Coalitions with limited coordination

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Abstract

We study economies endowed with a market structure, where coalitions can form within each market but have no enforcement possibilities across markets. A standard cooperative game cannot be defined in this case. We develop a certain equilibrium notion which ties together the cores of the different markets. We provide an existence result and an application to economies with asymmetric information.

Keywords Core · Multiple proposals · Incomplete coordination

JEL Classification C71 · D50 · D82

1 Introduction

The standard definition of the core of an exchange economy, as formalized in Debreu and Scarf (1963), is based on the idea of blocking: any group of agents can ‘block’ a proposed allocation by involving in trades among themselves, which are improving for the members of the group over the proposed allocation. The core is defined then as a feasible allocation which is not blocked by any coalition. There is one important detail in this definition that we will focus on: it involves *one* proposed allocation (or, equivalently, one set of net trades) which is then tested against deviations by coalitions.

This scenario reflects the idea that there is a single ‘clearing house’ where all exchange takes place. Let us consider now the scenario where there are several ‘clearing houses’ where trades are being negotiated, i.e., coalitions are faced with multiple trade proposals. In such case one has to decide the extent of coordination power of coalitions: if coalitions are assumed able to coordinate the activities of their members

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throughout the spectrum of proposals (complete coordination) then proposals can be jointly evaluated and accepted or blocked. In such a case the outcome of the negotiations is the same as in the standard definition of the core. If on the other hand the coordination powers of coalitions are limited (incomplete coordination) then one is faced with the problem of developing a convincing equilibrium concept.

The central idea in this paper is that a coalition may not be able to enforce cooperation in all negotiations. Even if such a possibility exists, it is not clear that agents would prefer such an institutional arrangement over a more loose agreement that binds them together in a subset of the negotiations. In other words, it might be in the strategic interest of agents to cooperate only in a specified subset of the proposals.

The idea of incomplete coordination has already cropped up in many guises in the literature. One reference to the issue, which is not the earliest we found but refers to historical considerations relating to incomplete coordination is Vind (1995). The author there motivates his definition of an exchange equilibrium¹ as follows:

The members of a coalition—in the definition of the core—have to give up all previous exchanges outside the coalition and have to refrain from having exchanges outside the coalition. These requirements are against the whole spirit of classical economics, nothing which can be interpreted as those limitations on the actions available to consumers can be found in Edgeworth, in fact they are explicitly ruled out. [Vind (1995), page 1735]

We refrain from making any scholarly remarks on the subject, but the relevance of incomplete coordination for the concept of an exchange equilibrium is unmistakable.

The issue has appeared more often in the context of dynamic economies. It is well known since the early work of Gale (1978) and Repullo (1988), the later work of Koutsougeras (1999) and Koutsougeras and Shafer (1993), which focuses on asset markets, and Koutsougeras (1998) which applies these issues in asymmetric information economies up to the recent work of Herings et al. (2002), that in a dynamic framework coalitions may form at any point in time in order to block intertemporal allocations. Briefly, the dynamic framework allows for the possibility of *transient* cooperation, namely that agents may be cooperating in some periods and not in others. Although, the temporal framework embodies some stylized facts specific to that context, which further complicate matters, the relevance of incomplete coordination is again unmistakable.

The issue however, arises even in the standard static general equilibrium pure exchange framework once one allows for multiple ‘clearing houses’ where proposals are made or, alternatively, for multistage negotiations. In general, the issue appears whenever one introduces some institutional structure to the exchange among coalitions. In this paper we consider a standard static general equilibrium model, where the need for multiple negotiations arises from the fact that there are multiple proposals for net trades. We like to think of multiple ‘clearing houses’ or ‘trading posts’ proposing net trades. The span of net trades proposed in each trading post could be any linear subspace of the commodity space. One example is to allow the whole space for each trading post, i.e., all commodities can be traded in each trading post. Another example

¹ In fact, versions of this concept had been developed earlier in Vind (1983).

is to partition the set of commodities into subsets and identify trading posts with each subset of commodities, i.e., only a certain subset of commodities can be traded in each trading post.² Above and beyond all combinations of subspaces of net trades, with perhaps several common dimensions may be considered.

