker is matched if $k$ other workers contact the same firm,[15] which gives a worker's expected match probability as

$$\sum_{k=0}^{\infty} \frac{1}{1+k} e^{-q_j} \frac{q_j^k}{k!} = \sum_{k=0}^{\infty} e^{-q_j} \frac{q_j^{k+1}}{(k+1)!} \frac{1}{q_j} = \frac{1 - e^{-q_j}}{q_j}.$$

Although all workers who contact a firm have the same match probability $1/n$ with the firm after the number of workers $n$ is realized, the expected number of competing

[14] These are mathematically known as *Radon-Nikodym derivatives*.

[15] The meeting technology between new workers is *urn-ball* and that between informed workers and their firms is *one-for-one*, but matching features rivalry between uninformed and informed workers (see Eeckhout and Kircher (2010)).

workers $k$ is higher for a new worker than for an old worker if both contact the same active firm. In expectation, a new worker who contacts an active firm (i) competes with the old worker with probability $c$, and additionally (ii) competes with $k$ new workers with probability $e^{-q_1}\frac{q_1^k}{k!}$; the old worker only faces competition from new workers. Altogether, we can show that the match probability of a new worker with an active firm is[16]

$$\frac{1-e^{-q_1}}{q_1} - \underbrace{\frac{1-e^{-q_1}-q_1e^{-q_1}}{q_1^2}c}_{=:\alpha_1^w},$$

which is lower than the match probability of the old worker if he contacts his previous firm. In other words, previous workers encounter lower matching frictions in their firms despite them not receiving priority treatment. Given that new and old workers have the same outside options, this implies that old workers with match values below the mean $1/2$ are cheaper to hire than new workers with average match values. Active firms who have the opportunity to rehire old workers are thus better off than inactive ones who must attract new workers. Since all matches are equally valuable to firms, we later find that active firms offer lower wages, i.e., $w_1 < w_0$.

### 3.1.4 Endogenous state

To compare the welfare effects of uniform and targeted wages, we endogenize $\rho$ by analyzing worker flows in a repeated labor matching model. This allows us to derive the steady state value of $\rho$, which plays a crucial role in the subsequent welfare analysis.

To this end, we consider worker flows among active and inactive firms in a repeated labor matching model where firm-worker match values are independent across time periods. At the beginning of period $t$, there are $\rho_t$ active firms per worker and $\nu - \rho_t$ inactive firms per worker. Using the derived match probabilities, we obtain that the fraction of active firms that stay active is $\left(1 - e^{-q_1}(1-c)\right)\rho_t$ and the fraction of inactive firms that become active is $\left(1 - e^{-q_0}\right)(\nu - \rho_t)$.

By applying this analysis to each period, we can derive the steady state value of $\rho$, which represents the fraction of matched workers in the economy. The fraction of active firms and informed workers at the beginning of the following period $t+1$ is hence given by

$$\rho_{t+1} = \left(1 - e^{-q_0}\right)(\nu - \rho_t) + \left(1 - e^{-q_1}(1-c)\right)\rho_t, \tag{2}$$

which immediately gives us the steady-state value of $\rho$,[17]

$$\rho = \frac{1-e^{-q_0}}{1-e^{-q_0}+e^{-q_1}(1-c)}\nu. \tag{3}$$

16 The details of derivations are provided in Appendix.

17 In labor markets, Eq. (3), corresponds with the *Beveridge curve* i.e., a negative relationship between the number of vacancies and the unemployment rate. In Eq. (3), $1 - e^{-q_0}$ denotes the vacancy filling rate and $e^{-q_1}(1-c)$ captures the vacancy opening rate.

This construction with independent worker match values, where a firm rehires the same worker with a probability of $c$, determines a lower bound on $\rho$. Employment rate increases if worker preferences for firms are persistent. For example, if we assume that a fraction $\sigma$ of workers always prefer to stay with the same firm ($s = 0$), while a fraction $(1 - \sigma)$ will draw a new match value with the firm, we obtain a higher benchmark employment rate:

$$\frac{1 - e^{-q_0}}{1 - e^{-q_0} + e^{-q_1}(1 - c)(1 - \sigma)} \nu.$$

However, since improving matching efficiency under targeted wages setting will continue to increase $\rho$ compared to uniform wage setting, our welfare analysis in Sect. 5 would exhibit minimal change due to this modification.

## 3.2 Equilibrium

### 3.2.1 Worker's problem

A worker's utility from contacting a firm is determined by the firm's wage offer, the worker's match value, and the match probability with the firm. Given these factors, the worker selects a contact strategy that maximizes his expected utility.

There are three application strategies available to a worker: (i) contacting a new inactive firm, (ii) contacting a new active firm, and (iii) contacting the worker's earlier active firm. The utilities of these strategies are given, respectively, by

$$\begin{aligned} v_0 &= \frac{1 - e^{-q_0}}{q_0} (w_0 - 1/2), \\ v_1 &= \left( \frac{1 - e^{-q_1}}{q_1} - \alpha_1^w c \right) (w_1 - 1/2), \\ v_k &= \frac{1 - e^{-q_1}}{q_1} (w_1 - s_k). \end{aligned} \tag{4}$$

Overall, a worker prefers to contact a firm that offers a higher wage. However, as other workers are also likely to contact such firms, the worker must consider the trade-off between the wage and the probability of being matched. Therefore, new workers optimally mix their contact strategy among different unknown firms.

### 3.2.2 Firm's problem

Figure 1 depicts the feasible matching patterns between workers (rectangles) and firms (squares). Wage setting being uniform, an active firm either makes (i) a (lower) wage that is attractive only to its previous worker (perfect sorting), or (ii) a (higher) wage

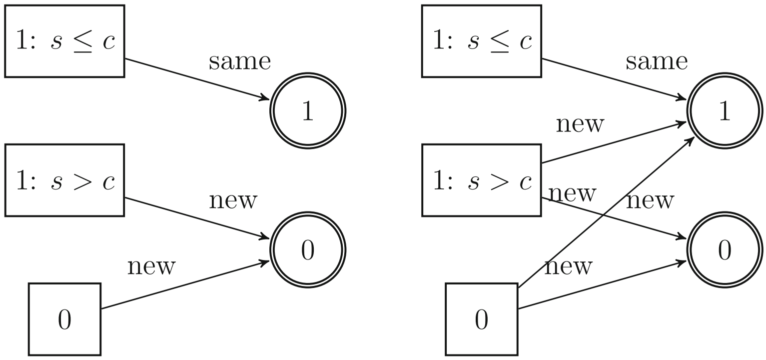


**Fig. 1** Matching by type: perfect sorting (left) and partial sorting (right)

that attracts both new workers and a previous worker (partial sorting). An inactive firm attracts only new workers.[18]

In line with the market utility approach (Moen 1997; Acemoglu and Shimer 1999), equilibrium wage strategies are solved by treating firms as "market utility takers" who maximize their profits by regarding workers' outside options as exogenous. The idea is that, to make contacting a firm optimal for a worker, a firm must provide a worker his market utility, $V_0$. Market utility is endogenously determined and represents a worker's indirect utility of pursuing his optimal contact strategy. If a firm makes a higher offer, a worker's utility of choosing it goes up, thus attracting more applicants to the firm – until the final marginal worker's utility of approaching the firm precisely equals the market utility $V_0$.

*Inactive firms*. An inactive firm's marginal applicants are always new workers. The problem of an inactive firm can thus be stated as follows.

$$J_0 = \max_{w_0} \quad (1 - e^{-q_0})(1 - w_0) \tag{5}$$

$$\text{s.t.} \quad v_0 = \frac{1 - e^{-q_0}}{q_0}(w_0 - 1/2) = V_0. \tag{6}$$

If an inactive firm manages to match with a worker, it obtains the value of matching, which equals unity, net of the compensation $w_0$ to the worker for the lost outside option $V_0$, the expected search costs, $\frac{1-e^{-q_0}}{q_0}$, and the expected match disutility of work, $1/2$. The inverse relationship between the wage and the probability of matching defines a one-to-one mapping between $w_0$ and $q_0$. We can thus think that, instead of choosing the wage $w_0$, an inactive firm selects the unique queue length $q_0$ associated with this

[18] No equilibrium where active firms only attract new workers and exclude all previous ones exists because attracting previous workers with the lowest match value is always more profitable than attracting new workers.

wage $w_0$. By solving the wage from (6) and inserting it into (5), we obtain

$$J_0 = \max_{q_0} \quad (1 - e^{-q_0}) \left( 1 - \frac{V_0}{\frac{1-e^{-q_0}}{q_0}} - \frac{1}{2} \right). \tag{7}$$

*Active firms*. In the perfect sorting equilibrium, an active firm is contacted only by its previous matched worker. The marginal informed worker with match value $s = c$ is indifferent between contacting the previous firm and a different firm. The other inframarginal (extramarginal) informed workers strictly prefer to approach the same firm again (some other firm). In the partial sorting equilibrium, on the other hand, an active firm has two types of marginal applicants: its previous matched worker and possible new matches. Encapsulating both possibilities, the problem of an active firm becomes

$$J_1 = \max_{w_1} (1 - e^{-q_1}(1 - c)) (1 - w_1) \tag{8}$$

$$\text{s.t.} \quad v_k[c] = \frac{1 - e^{-q_1}}{q_1} (w_1 - c) = V_0 \quad \text{and} \tag{9}$$

$$v_1 = \left( \frac{1 - e^{-q_1}}{q_1} - \alpha_1^w c \right) (w_1 - 1/2) = V_0, \quad \text{if} \quad q_1 > 0, \tag{10}$$

$$v_1 = \left( \frac{1 - e^{-q_1}}{q_1} - \alpha_1^w c \right) (w_1 - 1/2) \le V_0, \quad \text{if} \quad q_1 = 0. \tag{11}$$

We can think that an active firm chooses the profit-maximizing combination of a cutoff level $c$ and a queue length $q_1$

$$J_1 = \max_{c, q_1} \quad (1 - e^{-q_1}(1 - c)) \left( 1 - \frac{V_0}{\frac{1-e^{-q_1}}{q_1}} - c \right), \tag{12}$$

subject to the following constraint that relates the cutoff level $c$ and the queue length $q_1$ to a unique wage offer $w_0$

$$1/2 + \frac{V_0}{\frac{1-e^{-q_1}}{q_1} - \alpha_1^w c} = c + \frac{V_0}{\frac{1-e^{-q_1}}{q_1}} \, (= w_1) \,, \qquad \text{if} \quad q_1 > 0, \tag{13}$$

$$1/2 + \frac{V_0}{1 - c/2} \ge c + V_0 \, (= w_1) \,, \qquad \text{if} \quad q_1 = 0. \tag{14}$$

An active firm solves a tradeoff between sorting and screening as discussed in Eeckhout and Kircher (2010). On the one hand, the firm can post a higher (partial sorting) wage satisfying (13), which can attract new workers and is appealing to the majority of its previous matched workers (i.e., $c > 1/2$). On the other hand, the firm can offer a lower (perfect sorting) wage fulfilling (14), but then it can only attract a minority of its previous matched workers (i.e., $c \le 1/2$).

**Lemma 1** *If $c \leq 1/2$, then $q_1 = 0$ or, equivalently, if $q_1 > 0$, then $c > 1/2$.*

This result occurs because previous workers have a statistically higher probability of matching with their old firms. Previously matched workers are therefore willing to contact the same firm under higher match disutility $c$ than the mean, $1/2$.

As discussed previously, the finding does not rely on priority treatment of previous workers but arises because they represent a more efficient form of matching; explicit priority treatment as in Shi (2016) would reinforce repeated matching.

We obtain more precise predictions from the first-order condition of an active firm's problem

$$\frac{1 - V_0 - c}{c} = 1 \text{ for } q_1 = 0, \tag{15}$$

$$\frac{e^{-q_1}(q_1'(w_1)(1-c)+1)}{1 + \alpha_1^w \frac{V_0}{\left(\frac{1-e^{-q_1}}{q_1}\right)^2} q_1'(w_1)} \frac{1 - \frac{V_0}{\frac{1-e^{-q_1}}{q_1}} - c}{1 - e^{-q_1}(1-c)} = 1 \text{ for } q_1 > 0. \tag{16}$$

Conditions (15) and (16) are counterparts of the *Lerner index*, representing a firm's labor market power. They show that firms optimally keep their (wage) elasticity of match supply at unity.[19] Therefore, when the market utility $V_0$ becomes higher and the match supply more elastic across workers, an active firm compensates for this change by starting to target previous workers with lower match values and less elastic match supply. Conversely, when market utility $V_0$ becomes smaller and match supply less elastic for all workers, an active firm starts to target new workers and previous workers with higher match values to maintain constant elasticity of match supply. In other words, perfect worker sorting arises when firm competition is so intense that attracting new matches would require significant profit sacrifices.