In such a context, the inability of coalitions to keep their members together in all negotiations suggests that one cannot define a single cooperative game. The reason is that in order to do so, one should be able to define what the possibilities of a coalition are, i.e., what a coalition can do on its own. But this is not possible because, given the limited enforcement of coalitions, their possibilities depend on activities of its members outside the coalition. Therefore, one can define cooperative games *conditional* on the activities of its members outside the coalition. With respect to the scenario laid out above this means that, the best we can hope is to define one *conditional cooperative game* for each of the trading posts in the model. The key issue is how those games are to be tied together. Here, we focus on the properties of a particular way to tie together the cores of the respective trading posts, which has appeared repeatedly in various contexts in the literature referred to above.

2 The model

Let H be a finite set of agents and L a finite set of commodity types in the economy. The consumption set of each agent is identified with $\mathfrak{R}_+^L$. Each individual is characterized by a preference relation, which is representable by a utility function $u_h : \mathfrak{R}^L \rightarrow \mathfrak{R}$, and an initial endowment $e_h \in \mathfrak{R}_+^L$. An economy is defined as $\mathcal{E} = \{(\mathfrak{R}_+^L, u_h, e_h) : h \in H\}$.

The institutional structure of the economy is described as follows: there is a set of ‘clearing houses’ T where net trades are being proposed. The reader may like to think of a clearing house as a ‘trading post’ where net trades are negotiated.

A net trade $z^t \in \mathfrak{R}^{LH}$ proposed in ‘market place’ t is *feasible* if $\sum_{h \in H} z_h^t = 0$. A *profile* is a $\mathbf{z} = (z^t)_{t \in T} \in \mathfrak{R}^{LHT}$ of feasible net trades in each market place. Preferences over profiles of net trades $\mathbf{z} \in \mathfrak{R}^{LHT}$ are naturally induced by the utility functions of individuals: $U_h(\mathbf{z}_h) = u_h(e_h + \sum_{t \in T} z_h^t)$. We define the following:

Definition 1 Given a profile of net trades $\mathbf{z}$ we say that a coalition $S \subseteq H$ blocks $z^t \in \mathfrak{R}^{LH}$ which is proposed in market place t , if there is a $\hat{z}^t \in \mathfrak{R}^{LS}$ such that $\sum_{h \in S} \hat{z}_h^t = 0$ and $U_h(\mathbf{z}_h^{-t}, \hat{z}_h^t) > U_h(\mathbf{z}_h)$ for all $h \in S$, where $\mathbf{z}_h^{-t} = (z_h^r)_{r \neq t}$.

Definition 2 Given a profile of net trades $\mathbf{z}$, the core of the market place t , denoted as $C(\mathbf{z}^{-t})$, is the set of feasible net trades in the market place t that are not blocked by any coalition.

We denote by $\mathcal{C}(\mathbf{z}^{-t})$ the set of payoffs corresponding to the elements of $C(\mathbf{z}^{-t})$

² While we are it, we can restrict the span to certain subsets of the agents, i.e., certain markets reserved for particular groups.

Definition 3 The profile $\mathbf{z}$ is a **core-equilibrium** of the economy $\mathcal{E}$ characterized by T institutions, denoted $C^T(\mathcal{E})$, if and only if it is individually rational and

$$(U_h(\mathbf{z}_h))_{h \in H} \in \bigcap_{t \in T} C(\mathbf{z}^{-t})$$

The interpretation that we have in mind is as follows: there are T independent institutions where net trades are negotiated. We interpret them as ‘market places’ or as ‘clearing houses’, hence our requirement that the proposed net trades in each market place clear the markets for the commodities traded in that market place. The proposals are jointly evaluated on a conditional basis. Coalitions are of a limited scope: they may form in each market place in order to block a proposal in that market place, but cannot enforce cooperation outside that market place, i.e., it has no jurisdiction in other markets. Of course, this is just a benchmark case which makes the exposition of our ideas more simple. On a different level one could imagine coalitions with less limited scope, that could cooperate in several market places at once. This issue is still captured, at least in part, by the definition above. Simply notice that the proposals in some market place may regard net trades among a subset of agents only, or a subset of the commodities. There are many combinations of trading groups and subsets of traded commodities in each market place. The main issue regarding the relevance of institutions would continue to be essential as long as cooperation is incomplete in the sense that coalitions, perhaps for strategic reasons, do not have jurisdiction throughout the range of markets. This notion has cropped up, with minor differences, in various contexts such as in Repullo (1988) or Zhao (1992).