**Definition 1** An equilibrium is a tuple of (i) firms' optimal wage strategies, $w_0$ and $w_1$, (ii) workers' optimal contact strategies, $q_0$, $q_1$, and $c$, (iii) consistent with steady-state values of $\rho$ and $V_0$.

**Proposition 1** *There exist exogenous $\underline{\nu} < 1$ and $\overline{\nu} > 1$ and endogenous $v_0'$ such that (i) if $\nu \geq \overline{\nu}$, a perfect sorting equilibrium exists, where $V_0 \geq v_0'$; (ii) if $\nu \leq \underline{\nu}$, a partial sorting equilibrium exists, where $V_0 \leq v_0'$; (iii) if $\nu \in \left(\underline{\nu}, \overline{\nu}\right)$, a mixture of perfect sorting and partial sorting equilibrium exists, where $V_0 = v_0'$; in all these cases $w_0 > w_1$.*[20]

The equilibrium has a number of interesting properties.[21]

1. *Endogenous worker segmentation*. When workers are scarce ($\nu > \overline{\nu}$), markets become perfectly segmented as old and new workers always contact different firms.

[19] If $q_1 = 0$, a firm equates the profit per worker with the probability of employment and, if $q_1 > 0$, a firm puts a higher weight on employment probability than per worker profit.

[20] Our numerical analysis confirms that a unique equilibrium exists for all $\nu > 0$. It also shows that the bounds are $\underline{\nu} \approx 0.94 < \overline{\nu} \approx 1.28$ and the $V_0$ within these bounds $v_0' \approx 0.15$.

[21] More thoroughly described in Online Appendix.

2. *Endogenous worker churn*. When old workers are easily replaced by new workers ($\nu < \underline{\nu}$), firms make offers that attract both at the same time, leading to worker *churn*.
3. *Complementary wage strategies*. When $\underline{\nu} < \nu < \overline{\nu}$, partial sorting and perfect sorting have similar effects on active firms' profits, resulting in firms mixing these strategies.
4. *Attractive introductory wage*. To compensate new workers for their higher search costs, the *introductory wage* $w_0$ required to attract new workers exceeds the *continuation wage* $w_1$ necessary to renew the match.

The first three findings illustrate how firms address coordination problems in the absence of reliable labor supply prediction. The last finding may appear counter-intuitive in labor matching contexts, where wages typically increase with tenure. However, this feature becomes apparent once we consider that our model overlooks possible, unobserved or observable, worker output differences. On the one hand, Postel-Vinay and Robin (2002) find that low-skilled wage-setting remains unaffected by unobserved output differences. On the other hand, performance wage models such as piece rates are applicable to compensate for observable output differences. In the labor matching platforms we consider, workers are mostly employed in tasks where firms can observe their output. The higher wage of new workers can be interpreted as a start-up bonus, prevalent in labor matching platforms.

## 4 Targeted wages

We proceed to characterize the equilibrium in a market where targeted wage setting is introduced. An active firm can thus observe its previous worker's match value, $s_k$, which suffices for efficient forecasting of the firm's labor supply. This gives rise to first-degree wage discrimination with previous workers. We flag the variables by a hat ($\wedge$) under targeted wage setting.

Let us consider an active firm denoted by $k$. Because this firm can observe the match value, $s_k$, it can offer compensating wage differentials. The firm acknowledges that it cannot retain its old worker if the targeted wage fails to cover the match value and the market utility, i.e., $w_k < s_k + \hat{V}_0$. If the firm prefers to rehire its old worker, the firm must therefore increase the wage offer until $w_k = s_k + \hat{V}_0$, at which point the worker is indifferent between approaching it and some other firm.

Due to this wage discrimination, an informed worker receives the market utility, $\hat{V}_0$, also when it applies to its previous firm. By contrast, because the firm offers its worker a higher wage for a larger match value, the active firm's profit, $\hat{J}_k$, is variable and depends on the match value $s_k$. Specifically, the active firm's profit in repeated matching is $\hat{J}_k = 1 - (\hat{V}_0 + s_k)$.

To ensure that an active firm's payoff from making a targeted wage offer to retain old workers is higher than the payoff of making a general wage offer to attract new workers, the optimal wage strategy follows a cutoff rule. An active firm offers a targeted wage $w_k$ if $s_k \leq \hat{c}$ and a general wage $w_0$ if $s_k > \hat{c}$. The targeted offer thus varies between $\hat{V}_0$ for $s_k = 0$ and $\hat{V}_0 + \hat{c}$ for $s_k = \hat{c}$. For larger match values, $s_k > \hat{c}$, a general offer is made. In this case, an active firm's problem is identical to an inactive firm's problem whose solution is denoted by $\hat{w}_0$. The general wage offer is hence $\hat{w}_0$. The cutoff can be derived by observing that a firm has to be indifferent between offering a targeted wage $\hat{V}_0 + \hat{c}$ and a general wage $\hat{w}_0$ when $s_k = \hat{c}$, which gives

$$\begin{aligned} 1 - \hat{V}_0 - \hat{c} = J_0 &= \left(1 - e^{-\hat{q}_0}\right)\left(1 - \hat{w}_0\right) = \left(1 - e^{-\hat{q}_0}\right)\left(\frac{1}{2} - \frac{\hat{V}_0}{\frac{1-e^{-q_0}}{q_0}}\right) \\ &= \frac{1}{2}\left(1 - e^{-\hat{q}_0} - \hat{q}_0 e^{-\hat{q}_0}\right). \end{aligned} \tag{17}$$

Notably, a firm's problem of choosing $\hat{w}_0$ has not changed from Sect. 3. Thereby, $\hat{c}$ can be obtained by combining (17) with the first-order condition $\hat{V}_0 = e^{-\hat{q}_0}/2$ for

$$\hat{c} = \frac{1 + \hat{q}_0 e^{-\hat{q}_0}}{2}. \tag{18}$$

This shows that the cutoff must be higher than the expected match value $1/2$. Because the ability to predict labor supply turns firms into residual claimants of matching surplus, this result arises here independent of the level of competition $\nu$, unlike in Sect. 3 with uniform wage setting, where $c > 1/2$ and $q_1 > 0$ for $\nu < 1$. The underlying explanation is that, given the right efficient incentives, firms are willing to rematch with old workers with lower than average match value because new workers require compensation for their higher search costs.[22] Conversely, an old worker whose match value equals the mean prefers to continue with his firm, where he gets matched with a higher probability than at other firms. Specifically, a worker is willing to re-approach the firm even if (i) his match value $s_k$ slightly exceeds the average match value $1/2$ and (ii) his wage $\hat{w}_k[s_k]$ falls slightly short of the wage $\hat{w}_0$, available at other (vacant) firms.

To derive an equilibrium under targeted wage setting, we note that the queue length $\hat{q}_0$ to firms with wages $\hat{w}_0$ is

$$\hat{q}_0 = \frac{1 - \hat{\rho}\hat{c}}{\nu - \hat{\rho}\hat{c}} \tag{19}$$

since there are $\nu - \hat{\rho}\hat{c}$ firms who offer $\hat{w}_0$ and the fraction of workers who approach these firms is $1 - \hat{\rho} + \hat{\rho}\left(1 - \hat{c}\right)$.

The steady-state rate of employment $\hat{c}$ can thus be derived as

$$\hat{\rho} = \left(\nu - \hat{\rho}\hat{c}\right)\left(1 - e^{-\hat{q}_0}\right) + \hat{\rho}\hat{c}. \tag{20}$$

[22] In Heinsalu (2023), consumers also learn their valuation before product availability and the optimal price falls in the search cost of the consumers.

This together with (19) yields the cutoff $\hat{c}$ as a function of $\hat{q}_0$

$$\hat{c} = \frac{1 - \nu \hat{q}_0}{\nu \left(1 - e^{-\hat{q}_0}\right) - \hat{q}_0 \nu + e^{-\hat{q}_0}}. \tag{21}$$

**Proposition 2** *There exists a unique perfect sorting equilibrium for all* $\nu$*, where* $\hat{w}_k(0) < \hat{w}_0 < \hat{w}_k(\hat{c})$.

Thus, the targeted wages of old workers can either exceed or remain below the general wages of new workers.

## 5 Equilibrium comparison

We next contrast labor platforms which differ in whether wages are uniform or targeted.[23] This permits us to consider the welfare effects of more targeted wage setting in labor matching platforms, which is already the reality in delivery platforms and a possible future scenario in freelance platforms.

### 5.1 Welfare

To compare the performance of markets to an efficient benchmark, we consider as standard the problem of a fictional social planner. The planner maximizes the surplus created by matching workers with suitable firms. The exercise provides a measure of maximum welfare in a market with search frictions. The planner's problem is constrained by similar coordination frictions as we have assumed for workers. This requires, in particular, that workers contact all inactive firms with the same probabilities, defining $q_0^*$, and contact new active firms with the same probabilities, defining $q_1^*$. The asterisk $(*)$ denotes the solution to the planner's problem. An important assumption is that the planner can observe match values $s$ under targeted wage setting but not under uniform wage setting.

If the matching platform does not support targeted wage setting, the planner's problem is thus simply that of choosing the optimal levels of $q_0^*$, $q_1^*$, and $c^*$. The problem is even simpler when targeted wage setting is enabled by the platform. The planner can observe the match values of all matched workers and decide whether to rematch the worker with the firm. The planner clearly has no reason to direct new workers to a firm without letting the previous matched worker go. Because the optimal queue length at active firms is $q_1^* = 0$, the problem thus boils down to choosing the optimal cutoff $c^*$.[24]

Comparison of $c$ and $\hat{c}$ to $c^*$ juxtaposes the maximum welfare with platform welfare. The applications of new workers are distributed evenly among inactive firms and active

[23] The details of the analysis are presented in Appendix (Propositions 4 and 5) and in Online Appendix (Proposition 3).

[24] The planner's problem is different under uniform wages (where $c$ must satisfy the matched worker's incentive compatibility constraint) and under targeted wages (where the planner can decide $c^*$ because match values are observable). The additional constraint with uniform wage setting implies a higher maximal welfare with targeted wage setting.

firms who are not able to retain their matches. As before, this defines queue length $q_0^*$ as the ratio of workers to firms. The problem of the planner can therefore be expressed as choosing the optimal $c_t^*$ for a pre-determined $\rho_{t-1}$:

$$W_t = \max_{c_t} \left(1 - e^{-q_{0t}(c_t)}\right) (\nu - \rho_{t-1} c_t) \frac{1}{2} + \rho_{t-1} c_t \frac{2 - c_t}{2},$$

where $\rho_{t-1}$ needs to satisfy a steady-state condition, similar to (20), and an adding-up condition $q_{0t} = (1 - \rho_{t-1} c_t) / (\nu - \rho_{t-1} c_t)$, much like in (19). Thus, the first-order condition of the planner's problem is essentially identical to (18)

$$\frac{\rho^*}{2} \left(1 - 2c^* + q_0^* e^{-q_0^*}\right) = 0. \tag{22}$$

**Proposition 3** *The equilibrium is socially (constrained) efficient with targeted wage setting: $c^* = \hat{c}$.*

Figure 2 describes the effects of improved wage targeting on cutoffs and total platform welfare.

**Corollary** *Welfare is higher with targeted wage setting for all considered levels of market tightness $\nu$ relative to the setting of uniform wages.*

This shows that the opportunity to observe the match value $s_k$ and offer the worker a targeted wage $w_k$ completely eliminates the matching problems that arise under uniform wage setting. Targeted wage setting removes information rents and aligns private and social incentives. This manifests in the social planner implementing exactly the same cutoff as firms naturally employ under targeted wage setting.

Specifically, because an active firm knows how much it must pay to match with the same worker repeatedly, it becomes a residual claimant of surplus not only with new workers but also with its previous one, whose private match value information would otherwise drive a wedge of information rents between private and social payoffs. This encourages the firm to reduce otherwise excessive worker churn. Furthermore, in markets where workers are abundant, an additional efficiency improvement is that, because an active firm knows when its prospects of repeated matching are weak, the firm no longer needs to target new workers simultaneously. Thus, no discord between previous and new matches prevails. Altogether, this results in higher welfare in labor matching platforms with targeted wage setting.

The ability to predict labor supply also gives firms incentives to embrace longer informal employment relationships. This manifests in the fact that, under targeted wage setting, active firms choose a higher cutoff, $\hat{c} > c$, for all $\nu$, and do not attract new workers, $\hat{q}_1 = 0 < q_1$, for $\nu < \underline{\nu}$. A testable regularity provided by our model is hence that, if a platform improves its targeting facilities, firms should start investing more in repeat matches.

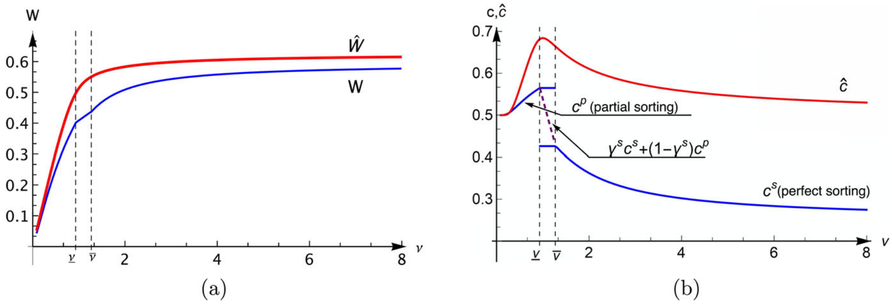


**Fig. 2** Comparison of welfare (a) and cutoffs (b)

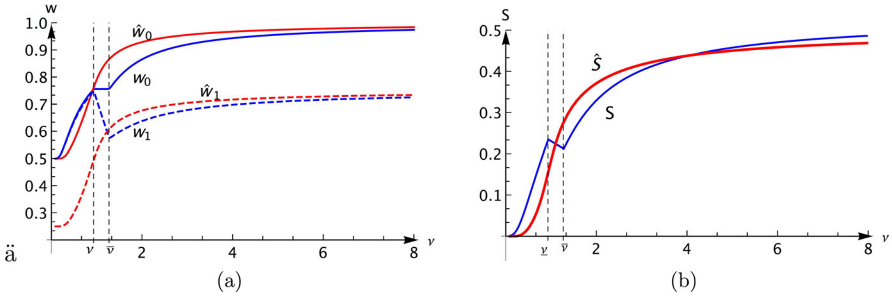


**Fig. 3** Comparison of expected wages (a) and worker welfare (b)

## 5.2 Workers

As equilibrium wages are inherently linked to market tightness $\nu$, the impact of more targeted wage setting on workers varies across markets. Market tightness influences a worker's outside option in the market, $V_0 = e^{-q_0}/2$, which is increasing in the level of firm competition for new workers, measured by the queue length $q_0$. The queue length depends on whether wage setting is targeted.

In particular, under uniform wages, the queue length is given by:

$$q_0 = \frac{1 - \rho c}{\nu - \rho} - \frac{\rho}{\nu - \rho} q_1. \tag{23}$$

By comparison, under targeted wages, the queue length changes to:

$$\hat{q}_0 = \frac{1 - \hat{\rho}\hat{c}}{\nu - \hat{\rho}\hat{c}}. \tag{24}$$

Since wages $w_0$ ($\hat{w}_0$) and worker outside options $V_0$ ($\hat{V}_0$) decrease as $q_0$ ($\hat{q}_0$) increases, we observe that targeted wage setting can have either positive or negative effects on wages (shown in Fig. 3a) and workers (shown in Fig. 3b), depending on $\nu$.

Different mechanisms are at work here. First, because targeted wage setting enables firms to tailor wages to labor supply, wages become more dispersed in equilibrium, with different wages targeting different workers. This wage dispersion effect is similar to Bulow and Levin (2006). Moreover, we find that targeted wage setting allows firms to reach for new workers immediately when they learn about low match probability with old workers. This improved market dynamism will expand the supply of vacancies that target new workers from $\nu - \rho$ to $\nu - \hat{\rho}\hat{c}$, putting upward pressure on wages $w_0$ and $w_1$. Also, because the number of firms that offer the higher introductory wage $w_0$ relative to the lower continuation wage $w_1$ increases under targeted wage setting, the average wage payment goes up.

Second, targeted wage setting also eliminates firms' incentives to attract informed and new workers at the same time, to insure themselves against production breaks because firms know when their previous workers are interested in repeated matching. In markets where workers outnumber vacancies, new workers are thus no longer divided between two different submarkets, represented by $q_0$ and $q_1$, but need to fit into one such submarket, encapsulated in $\hat{q}_0$. This will tend to increase the supply of workers to the submarket for new jobs and put downward pressure on wages for $\nu < 1$.

Finally, Proposition 4 shows that targeted wage setting augments employment because $\hat{\rho}$ and $\hat{c}$ increase for all $\nu$. These changes can either increase or decrease wages by affecting competition among firms. As can be seen from (23) and (24), if $\nu < 1$, then $\hat{q}_0$ is increasing in $\hat{\rho}$ (and $\hat{c}$), whereas, if $\nu > 1$, then $\hat{q}_0$ is decreasing in $\hat{\rho}$ (and $\hat{c}$). The explanation for this polarized finding is that the availability of the shorter market side becomes more limited in the market with increasing employment. Therefore, we find that targeted wage setting increases both introductory and continuation wages, $w_0$ and $w_1$, if there are more jobs than workers, but decreases wages otherwise.

**Proposition 4** *Targeted wage setting (i) decreases labor turnover and (ii) increases employment probability:* $\hat{c} > c$ *and* $\hat{\rho} > \rho$ *for all* $\nu$.

Employment rate under uniform wage setting is

$$\rho = \frac{1 - e^{-q_0}}{1 - e^{-q_0} + e^{-q_1}(1 - c)}\nu, \tag{25}$$

but increases under targeted wage setting to

$$\hat{\rho} = \frac{1 - e^{-\hat{q}_0}}{1 - e^{-\hat{q}_0} - (1 - \hat{c})(1 - e^{-\hat{q}_0}) + 1 - \hat{c}}\nu. \tag{26}$$

The primary reason why employment rate increases is that firms shift from being active to being inactive with a reduced probability of $1 - \hat{c}$ vs. $e^{-q_1}(1 - c)$ when tenure becomes longer. Additionally, firms switch back to being active immediately with a positive probability of $(1 - \hat{c})(1 - e^{-\hat{q}_0})$ by hiring a new worker. These effects are further reinforced by faster matching due to more relaxed firm competition for $\nu < 1$.

**Proposition 5** *Wage effects of targeted wage setting depend on market conditions: if* $\nu > (<)1$*, then* $\hat{w}_0 > (<)w_0$ *and* $E[\hat{w}_1] > (<)w_1$.

Whether the average worker is better off under uniform or targeted wage setting depends on firm competition. Our numerical analysis presented in Fig. 3b indicates that targeted wage setting increases worker payoff for $\nu \in (1, 4)$ but decreases worker payoffs for $\nu < 1$ and $\nu > 4$. This happens because targeting reduces competition intensity when it is already relaxed, but accelerates intense competition, as discussed previously. Thus, wages are increased for $\nu > 1$ but reduced for $\nu < 1$. However, when competition for workers becomes stronger (i.e., $\nu > 4$), most workers are paid the lower continuation wages rather than the higher introductory wages. The average wage thus starts to decrease, although the expected wages for both new and old workers continue to increase. This statistical possibility is known as Simpson's paradox.[25] As a result, although both marginal wages, $\hat{w}_0$ and $\hat{w}_1$, are higher under targeted wage setting (for $\nu > 1$), we find that the average wage payment can diminish (for $\nu > 4$).

In accordance with our model predictions, Castillo (2022) finds that targeted wage setting in the form of surge pricing benefits the average *Uber* worker who targets labor supply selectively (e.g., works only for $1 < \nu < 4$) but hurts those who work long hours (i.e., work also for $\nu < 1$ or $\nu > 4$). Our results are also aligned with Ming et al. (2019); Buchholz (2022) who find that targeted wage setting in ride-hailing and taxi markets can benefit all parties.

## 6 Conclusion

The "OECD Handbook on Competition Policy in the Digital Age," OECD (2022), emphasizes the significance of worker intermediation in digital environments as a notable new domain for modern competition policy (OECD 2019).

> [C]ompetition law may have a role in disciplining monopsony power that is artificially created, maintained or exploited in labour input markets, although competition authorities have so far largely overlooked these markets. When monopsony issues derive from the employer's business model itself, like for platform workers, or from natural factors such as matching, coordination or other labour market frictions, competition advocacy or other tools may be more apt to assist in the correction of these market failures. (OECD 2019, p. 10.)

Historically, labor market issues have predominantly fallen within the purview of specialized labor legislation. However, as discussed in OECD (2019), the situation has changed with the emergence of self-employed platform workers.

To our knowledge, this paper is pioneering in addressing the effects of more refined wage setting via AI on the competition among firms in labor matching platforms. This paper is also the first to explore the distributional consequences of improved matching functionalities, which allow the elimination of frictions in the matching platform, improving the platform's ability to function as an intermediary.

Our results indicate that the impact of AI on workers is uncertain: the deployment of enhanced wage targeting capabilities can have both pro- and anti-competitive effects.

[25] In a dynamic setting the incentive for investing in future matches can be strong. Online Appendix shows that, even if worker tastes are IID, matched workers may cross-subsidize new recruits, e.g., $\hat{w}_0 > 1/2 > E(\hat{w}_1)$ for $(\nu, \delta) = (0.5, 0.8)$.

While efficiency gains may be automatically redistributed to workers in some cases, since more firms compete for new workers, adverse effects on workers may call for the designing of compensation mechanisms in other cases.

Favorable effects on workers are somewhat surprising because the additional information on labor supply allows firms to make discriminatory wage offers to workers. This turns firms into the residual claimants of the matching surplus, thus improving efficiency but transferring the surplus from workers to firms. However, although the immediate effect of targeted wage setting is either harmful or neutral to informed workers, in line with Holmes (1989), we demonstrate that the pro-competitive effects on wages may offset this harm by increasing outside worker options. This requires that workers are on the shorter market side.

Our paper offers vital insights for policymakers aiming to ensure that the gains from technological advances are distributed equitably. We conclude by reviewing some extensions that we have considered and pinpointing certain limitations of our modeling approach.

### 6.1 Non-uniform match value distribution

While the main text is focused on a uniform match value distribution, we have also considered various extensions to non-uniform match values. Overall, without uniform distribution the uniqueness of equilibrium is hard to prove but, according to our additional studies, the general insights of our paper still apply for more general distributions.

Specifically, our main welfare comparison of targeted and uniform wages on worker welfare hinges on positive effects of targeted wages on the matching efficiency and therefore employment. An increase in the employment rate implies that the 'shorter market side becomes shorter', indicating that targeting is anti-competitive for $\nu < 1$ (leading to reduced wages and $V_0$) and pro-competitive for $\nu > 1$ (resulting in augmented wages and $V_0$).

To clarify what we mean by this, suppose there are 5 vacancies and 4 workers of which 2 are employed under uniform wages and 3 are employed under targeted wages. In this case, the ratio of unemployed workers to unfilled vacancies is $\frac{2}{3}$ with uniform wages and is lower at $\frac{1}{2}$ with targeted wages. This mechanism puts upward pressure on wages under targeting for $\nu > 1$ while the opposite mechanism reduces wages under targeting for $\nu < 1$.

The mechanism is robust and survives in all of the analyses that we have performed for different match value distributions. However, we do find that the skewness and kurtosis of the match value distribution alter equilibrium wages. Namely, if the distribution of $c$ is right-skewed the expected disutility of work is lower. A lower wage thus suffices to attract new and old workers, which reduces the offered $w_0$ and $w_1$; the opposite effect increases wages for a left skewed distribution of $c$.

### 6.2 Entry into labor platform

The main text studies a labor platform with an exogenous $\nu$ and examines the effects of targeted wages relative to this $\nu$. However, the market tightness in actual platforms is endogenous, influenced by firms' and workers' incentives to join a platform, including outside options and entry costs. Moreover, the introduction of targeting in the platform changes the payoffs of firms and workers. The resulting $\hat{\nu}$ under targeted wage setting can thus differ from the original $\nu$ established under uniform wage setting.