3 A model of exchange in subsets of commodities

Consider a partition of the set of commodities L , into nonempty subsets $(A^t)_{t \in T}$, where $\#A^t \geq 2$. We associate a trading post with each subset of commodities in the partition of L . The interpretation is that in the trading post t (only) commodities in each block A^t can be traded against one another. We like to think of the partition of the set of commodities into subsets, as representing a lack of means to contract exchanges across the whole array of commodities. Coalitions are assumed to be subject to the same constraints in contracting and so they are able to make binding agreements in each trading post t but have no enforcement powers across posts.³ In other words, a complete description of a coalition requires a specification of a particular trading post that it is associated with. This setup does not allow the development of a single cooperative game, because the possibilities of coalitions (e.g., their NTU value) cannot be defined without reference to activities outside their scope. For this reason the standard cooperative concepts do not fit this context. Instead, this context calls for the development of a family of ‘conditional’ cooperative games, each associated with trades in each trading post, which are tied together.

³ The case where coalitions may have at their disposal varying means of contracting and thus can operate across alternative trading posts is a formidable challenge. We leave this issue aside for the moment.

It will be convenient to express trades in each trading post $t = 1, 2, \dots, T$ as follows. For each t consider the $L \times L$ projection matrix⁴

$$\chi_{A_t} = \begin{pmatrix} c_1 & 0 & \cdots & 0 \\ 0 & \cdots & c_i & \cdots & 0 \\ 0 & \cdots & & & c_L \end{pmatrix}, \text{ where } c_i = \begin{cases} 1 & \text{if } i \in A_t \\ 0 & \text{if } i \notin A_t \end{cases} \quad (1)$$

The set of available net trades for an individual $h \in H$ in the trading post t , is given by:

$$Z_h^t = \left\{ z \in \mathbb{R}^L : z = w\chi_{A_t}, \text{ for } e_h + w \in \mathbb{R}_+^L \right\}$$

The net trades an individual $h \in H$ across all trading posts $t = 1, 2, \dots, T$ can be summarized by an array $\mathbf{z}_h = (z_h^t)_{t=1}^T \in \prod_{t=1}^T Z_h^t \subset \mathbb{R}^{LT}$. As in the setup of our model, such an array of net trades $\mathbf{z}_h$ gives rise to an overall net trade $z_h = \sum_{t=1}^T z_h^t \in \mathbb{R}^L$ and preferences over arrays of net trades are induced in a natural way by $U_h : \prod_{t=1}^T Z_h^t \rightarrow \mathbb{R}$, which is defined as $U_h(\mathbf{z}_h) = u_h(e_h + \sum_{t=1}^T z_h^t)$.

The set of feasible net trades among any group $S \subseteq H$ in each trading post t is thus

$$F_S^t = \left\{ z \in \prod_{h \in S} Z_h^t : \sum_{h \in S} z_h = 0 \right\}$$

Let $Z = \prod_{i=1}^T F_H^t \subset \mathbb{R}^{LHT}$ denote the set of possible profiles of net trades, with typical element $\mathbf{z} \in Z$. In what follows we will consider economies where preferences satisfy the following assumption:

Assumption 1 For each $h \in H$, $u_h(\cdot)$ is strictly concave, differentiable and non satiated.