We discuss next how accounting for changing market tightness alters our key results. The impact on workers depends on whether $\nu \in (1, 4)$, $\nu \leq 1$, or $\nu \geq 4$.

First, because workers and firms share the welfare gains of targeted wage setting for $\nu \in (1, 4)$, the introduction of targeting encourages more firms and workers to enter the platform. This can have ambiguous effects on market tightness $\hat{\nu}$ relative to the original $\nu$. However, if this leads, on the first iteration, to a lower $\hat{\nu}_1 < \nu$ and reduced worker surplus, the second iteration will have a higher $\hat{\nu}_2 > \nu$ because some workers exit, etc. As a result, given that more workers enter the platform only if their payoffs increase, we observe that targeted wage setting cannot reduce the worker surplus share.

Second, because firms obtain the efficiency gains of more targeted wage setting for $\nu \leq 1$ and $\nu \geq 4$, the number of firms over workers in the platform tends to increase in these cases. As the worker surplus share increases with the firm-to-worker ratio $\hat{\nu}$ when the platform algorithm enables targeted wage setting, the change in the market composition tends to benefit workers for $\nu \leq 1$ and $\nu \geq 4$. We believe these positive effects of firm entry into the platform can represent a significant additional channel through which the introduction of targeted wage setting may benefit platform workers.[26]

**Supplementary Information** The online version contains supplementary material available at https://doi.org/10.1007/s00199-024-01566-9.

**Funding** Open Access funding provided by University of Helsinki (including Helsinki University Central Hospital).



[26] The decision when to work for platforms, such as *Wolt* or *Uber*, might be hence utterly important for worker payoffs.

# A Appendix

To abbreviate expressions, we use the following notation for match probabilities and auxiliary variables:

$$\beta_j^w(q_j) := \frac{1-e^{-q_j}}{q_j},$$
$$\alpha_1^w(q_1) := \frac{1-e^{-q_1}-q_1 e^{-q_1}}{q_1^2}.$$

The total number of firms is denoted by $N^f$ and the total number of workers by $N^w$, where $\nu = N^f/N^w$.

## A.1 Derivation of match probabilities

The following methodology is adopted from the working paper version of Shi (2016). First, we derive the probability $g_0(1)$ that a new worker is matched with a firm of type 0. In addition to the worker we consider, there are $i \leq (1-\rho) N^w - 1$ uninformed workers. They contact the firm with probability $C^i_{(1-\rho)N^w-1}(1-\theta_{00})^{(1-\rho)N^w-1}\theta^i_{00}$. There are also $j \leq \rho N^w(1-c)$ informed workers who were matched the previous period but have a low match value with their matched firms $s \geq c$. Their probability of contacting the firm is $C^i_{\rho N^w}(1-\theta_{10}(1-c))^{\rho N^w - j}(\theta_{10}(1-c))^j$. $C^n_k = \frac{n!}{k!(n-k)!}$ denotes the binomial coefficient. This allows us to derive $g_0(1)$ by considering the related function

$$g_0(x) = \sum_{i=0}^{(1-\rho)N^w-1}\sum_{j=0}^{\rho N^w}\frac{C^i_{(1-\rho)N^w-1}(1-\theta_{00})^{(1-\rho)N^w-1-i}(x\theta_{00})^i}{i+j+1}$$
$$\times\left[C^j_{\rho N^w}(1-(1-c)\theta_{10})^{\rho N^w-j}(x(1-c)\theta_{10})^j\right],$$

which can be differentiated with respect to its argument to get

$$\frac{\partial x g_0(x)}{\partial x} = \sum_{i=0}^{(1-\rho)N^w-1} C^i_{(1-\rho)N^w-1}(1-\theta_{00})^{(1-\rho)N^w-1-i}(x\theta_{00})^i$$
$$\times\sum_{j=0}^{\rho N^w} C^j_{\rho N^w}(1-(1-c)\theta_{10})^{\rho N^w-j}(x(1-c)\theta_{10})^j$$
$$= (1-\theta_{00}+x\theta_{00})^{(1-\rho)N^w-1}(1-(1-c)\theta_{10}+x(1-c)\theta_{10})^{\rho N^w}.$$

Note that $xg_0(x)$ and $g_0(x)$ are bounded and thereby integrable for all $x \in (0,1)$, which gives

$$xg_0(x) = \int_0^x (1-(1-y)\theta_{00})^{(1-\rho)N^w-1}(1-(1-y)(1-c)\theta_{10})^{\rho N^w}\,dy.$$

We proceed by plugging in contact probabilities $\theta_{00}$ and $\theta_{10}$.

$$xg_0(x) = \int_0^x \left(1 - (1-y)\frac{q_{00}}{(1-\rho)N^w}\right)^{(1-\rho)N^w-1}\left(1-(1-y)\frac{q_{10}}{\rho N^w}\right)^{\rho N^w} dy,$$

and take the limit $N^w \to \infty$ to obtain the following expression

$$xg_0(x) = \int_0^x e^{-q_{00}(1-y)-q_{10}(1-y)}dy = \int_0^x e^{-q_0(1-y)}dy = \frac{e^{q_0(x-1)} - e^{-q_0}}{q_0}.$$

Now we set $x = 1$. This shows that the match probability $g_1(1)$ for a new worker contacting a firm of type 0 equals $\left(1 - e^{-q_0}\right)/q_0$. Because an informed worker who contacts the same firm repeatedly is in the same situation with the competition from new workers in $q_1$, the above calculation also shows that the match probability for an informed worker who contacts the same firm repeatedly equals $\left(1 - e^{-q_1}\right)/q_1$. We proceed to derive the match probability $g_1(1)$ of a new worker conditional on contacting a firm of type 1.

$$\begin{aligned} g_1(x) = & \sum_{i=0}^{(1-\rho)N^w-1}\sum_{j=0}^{\rho N^w-1} \frac{cxC^i_{(1-\rho)N^w-1}(1-\theta_{01})^{(1-\rho)N^w-1-i}(x\theta_{01})^i}{i+j+2} \\ & \times \left[C^j_{\rho N^w-1}(1-(1-c)\theta_{11})^{\rho N^w-j}(x(1-c)\theta_{11})^j\right] \\ & + \sum_{i=0}^{(1-\rho)N^w-1}\sum_{j=0}^{\rho N^w-1} \frac{(1-c)C^i_{(1-\rho)N^w-1}(1-\theta_{01})^{(1-\rho)N^w-1-i}(x\theta_{01})^i}{i+j+1} \\ & \times \left[C^j_{\rho N^w-1}(1-(1-c)\theta_{11})^{\rho N^w-j}(x(1-c)\theta_{11})^j\right]. \end{aligned}$$

As before, we take the derivative of $xg_1(x)$ with respect to $x$

$$\begin{aligned} \frac{\partial xg_1(x)}{\partial x} = & cx\left(\sum_{i=0}^{(1-\rho)N^w-1} C^i_{(1-\rho)N^w-1}(1-\theta_{10})^{(1-\rho)N^w-1-i}(x\theta_{10})^i\right. \\ & \left.\times \sum_{j=0}^{\rho N^w-1} C^j_{\rho N^w-1}(1-(1-c)\theta_{11})^{\rho N^w-j}(x(1-c)\theta_{11})^j\right) \\ & + (1-c)\left(\sum_{i=0}^{(1-\rho)N^w-1} C^i_{(1-\rho)N^w-1}(1-\theta_{01})^{(1-\rho)N^w-1-i}(x\theta_{01})^i\right. \\ & \left.\times \sum_{j=0}^{\rho N^w-1} C^j_{\rho N^w-1}(1-(1-c)\theta_{11})^{\rho N^w-j}(x(1-c)\theta_{11})^j\right) \\ = & cx(1-\theta_{01}(1-x))^{N^w(1-\rho)-1}(1-(1-c)\theta_{11}(1-x))^{\rho N^w-1} \end{aligned}$$

$$+ (1-c)\left(1-\theta_{01}(1-x)\right)^{N^w(1-\rho)-1}\left(1-(1-c)\,\theta_{11}(1-x)\right)^{\rho N^w-1},$$

and substitute in the expression the contact probabilities $\theta_{01}$ and $\theta_{11}$, which gives

$$\begin{aligned} xg_1(x) = &\int_0^x (1-c+yc)\left(1-(1-y)\frac{q_{10}}{(1-\rho)N^w}\right)^{(1-\rho)N^w-1} \\ &\times\left(1-(1-y)\frac{q_{10}}{\rho N^w-1}\right)^{\rho N^w-1} dy. \end{aligned}$$

By taking the limit $N^w \to \infty$ and plugging in queue length $q_1$, we now find that

$$\begin{aligned} xg_1(x) &= \int_0^x (1-c+yc)\, e^{-q_1(1-y)} dy \\ &= (1-c)\frac{-e^{-q_1}+e^{-q_1(1-x)}}{q_1} + c\frac{e^{-q_1}+e^{-q_1(1-x)}(q_1x-1)}{q_1^2}. \end{aligned}$$

Setting $x = 1$ results in the following expression

$$g_1(1) = (1-c)\frac{1-e^{-q_1}}{q_1} + c\frac{e^{-q_1}-1+q_1}{q_1^2} = \frac{1-e^{-q_1}}{q_1} + c\frac{e^{-q_1}(1+q_1)-1}{q_1^2}.$$

### A.2 Proofs

***Proof of Lemma 1*** As $\frac{1-e^{-q_1}}{q_1} - \alpha_1^w c < \frac{1-e^{-q_1}}{q_1}$, no solution $(c, q_1)$ of Eq. (13) has $c \leq 1/2$ and $q_1 > 0$. □

***Proof of Proposition 1*** The proof of Proposition 1 relies on Lemmata A2-A7. We begin with *perfect sorting*.

In a perfect sorting equilibrium, the equilibrium levels of $q_0$ and $c$ are determined by the first-order conditions of (7) and (8):

$$\frac{e^{-q_0}}{2} - V_0 = 0, \qquad V_0 = \frac{e^{-q_0}}{2} < \frac{1}{2}, \tag{A1}$$

$$1 - 2c - V_0 = 0, \qquad c = \frac{1}{2} - \frac{V_0}{2} < \frac{1}{2}. \tag{A2}$$

**Lemma A2** ***(Perfect sorting - Fixed point)***

*By differentiating the profit function (12) implicitly with respect to $q_1$, the first-order condition of an active firm's problem can thereby be written as*

$$\begin{aligned} &e^{-q_1}\left(1-c+\frac{\partial c}{\partial q_1}\right)\left(1-\frac{V_0}{\beta_1^w}-c\right) \\ &\quad -\left(1-e^{-q_1}(1-c)\right)\left(\frac{\partial c}{\partial q_1}+\frac{V_0\alpha_1^w}{\left(\beta_1^w\right)^2}\right) = 0, \end{aligned} \tag{A3}$$

*where we have used the fact that* $\partial\beta_1^w/\partial q_1 = -\alpha_1^w$.

*For any* $\nu > 1$ *there is a unique pair of* $c$ *and* $q_0$ *that satisfies the first-order conditions (A1) and (A2), the adding-up condition, and the steady-state condition.*

***Proof*** We set $q_1 = 0$ in (1) and obtain

$$q_0 = \frac{N^w(1-\rho) + N^w\rho(1-c)}{N^f - \rho N^w} = \frac{1-\rho c}{\nu-\rho}, \text{ which gives the expression}$$
$$\rho = \frac{1-\nu q_0}{c-q_0}. \tag{A4}$$

Further, the same replacement in (3) gives us

$$\rho N^w = \left(1-e^{-q_0}\right)\left(N^f - \rho N^w\right) + \rho N^w c, \text{ which can be rewritten as}$$
$$\rho = \nu\frac{2-e^{-q_0}}{1-e^{-q_0}-c}. \tag{A5}$$

Joining (A4) and (A5) results in

$$\frac{1-\nu q_0}{c-q_0} = \nu\frac{1-e^{-q_0}}{2-e^{-q_0}-c} \text{ that we solve for } c:$$
$$c = 1 - \frac{1}{1+\xi}, \text{ where } \xi = \frac{2-e^{-q_0}-\nu q_0}{(\nu-1)(1-e^{-q_0})}. \tag{A6}$$

The first-order conditions (A1) and (A2) give $c = 1 - \frac{2+e^{-q_0}}{4}$. Thus, the following two equations must hold in perfect sorting.

$$c = 1 - \frac{2+e^{-q_0}}{4}, \tag{A7}$$
$$c = 1 - \frac{1}{1+\xi}, \quad \xi = \frac{2-e^{-q_0}-\nu q_0}{(\nu-1)(1-e^{-q_0})}. \tag{A8}$$

Our task is to prove that the curves (A7) and (A8) cross on the plane $q_0 \times c$. The intersection point determines the equilibrium value of $q_0$.

We denote the LHS of (A7) by $c_1$. Note that $c_1$ equals $1/4$ if $q_0 = 0$ and $\lim_{q_0\to\infty} c_1 = 1/2$. The RHS of (A7) is increasing in $q_0$.

Next, we denote the LHS of (A8) by $c_2$. Because $\nu > 1$, the derivative of $c_2$ with respect to $q_0$ has the same sign as

$$(\nu-1)\left(1-e^{-q_0}\right)^2\frac{\partial\xi}{\partial q_0} = \left(1-e^{-q_0}\right)\left(e^{-q_0}-\nu\right) - e^{-q_0}\left(2-e^{-q_0}-\nu q_0\right)$$
$$= -e^{-q_0} + \nu\left(e^{-q_0} - 1 + e^{-q_0}q_0\right). \tag{A9}$$

Because $e^{-q_0} - 1 + e^{-q_0}q_0 < 0$, we conclude that (A9) is negative and $c_2$ is decreasing in $q_0$.

In addition, we observe that the denominator of $c_2$ equals zero if

$$q_0 = q_c = \left(1 + \nu + \nu W\left(-e^{-1-1/\nu}\right)\right)/\nu,$$

where $W$ is a Lambert-W function. Thus, the function $c_2(q_0)$ is discontinuous at point $q_c$. We will look to the left from $q_c$ further on.

We note that $\lim_{q_0 \to 0} c_2 = 1$, and $\lim_{q_0 \to q_c} c_2 = -\infty$. Thus, $c_2$ crosses with $c_1$ once, which guarantees the uniqueness of a fixed point. □

**Lemma A3** *(**Perfect sorting - Wages**) In the perfect sorting setting with $\nu > 1$, $w_1 < w_0$.*

***Proof*** The difference between the wages posted by firms of type 0 and 1 is

$$\begin{aligned} w_0 - w_1 = \frac{V_0}{\beta_0^w} + \frac{1}{2} - V_0 - c = \frac{V_0}{\beta_0^w} - \frac{1}{2} \\ -V_0 + \frac{1}{2} + \frac{V_0}{2} = V_0\left(\frac{1}{\beta_0^w} - \frac{1}{2}\right) > 0. \end{aligned} \tag{A10}$$

□

**Lemma A4** *The value of $c$ that solves (13) continuous in $q_1$ when $q_1 < q_0$.*

***Proof*** The indifference condition (13) can be rewritten as

$$\begin{aligned} (1-c)^2 \alpha_1^w - (1-c)\left(\frac{3\alpha_1^w}{2} - \beta_1^w + \frac{V_0\alpha_1^w}{\beta_1^w}\right) \\ + \frac{V_0\alpha_1^w}{\beta_1^w} + \frac{1}{2}\alpha_1^w - \frac{\beta_1^w}{2} = 0. \end{aligned} \tag{A11}$$

The LHS of (A11) is a convex second-degree polynomial of $1-c$. If $1-c=0$, then the LHS of the equation is negative because

$$\begin{aligned} &\frac{V_0\alpha_1^w}{\beta_1^w} + \frac{1}{2}\left(\alpha_1^w - \beta_1^w\right) \\ &= \frac{V_0\alpha_1^w}{\beta_1^w} - \frac{1}{2}\left(\frac{e^{-q_1} - 1 + q_1}{q_1^2}\right) \\ &< \frac{e^{-q_1}\left(1 - e^{-q_1}(1+q_1)\right)}{2\left(1 - e^{-q_1}\right)q_1} - \frac{1}{2}\left(\frac{e^{-q_1} - 1 + q_1}{q_1^2}\right) < 0. \end{aligned}$$

We used the definitions of $\alpha_1^w$ and $\beta_1^w$ to obtain the second line. The third line was obtained by using the definitions of $\alpha_1^w$ and $\beta_1^w$ and the fact that $V_0 = e^{-q_0}/2 < e^{-q_1}/2$.

This shows that $1-c$ is the larger of the two roots (the smaller one is to the left form zero) of the second-order equation that is of our interest. The root lies below $1/2$ ($c$ is above $1/2$) because

$$\frac{\alpha_1^w}{4} - \frac{3\alpha_1^w}{4} + \frac{\beta_1^w}{2} - \frac{V_0\alpha_1^w}{2\beta_1^w} + \frac{V_0\alpha_1^w}{\beta_1^w} + \frac{1}{2}\alpha_1^w - \frac{\beta_1^w}{2} = \frac{V_0\alpha_1^w}{2\beta_1^w} > 0.$$

More specifically, this root equals

$$\begin{aligned} 1-c &= \frac{\frac{3}{2}\alpha_1^w - \beta_1^w + \frac{V_0\alpha_1^w}{\beta_1^w} + \sqrt{D}}{2\alpha_1^w} = \frac{3\alpha_1^w\beta_1^w - 2\left(\beta_1^w\right)^2 + 2V_0\alpha_1^w + 2\beta_1^w\sqrt{D}}{4\alpha_1^w\beta_1^w} \\ &= \frac{3}{4} - \frac{\beta_1^w}{2\alpha_1^w} + \frac{V_0}{2\beta_1^w} + \frac{\sqrt{D}}{2\alpha_1^w}, \end{aligned} \tag{A12}$$

or

$$c = 1 - \left(\frac{3}{4} - \frac{\beta_1^w}{2\alpha_1^w} + \frac{V_0}{2\beta_1^w} + \frac{\sqrt{D}}{2\alpha_1^w}\right), \tag{A13}$$

where the discriminant is given by

$$D = \left(-\frac{3\alpha_1^w}{2} + \beta_1^w - \frac{V_0\alpha_1^w}{\beta_1^w}\right)^2 + 4\alpha_1^w\left(-\frac{V_0\alpha_1^w}{\beta_1^w} - \frac{1}{2}\alpha_1^w + \frac{\beta_1^w}{2}\right).$$

Further,

$$\begin{aligned} \frac{D}{4\left(\alpha 1^w\right)^2} &= \frac{1}{4\left(\alpha_1^w\right)^2}\left(-\frac{3\alpha_1^w}{2} + \beta_1^w - \frac{V_0\alpha_1^w}{\beta_1^w}\right)^2 + \frac{4\alpha_1^w}{4\left(\alpha_1^w\right)^2}\left(-\frac{V_0\alpha_1^w}{\beta_1^w} - \frac{1}{2}\alpha_1^w + \frac{\beta_1^w}{2}\right) \\ &= \frac{1}{4}\left(-\frac{3}{2} + \frac{\beta_1^w}{\alpha_1^w} - \frac{V_0}{\beta_1^w}\right)^2 + \left(-\frac{V_0}{\beta_1^w} - \frac{1}{2} + \frac{\beta_1^w}{2\alpha_1^w}\right). \end{aligned} \tag{A14}$$

Clearly, the expression (A14) is continuous in $q_1$. In addition, the derivative of (A14) with respect to $V_0$ is negative:

$$-\frac{1}{2\beta_1^w}\left(\frac{\beta_1^w}{\alpha_1^w} + \frac{1}{2} - \frac{V_0}{\beta_1^w}\right) < -\frac{1}{2\beta_1^w}\left(\frac{\left(1-e^{-q_1}\right)q_1}{1-e^{-q_1}-e^{-q_1}q_1} + \frac{1}{2} - \frac{e^{-q_1}q_1}{2\left(1-e^{-q_1}\right)}\right) < 0.$$

Therefore, (A14) is greater than after setting $V_0/\beta_1^w = e^{-q_1}/\left(2\beta_1^w\right)$, which gives a positive expression:

$$\frac{1}{4}\left(-\frac{3}{2} + \frac{\beta_1^w}{\alpha_1^w} - \frac{e^{-q_1}}{2\beta_1^w}\right)^2 + \left(-\frac{e^{-q_1}}{2\beta_1^w} - \frac{1}{2} + \frac{\beta_1^w}{2\alpha_1^w}\right) > 0.$$

Because $D > 0$ and continuous in $q_1$, and $\beta_1^w/\alpha_1^w$ and $1/\beta_1^w$ are continuous in $q_1$, we conclude that (A12) is a continuous function of $q_1$. □

**Lemma A5** ***(Partial sorting - Fixed point)*** *For any $\nu > 1$ there is a unique pair of $c$ and $q_0$ that satisfies the first-order conditions (A2) and (A3), the adding-up condition, and the steady-state condition.*

***Proof*** The essence of the proof is to show that there are two implicit relationships $q_0^1(q_1)$ and $q_0^2(q_1)$ that cross on the plane $q_0 \times q_1$ in the lower region where $q_1 < q_0$.

The first implicit relationship $q_0^1(q_1)$ comes from (A2) and (A3). From Lemma A4, we obtain that the derivative $\partial c/\partial q_1$ exists for all $q_1 < q_0$. Thus, there is a continuous implicit relationship $q_0^1(q_1)$ that is defined by (A3). To proceed, we next take the limit of the LHS of (A3) as $q_1 \to 0$, which gives us

$$\frac{14V_0^2 - 37V_0 + 9 + (3 - 17V_0)\sqrt{4V_0^2 - 20V_0 + 9}}{24\sqrt{4V_0^2 - 20V_0 + 9}} = 0. \tag{A15}$$

Eq. (A15) has a unique fixed point $V_0 = 0.218731$, which gives $q_0 = 0.826764 = -ln(2V_0)$.

On the other hand, if we set $q_1 = q_0$, we obtain a negative expression that approaches zero as $q_0$ approaches infinity. Thus, the implicit relationship $q_0^1(q_1)$ never crosses the 45-degree line although it approaches it asymptotically as $q_0 \to \infty$.

Next, we look at the second relationship $q_0^2(q_1)$ that comes from the steady state conditions. From (1) and (3), we obtain

$$\frac{\left(1 - e^{-q_0}\right)(\nu + q_1\nu - 1)}{e^{-q_1}(1 - \nu q_0) + \nu\left(1 - e^{-q_0}\right)} - 1 + c = 0, \tag{A16}$$

where $c$ is given by (A13). This defines an implicit relationship $q_0^2(q_1)$, which is not continuous for all $q_1$. In particular, this relation is not defined for

$$e^{-q_1} = \frac{\nu\left(1 - e^{-q_0}\right)}{1 - \nu q_0}. \tag{A17}$$

The RHS of (A17) is increasing in $q_0$. When we set $q_0 = q_1$ and move the terms on the RHS to the LHS, we obtain the following series of inequalities

$$\begin{aligned} e^{-q_1} - \frac{\nu\left(1 - e^{-q_1}\right)}{1 - \nu q_1} &> e^{-q_1} - \lim_{\nu\to\infty}\frac{\nu\left(1 - e^{-q_1}\right)}{1 - \nu q_1} \\ &= e^{-q_1} + \frac{1 - e^{-q_1}}{q_1} > 0. \end{aligned}$$

As a result, the implicit relationship $q_0^2(q_1)$ is continuous if $q_0 > q_1$.

Equation (A16) solved for $\nu$ gives

$$\nu = \frac{(1-c)\,e^{-q_1} + 1 - e^{-q_0}}{\left(1-e^{-q_0}\right)(q_1+c) + (1-c)\,e^{-q_1} q_0}. \tag{A18}$$

Then, we substitute in (A18) the value of $c$ from (A13), set $V_0 = e^{-q_0}/2$, and take the limit $q_1 \to 0$. This gives $\nu$ as a decreasing function of $q_0$. When $q_0$ approaches its previously derived fixed point value 0.826764, we find that $\nu$ approaches 1.447. Hence, If $\nu < 1.447$, then $q_0 > 0.826764$, which implies that $q_0^1(0) < q_0^2(0)$.