Let us consider an array of trades $\mathbf{z} \in Z$ and focus on the market place t . We consider the sub-economy $\mathcal{E}_{\mathbf{z}^{-t}} = \{(Z_h^t, U_h^t, e_h^t) : h \in H\}$ where: $A^t \subset L$ is the relevant subset of commodities, Z_h^t is the choice space, the preferences are $U_h^t(w) = U_h(w, \mathbf{z}_h^{-t})$ for $w \in Z_h^t$ and the endowment of each individual is $e_h^t = 0 \in Z_h^t$. We will proceed now to derive a cooperative exchange game for each sub-economy in the family generated in this way and define the cores of those cooperative games. The standard results for the non emptiness of the core of these games can be applied, but this is not sufficient for our purpose. Our objective is to find a profile $\mathbf{z} \in Z$ with the property that the family of cores so derived has a non empty intersection in utility space.

Let $\mathcal{H}$ denote the family of all nonempty subsets of H . We derive an NTU exchange game $(H, V_{\mathbf{z}^{-t}})$ in the trading post t , where $V_{\mathbf{z}^{-t}} : \mathcal{H} \rightarrow \mathbb{R}^H$, by defining:

$$V_{\mathbf{z}^{-t}}(S) = \left\{ w \in \mathbb{R}^H : w_h \leq U_h^t(y_h) \forall h \in S, \text{ for some } y \in F_S^t \right\}$$

⁴ This projection mapping fits the particular structure of trades proposed here. Alternative structures of trades can be expressed by means of appropriate projection mappings.

The core of the NTU game in the market t can be expressed as

$$\mathcal{C}(\mathbf{z}^{-t}) = V_{\mathbf{z}^{-t}}(H) / \bigcup_{S \in \mathcal{H}} \text{int } V_{\mathbf{z}^{-t}}(S)$$

In the sequel we will need to work with the associated family of TU games constructed as follows. Let Δ_H denote the $H - 1$ dimensional simplex. For a given $\lambda \in \Delta_H$, we define a (conditional) characteristic function $v_{\mathbf{z}^{-t}}^\lambda : \mathcal{H} \rightarrow \Re$ as follows:

$$\forall S \subseteq H, \quad v_{\mathbf{z}^{-t}}^\lambda(S) = \max \left\{ \sum_{h \in S} \lambda_h \cdot U_h^t(y_h) : y \in F_S^t \right\}$$

The function $v_{\mathbf{z}^{-t}}^\lambda(\cdot)$ is well defined. The TU exchange game in the market for commodity i , denoted $(H, v_{\mathbf{z}^{-t}}^\lambda)$, is defined by the sets:

$$V_{\mathbf{z}^{-t}}^\lambda(S) = \left\{ x \in \Re^H : \sum_{h \in S} \lambda_h \cdot x_h \leq v_{\mathbf{z}^{-t}}^\lambda(S) \right\}$$

The core of this game is defined as:

$$\hat{\mathcal{C}}(\lambda, \mathbf{z}^{-t}) = \left\{ x \in \Re^H : \sum_{h \in H} \lambda_h x_h \leq v_{\mathbf{z}^{-t}}^\lambda(H), \quad \sum_{h \in S} \lambda_h x_h \geq v_{\mathbf{z}^{-t}}^\lambda(S), \quad \forall S \subset H \right\}$$

We have the following lemma:

Lemma 1 $\hat{\mathcal{C}}(\lambda, \mathbf{z}^{-t})$ is nonempty, convex and compact. Moreover, the mapping $(\lambda, \mathbf{z}^{-t}) \mapsto \hat{\mathcal{C}}(\lambda, \mathbf{z}^{-t})$ is u.h.c.

Proof See “Appendix”.

We recall here the following fact that associates the TU game with the related NTU game:

Fact 1 If $x \in \hat{\mathcal{C}}(\lambda, \mathbf{z}^{-t})$ then the utility allocation x cannot be improved upon in the NTU game $(H, V_{\mathbf{z}^{-t}})$. However, it is possible that $x \notin V_{\mathbf{z}^{-t}}(H)$.