Thereafter, we set $q_1 = q_0$ in (A18) and obtain

$$\nu = \frac{(1-c)\,e^{-q_0} + 1 - e^{-q_0}}{\left(1-e^{-q_0}\right)(q_0+c) + (1-c)\,e^{-q_0} q_0}. \tag{A19}$$

The LHS of (A19) is a decreasing function of $q_0$ and there is a value of $q_0$ that solves (A19) for any $\nu < 1.447$. As a result, the implicit relationship $q_0^2(q_1)$, necessarily crosses the 45-degree line and thus the other implicit relationship $q_0^1(q_1)$. Therefore, a fixed point exists. □

**Lemma A6** ***(Partial sorting - Wages)*** *In the partial sorting setting with $\nu < 1.447$, $w_1 > w_0$ if $\beta_0^w < \beta_1^w - c\alpha_1^w$.*

***Proof*** The difference in wages is negative:

$$w_0 - w_1 = \frac{V_0}{\beta_0^w} + \frac{1}{2} - \frac{V_0}{\beta_1^w - c\alpha_1^w} - \frac{1}{2} = V_0\left(\frac{1}{\beta_0^w} - \frac{1}{\beta_1^w - c\alpha_1^w}\right) < 0. \tag{A20}$$

□

We have proved the existence of the fixed points and the order of wages related to those fixed points. Next, we proceed by showing the conditions under which either one or the other fixed point constitutes an equilibrium. This is done by checking for possible profitable deviations from one setting to the other.

**Lemma A7** *There is a unique value of $q_0 = \dot{q}_0$ that corresponds with a unique value of $V_0 = \dot{V}_0$ such that*

- *if $V_0 > \dot{V}_0$ ($q_0 < \dot{q}_0$), then a firm of type 1 deviates from the partial sorting setting to perfect sorting;*
- *if $V_0 < \dot{V}_0$ ($q_0 > \dot{q}_0$), then a firm of type 1 deviates from the perfect sorting setting to partial sorting.*

***Proof*** *Part I. Deviations from partial sorting.* Suppose that a firm of type 1 deviates to a $w_1^d$ such that no uninformed worker contacts the firm, and the informed worker is indifferent between contacting the deviant and any other firm. Thus, the payoff of the deviant is exactly the same as that in the perfect sorting setting and the deviant chooses $c^d$ to maximize

$$J^d = c^d\left(1 - c^d - V_0\right).$$

Now, the profit-maximizing $c^d$ equals $(1 - V_0)/2$ and the deviation profit is given by

$$J^d = \frac{1 - V_0}{2}\left(\frac{1 + V_0}{2} - V_0\right) = \frac{(1 - V_0)^2}{4}.$$

The deviation is unprofitable if $J^d < J_1$. The payoff of the firm if it follows the partial sorting setting is

$$J_1 = \left(1 - (1 - c)\, e^{-q_1}\right)\left(1 - c - \frac{V_0}{\beta_1^w}\right).$$

Hence, the deviation is not profitable if

$$\left(1 - (1 - c)\, e^{-q_1}\right)\left(1 - c - \frac{V_0}{\beta_1^w}\right) - \frac{(1 - V_0)^2}{4} > 0 \tag{A21}$$

We take the derivative of the LHS of (A21) with respect to $V_0$. The derivative is

$$\frac{\partial c}{\partial V_0}\left(e^{-q_1}\left(2 - 2c - \frac{V_0}{\beta_1^w}\right) - 1\right) - \frac{1 - (1 - c)\, e^{-q_1}}{\beta_1^w} + \frac{1 - V_0}{2}, \tag{A22}$$

where the value of $c$ is given by (A13); thus,

$$\frac{\partial c}{\partial V_0} = -\frac{1}{2\beta_1^w}\left(1 - \frac{\left(\frac{\alpha_1^w}{2} + \beta_1^w - \frac{V_0\alpha_1^w}{\beta_1^w}\right)}{\sqrt{\left(-\frac{3\alpha_1^w}{2} + \beta_1^w - \frac{V_0\alpha_1^w}{\beta_1^w}\right)^2 + 4\alpha_1^w\left(-\frac{V_0\alpha_1^w}{\beta_1^w} - \frac{1}{2}\alpha_1^w + \frac{\beta_1^w}{2}\right)}}\right). \tag{A23}$$

We then observe that

$$\left(\frac{\alpha_1^w}{2} + \beta_1^w - \frac{V_0\alpha_1^w}{\beta_1^w}\right)^2 - \left(-\frac{3\alpha_1^w}{2} + \beta_1^w - \frac{V_0\alpha_1}{\beta_1^w}\right)^2$$
$$- 4\alpha_1^w\left(-\frac{V_0\alpha_1^w}{\beta_1^w} - \frac{1}{2}\alpha_1 + \frac{\beta_1^w}{2}\right) = 2\alpha_1\beta_1^w > 0.$$

Therefore, the fraction in the parentheses of (A23) is greater that 1 and (A23) is positive.

Moreover, because $c \geq 1/2$ in this case, we find that

$$e^{-q_1}\left(2 - 2c - \frac{V_0}{\beta_1^w}\right) - 1 \leq e^{-q_1}\left(1 - \frac{V_0}{\beta_1^w}\right) - 1 < 0.$$

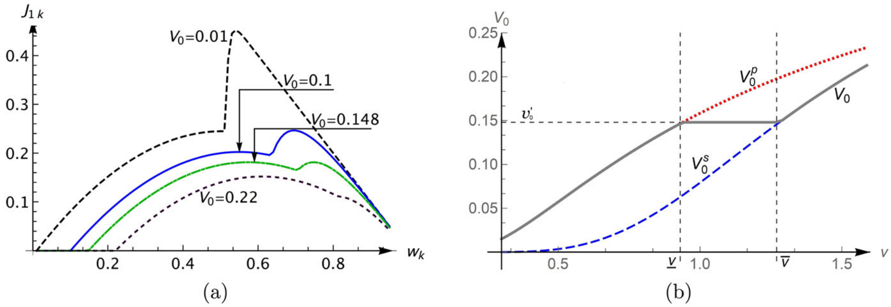


**Fig. 4** The profit of firm $k$ of type 1 for different $w_k$ and $V_0$ (Figure a) and $V_0$ assuming partial sorting and perfect sorting strategies for firms (Figure b)

Additionally, we note that $-\frac{1-(1-c)e^{-q_1}}{\beta_1^w} + \frac{1-V_0}{2}$ is decreasing in $V_0$. Setting $V_0 = 0$ in this expression, we obtain a negative function of $q_1$ (we use (A13) for $c$)

$$-\frac{1-\left(1-c|_{V_0=0}\right)e^{-q_1}}{\beta_1^w} + \frac{1}{2} < 0.$$

Hence, (A22) is negative and (A21) is decreasing in $V_0$. If we take $q_0 \to \infty$ so that $V_0 \to 0$, the LHS of (A21) is a positive function of $q_1$ for all $q_1$. However, assuming $q_0 = q_1$, the LHS of (A21) is a negative function of $q_1$ for $q_1 < 1.45103$. Thus there is a unique implicit relationship $q_0^a(q_1)$ such that (A21) equals zero.

Next, we take the first-order condition (A3) and set $q_1 = 1.451031$. By solving the equation, we get the corresponding value of $q_0 = 2.165399 > 1.451031$. Instead, by setting $q_1 = 0$ in (A3) we obtain $q_0 < 1$. Thus, the implicit relationship $q_0^1(q_1)$ obtained from (A3) is above the implicit relationship $q_0^a(q_1)$ obtained from setting (A21) equal to zero at the point $q_1 = 1.451031$. Moreover, if we take a limit $q_1 \to 0$ on the LHS of (A21), we get the following expression

$$\frac{1}{8}\left(-2V_0^2 + 3\sqrt{4V_0^2 - 20V_0 + 9} + 10V_0 - 9\right),$$

which approaches zero when $V_0$ approaches zero, or $q_0$ approaches infinity. Thus, $q_0^1(0) < q_0^a(0)$, which implies that the functions $q_0^1(q_1)$ and $q_0^a(q_1)$ cross on the plane $q_0 \times q_1$, where $q_1 \leq \min\{q_0, 1.451031\}$ and $q_0 > 0$.

As a result, we conclude that there is a fixed point $(\dot{q}_0, \dot{q}_1)$ such that both (A3) is satisfied and the LHS of (A21) equals zero. Therefore, if $q_0 > \dot{q}_0$, then the LHS of (A21) is positive, which makes the deviation unprofitable. However, if $q_0 < \dot{q}_0$, then a firm of type 1 wants to deviate to the best $w_1^d$.

The results are illustrated in Fig. 4a, where we plot the profit of a firm of type 1 as a function of its wage for different values of $V_0$. The payoff function has two peaks. The left peak corresponds with the wage under perfect sorting ($q_1 = 0$) and the right peak with the wage under partial sorting ($q_1 > 0$) in a candidate equilibrium, where market utility equals $V_0$. The left peak is clearly higher if $V_0$ is larger.

*Part II. Deviations from perfect sorting.* Suppose that a firm of type 1 deviates from the perfect sorting setting by setting a higher wage $w_1^d$ to attract uninformed workers for one period. In this case, the matched worker of the deviant firm gets a job at his last firm with probability $\beta_d^c$. As a result, the worker is indifferent between contacting and not contacting his previous firm if $s = 1 - c^d$, which is implicitly given by

$$\beta_d^w \left( w_1^d - c^d \right) = V_0.$$

Since an uninformed worker contacting the deviant firm gets matched with probability $-\alpha_d^w c^d + \beta_d^w$, he must also be indifferent between contacting the firm and any other firm, which gives us

$$w_1^d = \frac{1}{2} + \frac{V_0}{\beta_d^w - c^d \alpha_d^w}.$$

Merging the last two equations, we obtain

$$\beta_d^w \left( 1 - c^d - \frac{1}{2} + \frac{V_0}{\beta_d^w - c^d \alpha_d^w} \right) = V_0, \tag{A24}$$

which requires a sufficient cutoff, $c^d > 1/2$.

The profit of the deviant firm of type 1 can now be written as

$$J_1^d = \left( 1 - \left( 1 - c^d \right) e^{-q_d} \right) \left( 1 - c^d - \frac{V_0}{\beta_d^w} \right).$$

The profit of a non-deviating firm of type 1 in equilibrium equals

$$J_1 = c \left( 1 - c - V_0 \right) = c^2 = \frac{(1 - V_0)^2}{4},$$

where the second equality has been obtained by using the first-order condition of a firm of type 1.

Then, the deviation is not profitable if

$$\left( 1 - \left( 1 - c^d \right) e^{-q_d} \right) \left( 1 - c^d - \frac{V_0}{\beta_d^w} \right) - \frac{(1 - V_0)^2}{4} < 0. \tag{A25}$$

We observe that the inequalities (A25) and (A21) are identical except for their signs. Hence, we arrive at identical queue lengths as before $(\dot{q}_0, \dot{q}_1)$ at the bordering case determining whether the deviation is profitable or not. □

***The proof of Proposition 1*** Proofs of Lemmata A2–A7 provide the conditions for the existence of a fixed point and a condition regarding the profitability of deviations from the prescribed wages. In this proof, we continue the characterization of the equilibrium.

Because the optimal queue lengths are determined differently, the values of $\dot{q}_0$ and $\dot{q}_1$ can be attained at perfect and partial sorting settings under different values of $\nu$ as depicted in Fig. 4b.[27] In the following analysis, we denote by $\underline{\nu}$ the value of $\nu$ at which we obtain $(\dot{q}_0, \dot{q}_1)$ in the partial sorting setting and by $\overline{\nu}$ the corresponding value of $\nu$ which gives $\dot{q}_0$ in the perfect sorting setting. In particular,

$$\overline{\nu} = \frac{2 - \dot{c}^s - e^{-\dot{q}_0}}{q_0 + \dot{c}^s \left(1 - e^{-\dot{q}_0} - \dot{q}_0\right)} \quad \text{and} \tag{A26}$$

$$\underline{\nu} = \frac{(1 - \dot{c}^p)\, e^{-q_1} + 1 - \dot{q}_0}{e^{-q_1}\dot{q}_0 + q_1 \left(1 - e^{-\dot{q}_0}\right) + \dot{c}^p \left(1 - e^{-\dot{q}_0} - e^{-q_1}\dot{q}_0\right)}, \tag{A27}$$

where $\dot{c}^p$ and $\dot{c}^s$ are associated values of the cutoff level $c$ is each setting.

The derivative of $\underline{\nu}$ with respect to $q_1$ holding $\dot{c}^p$ constant has the same sign as

$$\begin{aligned}
&- \left(1 - \dot{c}^p\right) e^{-\dot{q}_1} \left[e^{-q_1}\dot{q}_0 + \dot{q}_1 \left(1 - e^{-\dot{q}_0}\right) + \dot{c}^p \left(1 - e^{-\dot{q}_0} - e^{-\dot{q}_1}\dot{q}_0\right)\right] \\
&\quad - \left(\left(1 - \dot{c}^p\right) e^{-\dot{q}_1} + 1 - e^{-\dot{q}_0}\right) \left[-e^{-\dot{q}_1}\dot{q}_0 + 1 - e^{-\dot{q}_0} + \dot{c}^p e^{-\dot{q}_1}\dot{q}_0\right] \\
&= \left(1 - e^{-\dot{q}_0}\right) \left[-\left(1 - \dot{c}^p\right) e^{-q_1} \left(\dot{q}_1 + \dot{c}^p + 1 + \dot{q}_0\right) - 1 + e^{-\dot{q}_0}\right] \\
&< 0.
\end{aligned}$$

Thus,

$$\begin{aligned}
\underline{\nu} &< \frac{(1 - \dot{c}^p)\, e^0 + 1 - e^{-\dot{q}_0}}{e^0\dot{q}_0 + \dot{c}^p \left(1 - e^{-\dot{q}_0} - e^0\dot{q}_0\right)} = \frac{2 - \dot{c}^p - e^{-\dot{q}_0}}{\dot{q}_0 + \dot{c}^p \left(1 - e^{-\dot{q}_0} - \dot{q}_0\right)} \\
&< \frac{2 - \dot{c}^s - e^{-\dot{q}_0}}{\dot{q}_0 + \dot{c}^s \left(1 - e^{-\dot{q}_0} - \dot{q}_0\right)} = \overline{\nu}.
\end{aligned}$$

This implies that there is a range of values $\nu$ for which firms of type 1 apply mixed strategies and choose the wage for perfect sorting with probability $\gamma^s$ and the wage for partial sorting with probability $1 - \gamma^s$. As discussed in the main text, within this range of values for $\nu$, the queue length of uninformed workers at firms of type 0 is fixed at $\dot{q}_0$ and the queue length of uninformed workers at firms of type 1 is either zero if a firm sets the wage for perfect sorting or remains fixed at $\dot{q}_1 > 0$ if the firm sets the wage for partial sorting. The corresponding values of the cutoff levels are also constant within the range: $\dot{c}^p$ and $\dot{c}^s$ for partial sorting and perfect sorting, respectively. As a result, we only need to identify $\gamma^s$ and determine how it behaves when $\nu$ changes from $\underline{\nu}$ to $\overline{\nu}$.

[27] Figure 4b shows the market utility derived by restricting firms to offer either the optimal wage for perfect sorting or the optimal wage for partial sorting, respectively. The gray line depicts market utility in equilibrium under different values of $\nu$.

The value of $\gamma^s$ is found by using the adding-up condition and the steady-state condition. They are here given by

$$(\nu-\rho)\left(1-e^{-\dot{q}_0}\right)+\dot{c}^s\gamma^s\rho+\left(1-\gamma^s\right)\left(1-e^{-\dot{q}_1}\left(1-\dot{c}^p\right)\right)\rho-\rho=0 \quad \text{(A28)}$$

$$\dot{c}^s\gamma^s\rho+\dot{c}^p\left(1-\gamma^s\right)\rho+(\nu-\rho)\dot{q}_0+\left(1-\gamma^s\right)\rho\dot{q}_1-1=0 \quad \text{(A29)}$$

The LHS of (A28) is linear in both $\gamma^s$ and $\rho$. It is decreasing in $\rho$ because its derivative with respect to $\rho$ is negative:

$$\begin{aligned}
&-2+e^{-\dot{q}_0}+\dot{c}^s+\left(1-\gamma^s\right)\left(1-e^{-\dot{q}_1}\left(1-\dot{c}^p\right)-\dot{c}^s\right)\\
&\quad\leq -2+e^{-\dot{q}_0}+\dot{c}^p+\left(1-\gamma^s\right)\left(1-e^{-\dot{q}_1}\left(1-\dot{c}^p\right)-\dot{c}^p\right)\\
&\quad= -1+e^{-\dot{q}_0}+\left(1-\dot{c}^p\right)\left(\left(1-\gamma^s\right)\left(1-e^{-\dot{q}_1}\right)-1\right)\leq 0
\end{aligned}$$

The derivative of the LHS of (A28) with respect to $\gamma^s$ is also negative:

$$\begin{aligned}
&-\left(1-e^{-\dot{q}_1}\left(1-\dot{c}^p\right)-\dot{c}^s\right)\rho\\
&\quad\leq -\left(1-e^{-\dot{q}_1}\left(1-\dot{c}^p\right)-\dot{c}^p\right)\rho\\
&\quad= -\left(1-\dot{c}^p\right)\left(1-e^{-\dot{q}_1}\right)\rho\leq 0.
\end{aligned}$$

Thus, the implicit relationship $\gamma^s(\rho)$ that we denote by $\gamma_1^s(\rho)$ is decreasing in $\rho$. Furthermore, the whole curve moves up when $\nu$ increases because the derivative of the LHS of (A28) with respect to $\nu$ is positive: $1-e^{-\dot{q}_0}\geq 0$.

The implicit relationship $\gamma_2^s(\rho)$ is obtained from (A29) and it is unique because (A29) is linear in both $\gamma^s$ and $\rho$. The derivative of the LHS of (A29) with respect to $\gamma^s$ is negative:

$$\dot{c}^s\rho-\dot{c}^p\rho-\rho\dot{q}_1\leq 0$$

In contrast, the derivative of the LHS of (A29) with respect to $\rho$ is positive

$$\begin{aligned}
&\dot{c}^s\gamma^s+\dot{c}^p\left(1-\gamma^s\right)-\dot{q}_0+\left(1-\gamma^s\right)\dot{q}_1\\
&\quad=\dot{q}_1-\dot{q}_0+\dot{c}^p-\gamma^s\left(\dot{q}_1+\dot{c}^p-\dot{c}^s\right)\\
&\quad\leq \dot{q}_1-\dot{q}_0+\dot{c}^p\approx -0.211654<0.
\end{aligned}$$

To derive the final inequality we solve numerically $\dot{q}_0$, $\dot{q}_1$, and $\dot{c}^p$ from (A3) and (A21), where the latter is applied as an equality. Hence, the implicit relationship $\gamma_2^s(\rho)$ is decreasing in $\rho$ and moves up when $\nu$ increases. As a result, if the solution to the system (A28) and (A29) exists, then $\gamma^s$ is increasing in $\nu$.

To be sure that $\gamma^s\in(0,1)$, we look at the solution of (A28) and (A29). We make use of the exact values of $\dot{q}_0$, $\dot{q}_1$, $\dot{c}^p$, $\dot{c}^s$, which we can easily derive numerically. The values are given by $\dot{q}_0 \approx 1.21707$, $\dot{q}_1 \approx 0.44050$, $\dot{c}^p \approx 0.56491$, $\dot{c}^s \approx 0.425976$. This immediately pins down the bordering values: $\underline{\nu} = 0.938389$ and $\overline{\nu} = 1.27988$.

Finally, we consider uniqueness by studying Eqs. (A28) and (A29). If $\gamma^s = 1$, the values of $\rho$ given by each equation are: $\rho_1 = 0.550816\nu > -1.26408 + 1.53847\nu = \rho_2, \forall \nu \in \left(\underline{\nu}, \overline{\nu}\right)$. If we set $\gamma^s = 0$, the corresponding values of $\rho$ are $\rho_1 = 0.715366\nu < -4.72467 + 5.75024\nu = \rho_2 \; \forall \nu \in \left(\underline{\nu}, \overline{\nu}\right)$.

When $\nu = \underline{\nu}$, $\gamma^s = 0$, and we thus conclude that for $\nu < \underline{\nu}$ there can be only a partial sorting equilibrium. If $\nu = \overline{\nu}$, then $\gamma^s = 1$ and we also conclude that for $\nu > \overline{\nu}$, there can be only a perfect sorting equilibrium. □

***Proof of Proposition 2*** To prove the existence of the equilibrium, we must show that there exist $\hat{q}_0 > 0$ and $\hat{c}$ that solve the first-order condition of firms and the steady-state condition.

Equation (21) defines the relationship between $\hat{c}$ and $\hat{q}_0$. This relation is denoted here by $\hat{c}_1(\hat{q}_0)$. Equation (18) provides the relationship between $\hat{c}$ and $\hat{q}_0$. We denote this second relationship by $\hat{c}_2(\hat{q}_0)$. In what follows we show that $\hat{c}_1$ and $\hat{c}_2$ cross in the plane $\hat{q}_0 \times \hat{c}$, where $\hat{q}_0 \in \mathbb{R}_+$ and $\hat{c} \in [1/2, 1]$.

The derivative of $\hat{c}_1$ with respect to $\hat{q}_0$ is

$$\frac{\partial \hat{c}_1}{\partial \hat{q}_0} = \frac{(1-\nu)\left(\nu\left(1 - e^{-\hat{q}_0} - \hat{q}_0 e^{-\hat{q}_0}\right) + e^{-\hat{q}_0}\right)}{\left[\hat{q}_0 \nu - \nu\left(1 - e^{-\hat{q}_0}\right) - e^{-\hat{q}_0}\right]^2}.$$

<u>Case I</u>. Let us first assume that $\nu > 1$. Then, $1 - \nu < 0$ and $\hat{c}_1$ is decreasing in $\hat{q}_0$ (we observe that $1 - e^{-\hat{q}_0} - \hat{q}_0 e^{-\hat{q}_0} > 0$). If $\hat{q}_0$ approaches zero, then $\hat{c}_1$ equals one. When $q \to 1/\nu$, the value of $\hat{c}_1$ approaches zero. The value of $\hat{q}_0$ which makes the denominator of the fraction equal to zero is $1 + W\left(\frac{1-\nu}{e\nu}\right) > 1/\nu$.

In addition, $1 + W\left(\frac{1-\nu}{e\nu}\right) < 1$. Thus, we consider $\hat{q}_0 < 1$ and obtain that the relationship $\hat{c}_2$ is increasing in $\hat{q}_0$. When $\hat{q}_0 = 0$, then $\hat{c}_2 = 1/2$. If $\hat{q}_0$ approaches one, then $\hat{c}_2 = \left(1 + e^{-1}\right)/2 > 1/2$. As a result, we conclude that if $\nu > 1$, then there is a unique crossing point of $\hat{c}_1$ and $\hat{c}_2$.

<u>Case II</u>. Now we take $\nu < 1$. Then $1 - \nu > 0$, and $\hat{c}_1$ is increasing in $\hat{q}_0$. When $\hat{q}_0$ approaches $1 + W\left(\frac{1-\nu}{e\nu}\right)$ from the left, then $\hat{c}_1$ approaches infinity. When $\hat{q}_0$ approaches $1 + W\left(\frac{1-\nu}{e\nu}\right)$ from the right, then $\hat{c}_1$ approaches minus infinity. Because the solution $\hat{c}$ must be less than one, we consider the values of $\hat{q}_0$ to the right from $1 + W\left(\frac{1-\nu}{e\nu}\right) > 1$ further on.

The relationship $\hat{c}_2$ is decreasing in $\hat{q}_0$ for $\hat{q}_0 > 1$. We have that $\lim_{\hat{q}_0 \to 1 + W\left(\frac{1-\nu}{e\nu}\right)} \hat{c}_2$ is finite and positive, $\lim_{\hat{q}_0 \to \infty} \hat{c}_2 = 1/2$ and $\lim_{\hat{q}_0 \to \infty} \hat{c}_1 = 1$. So, we find that there is a unique crossing point of $\hat{c}_1$ and $\hat{c}_2$ also when $\nu < 1$.

If $\nu \to 1$, then $1 + W\left(\frac{1-\nu}{e\nu}\right) = 1$, $\hat{q}_0 = 1$ and $\hat{c} = \left(1 + e^{-1}\right)/2$.

*Wages.* The wage of a firm of type 0 is $\hat{w}_0 = \frac{1}{2}\left(1 + \frac{e^{-\hat{q}_0}\hat{q}_0}{1 - e^{-\hat{q}_0}}\right)$. The highest wage of a firm of type 1 is $\hat{w}_{1\hat{c}} = \frac{1}{2}\left(1 + e^{-\hat{q}_0} + e^{-\hat{q}_0}\hat{q}_0\right)$. The difference $\hat{w}_0 - \hat{w}_{1\hat{c}}$ has the same sign as $e^{-\hat{q}_0} - 1 + \hat{q}_0 e^{-\hat{q}_0} < 0$. Now, the lowest wage that a firm of type 1 pays is $\hat{V}_0$. This entails that $\hat{w}_0 - \hat{V}_0$ is proportional to $1 - 2e^{-\hat{q}_0} + e^{-\hat{q}_0}\hat{q}_0 + e^{-2\hat{q}_0} > 0$ □

***Proofs of Propositions 4 and 5*** *Part I. The proof that $c < \hat{c}$.* First, we take $\nu > \overline{\nu}$. In this case, $c$ is between 1/4 and 1/2, whereas $\hat{c} \in [1/2, 1]$. Hence, $c < \hat{c}$.

Next, we move to $\nu < \underline{\nu}$. Replacing $q_1$ with $q_0$ in the equation that sums up the workers (Eq. (1)) we obtain

$$(\nu - \rho) q_1 + \rho q_1 + \rho c < 1 \quad \Rightarrow \quad \rho < \frac{1 - \nu q_1}{c}.$$

From the steady-state condition, because $q_1 < q_0$ we obtain

$$(\nu - \rho) \left(1 - e^{-q_1}\right) - e^{-q_1} (1 - c) \rho < 0. \tag{A30}$$

Further,

$$\begin{aligned} (\nu - \rho) \left(1 - e^{-q_1}\right) - e^{-q_1} (1 - c) \rho &= \nu \left(1 - e^{-q_1}\right) - \left(1 - e^{-q_1} c\right) \rho \\ &> \nu \left(1 - e^{-q_1}\right) - \left(1 - e^{-q_1} c\right) \frac{1 - \nu q_1}{c} \end{aligned} \tag{A31}$$

Because (A30) is negative, we obtain that the second line of (A31) is negative and

$$c < \frac{1 - \nu q_1}{\nu \left(1 - e^{-q_1} - e^{-q_1} q_1\right) + e^{-q_1}} < \frac{1 - \nu q_1}{\nu \left(1 - e^{-q_1} - q_1\right) + e^{-q_1}}. \tag{A32}$$

Note that the RHS of (A32) resembles (21) where $\hat{q}_0$ is replaced with $q_1$. Next, we apply a proof by contradiction. Assume that $\hat{c} < c$. This, together with (A32), would imply that $\hat{q}_0 < q_1 < q_0$ and $\hat{V}_0 > V_0$. By employing Eq. (13), we obtain that

$$c < \frac{1}{2} + \frac{\hat{V}_0}{\beta_1^w - c\alpha_1^w} - \frac{\hat{V}_0}{\beta_1^w} = \hat{c} - \hat{q}_0 \hat{V}_0 + \hat{V}_0 \frac{c\alpha_1^w}{\beta_1^w \left(\beta_1^w - c\alpha_1^w\right)}, \tag{A33}$$

where the equality was obtained by the fact that $\hat{c} = 1/2 + \hat{q}_0 \hat{V}_0$.

We take the value of $c$ given by (A13). If we replace $V_0$ with $e^{-q_1}/2$ on the RHS of that equation, then the equation becomes a strict inequality with '<' sign. The highest possible values of the RHS of the inequality (it is a function of $q_1$ only) is 0.596705. Hence, $c < 0.6$.

The fraction $\frac{\hat{V}_0 \alpha_1^w c}{\beta_1^w (\beta_1^w - \alpha_1^w c)}$ is increasing in $c$. As a result $\frac{\hat{V}_0 \alpha_1^w c}{\beta_1^w (\beta_1^w - \alpha_1^w c)} < \frac{\hat{V}_0 \alpha_1^w \cdot 0.6}{\beta_1^w (\beta_1^w - \alpha_1^w \cdot 0.6)}$. Moreover, to have $c < \hat{c}$, it must be true that

$$\hat{q}_0 < \frac{\alpha_1^w c}{\beta_1^w \left(\beta_1^w - \alpha_1^w c\right)} < \frac{\alpha_1^w \cdot 0.6}{\beta_1^w \left(\beta_1^w - \alpha_1^w \cdot 0.6\right)}$$

Further, we examine if the condition $\hat{q}_0 < \frac{0.6\alpha_1^w}{\beta_1^w (\beta_1^w - 0.6\alpha_1^w)}$ holds. The maximum of $\frac{0.6\alpha_1^w}{\beta_1^w (\beta_1^w - 0.6\alpha_1^w)}$ is 0.660141. When $\nu < 1$, $\hat{c}_2$ given in the proof of Proposition 2 is increasing in $\nu$. Thus, the solution $\hat{q}_0$ is decreasing in $\nu$. When $\nu = 0.94$, the value of

$\hat{q}_0$ that makes $\hat{c}_1 = \hat{c}_2$ is 1.15766. Therefore, we conclude that $\hat{q}_0 > \frac{0.6\alpha_1^w}{\beta_1^w(\beta_1^w - 0.6\alpha_1^w)}$ and $\hat{c} > c$.

*Part II. The comparison of wages of type* 0 *firms.* We begin with the proof of $\rho < \hat{\rho}$. First, we analyze values of $\nu > \overline{\nu}$ assuming the setting of Sect. 3. Then, we make the following unexpected change in the prediction, separations, and wages: all informed workers leave and contact new firms if $s_l > c$, and all firms of type 1 note this and set $w_0$ thereafter. If an informed worker contacts the same firm again, his firm matches with him at $w_1$. Hence, we have a setting of Sect. 4 with the wages and thresholds of Sect. 3.

With this change, the total number of workers who contact firms of type 0 remains constant. However, the number of firms offering $w_0$ increases by $\rho\,(1 - c)$. Thus, the queue length at a firm offering $w_0$ decreases to $(1 - \rho c)\,/\,(\nu - \rho + \rho\,(1 - c))$.[28]

The number of workers who are employed also changes by

$$\rho c + (1 - \rho c)\,\frac{1 - e^{-\frac{1-\rho c}{\nu-\rho c}}}{\frac{1-\rho c}{\nu-\rho c}} - (1 - \rho c)\,\frac{1 - e^{-\frac{1-\rho c}{\nu-\rho}}}{\frac{1-\rho c}{\nu-\rho}} - \rho c$$
$$= (1 - \rho c)\left(\frac{1 - e^{-\frac{1-\rho c}{\nu-\rho c}}}{\frac{1-\rho c}{\nu-\rho c}} - \frac{1 - e^{-\frac{1-\rho c}{\nu-\rho}}}{\frac{1-\rho c}{\nu-\rho}}\right).$$

The probability of matching $\beta_0^w$ is decreasing in $q_0$. As a result, we conclude that there are more workers who are employed after the change. We know that $c < 1/2 < \hat{c}$. Therefore, there are fewer informed workers that repeatedly contact firms of type 1 in Sect. 3 compared to the setting in Sect. 4. As a result, there are fewer workers who become employed in the setting of Sect. 3 compared to our new hybrid setting. In addition, there fewer workers who are employed in our hybrid setting compared to the setting in Sect. 4. Altogether, this entails that $\hat{\rho} > \rho$.

The ratio $(1 - \rho c)\,/\,(\nu - \rho c)$ is decreasing in $\rho c$. Because $c < 1/2 < \hat{c}$ and $\rho < \hat{\rho}$, we obtain that

$$q_0 > \frac{1 - \rho c}{\nu - \rho c} > \frac{1 - \hat{\rho}\hat{c}}{\nu - \hat{\rho}\hat{c}} = \hat{q}_0.$$

The result $q_0 > \hat{q}_0$ implies tougher competition for workers, which gives $V_0 < \hat{V}_0$ and $w_0 < \hat{w}_0$.

We then take the case of $\nu < \underline{\nu} < 1$ of Sect. 3 and make the same hybrid change as before. Now, the number of contacts that a firm of type 0 obtains increases to $(1 - \rho c)\,/\,(\nu - \rho c)$. Because $\nu < 1$, this ratio is increasing in $\rho c$.

The number of firms that are able to match with a worker increases after this change because the firms of type 1 whose informed workers have a low match value $s_l \leq c$ are matched with probability one as before. On the other hand, the rest of the firms of

[28] Because $\nu > 1$, the queue length decreases when there are more firms. If $\nu < 1$, then the same shift of some firms of type 1 would make the queues longer.

type 1 and the firms of type 0 get workers more often because their queues become longer.

In addition, we know that $\hat{c} > c$. Therefore, there is more hiring in Sect. 4, which shows up in $\hat{\rho} > \rho$. Therefore,

$$q_0 < \frac{1-\rho c}{\nu - \rho c} < \frac{1-\hat{\rho}\hat{c}}{\nu - \hat{\rho}\hat{c}} = \hat{q}_0.$$

In other words, longer queue lengths imply more relaxed hiring competition for workers in this case, which gives $w_0 > \hat{w}_0$ and $V_0 > \hat{V}_0$.

*Part III. The comparison of wages of type* 1 *firms for large* $\nu$. We have that $\nu > \overline{\nu}$ and use the fact that $V_0 = e^{-q_0}/2$ and obtain

$$w_1 = \frac{e^{-q_0}}{2} + c = \frac{2 + e^{-q_0}}{4}.$$

Similarly, we obtain

$$E\left[\hat{w}_1\right] = \hat{V}_0 + \frac{\hat{c}}{2} = \frac{1 + \hat{q}_0 e^{-\hat{q}_0} + 2e^{-\hat{q}_0}}{4}.$$

We observe that $w_1$ is decreasing in $q_0$ and $E[\hat{w}_1]$ is decreasing in $\hat{q}_0$. As a result, the difference between $w_1$ and $E[\hat{w}_1]$ is less than if we use the smallest $q_0$ and the greatest $\hat{q}_0$.

Because $c$ in (A8) is decreasing in $q_0$ and $c \leq 1/2$, the smallest $q_0$ is where $c = 1/2$ in (A8): $q_0 = q_z = \left(3 - \nu + \nu W\left(\frac{e^{1-\frac{3}{\nu}}(\nu-2)}{\nu}\right)\right)/\nu$. Additionally, we know that $\hat{c} \geq 1/2$, $\hat{c}$ is decreasing in $\hat{q}_0$ in (21). Thus, $\hat{q}_0 \leq \hat{q}_z = \left(2 - \nu + \nu W\left(\frac{e^{1-\frac{2}{\nu}}(\nu-1)}{\nu}\right)\right)/\nu$. Then we plot $1 + e^{-q_z} - \hat{q}_z e^{-\hat{q}_z} - 2e^{-\hat{q}_z}$ for $\nu > 1$, which shows that the difference is negative for $\nu > 11/10$ and $w_1 < E\left[\hat{w}_1\right]$.

*Part IV. The comparison of wages of type* 1 *firms for small* $\nu$. Next, we take $\nu < \underline{\nu}$. Because $c < \hat{c}$ and $\hat{V}_0 < V_0$, the following expression holds

$$E\left[\hat{w}_1\right] - w_1 = \hat{V}_0 + \frac{\hat{c}}{2} - \frac{V_0}{\beta_1^w} - c.$$

From the proof of Proposition A7 Part II we have that $c$, given by (A12) is increasing in $V_0$. Therefore, $-V_0/\beta_1^c - c$ is decreasing in $V_0$ and increasing in $q_0$. Hence,

$$\hat{V}_0 + \frac{\hat{c}}{2} - \frac{V_0}{\beta_1^w} - c \leq \hat{V}_0 + \frac{\hat{c}}{2} - c|_{V_0=0}$$

The expected wage $E\left[\hat{w}_1\right]$ is decreasing in $\hat{q}_0$. Because $\hat{q}_0 > q_1$, we obtain that

$$\hat{V}_0 + \frac{\hat{c}}{2} - c|_{V_0=0} \leq \frac{e^{-q_1}}{2} + \frac{1 + q_1 e^{-q_1}}{4} - c|_{V_0=0}$$

The last expression is less than zero for $q_1 > 1.455$. This shows that $E\left[\hat{w}_1\right] < w_1$. □

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