In this way we have a family of T cooperative games. We now turn to discuss some facts that will be useful to us in the sequel. Consider a solution to the following problem

$$z \in \arg \max \left\{ \sum_{h \in H} \lambda_h u_h(e_h + y_h) : \sum_{h \in H} y_h = 0, e + y \in \Re_+^{LH} \right\} \quad (2)$$

Consider now a solution $\mathbf{z} = (z^t)_{t=1}^T$ to the following family of parameterized problems:

$$z^t \in \arg \max \left\{ \sum_{h \in H} \lambda_h U_h^t(y_h) : y \in F_H^t \right\} \quad (3)$$

The following lemma, which asserts that the Pareto frontier can be reached in either case, makes use of the differentiability of utility functions. This will be a useful conclusion in the sequel.

Lemma 2 $\sum_{h \in H} \lambda_h U_h(\mathbf{z}_h) = \sum_{h \in H} \lambda_h u_h(e_h + z_h)$.

Proof See “Appendix”.

We can now state an existence result.

Theorem 1 Let $\mathcal{E}$ be an economy with preferences representable by differentiable, strictly concave, nonsatiated utility functions. Then there exists $\bar{\mathbf{z}} \in Z$ such that $\bigcap_{t=1}^T C(\bar{\mathbf{z}}^{-t}) \neq \emptyset$.

Proof Our proof proceeds by following for each game the existence proof for NTU-cores via ‘tracing’ the TU-cores for alternative weights λ , in order to take advantage of the convexity of the TU-core. The twist in the proof is to include two mappings (ψ and β in what follows) which ‘pool’ together the T games.

Define $W_h^t = \{w \in \mathfrak{R} : w = U_h^t(x_h) \geq U_h^t(e_h), x \in F_H^t\}$. Clearly, each of those sets is compact (because F_H^t is bounded below so each x_h is bounded above) and convex, so the same is true for the set $W = \prod_{t \in T} \prod_{h \in H} W_h^t$. Also let Δ_H , Δ_L and $\Delta_{\mathcal{H}}$ denote the $H - 1$, $T - 1$ and $2^H - 2$ dimensional simplices respectively. Finally define $\Omega = \Delta_H \times \Delta_T \times \Delta_{\mathcal{H}} \times W \times Z$.

Step I

We define a mapping that adjusts the weights of individuals in the associated TU games. Consider the correspondence $\phi : \Omega \rightarrow 2^{\Delta_H}$ defined as:

$$\phi(\lambda, \mu, \tau, w, \mathbf{z}) = \arg \max \left\{ \sum_{h \in H} p_h \left(\sum_{t=1}^T \mu^t w_h^t - U_h(\mathbf{z}_h) \right) : p \in \Delta_H \right\}$$

This correspondence is nonempty, convex valued and has a closed graph.

Next we define a correspondence that adjusts the weights of the T games in the family. Consider the correspondence $\psi : \Omega \rightarrow 2^{\Delta_T}$ defined as:

$$\psi(\lambda, \mu, \tau, w, \mathbf{z}) = \arg \max \left\{ \sum_{t=1}^T q^t \sum_{S \subseteq H} \tau_S v_{\mathbf{z}^{-i}}^{\lambda_{-i}}(S) : q \in \Delta_T \right\}$$

This correspondence is also nonempty, convex valued and has a closed graph.

We proceed by defining a correspondence $\beta : \Omega \rightarrow 2^{\Delta_{\mathcal{H}}}$, which adjusts the weights of coalitions in the family of T games as:

$$\beta(\lambda, \mu, \tau, w, \mathbf{z}) = \arg \max \left\{ \sum_{S \subseteq H} r_S \left[\bar{v}^\lambda(S) - \sum_{h \in H} \mu_h U_h(\mathbf{z}_h) \right] : r \in \Delta_{\mathcal{H}} \right\}$$

where $\bar{v}^\lambda(S) = \max_{t=1,2,\dots,T} v_{\mathbf{z}^{-t}}^\lambda(S)$ for each $S \subset H$. This correspondence is nonempty, convex valued and u.h.c, because $\bar{v}^\lambda(S)$ is continuous as the maximum of a finite number of continuous functions. Hence, $\beta(\cdot)$ has a closed graph.

We now define a correspondence that adjusts the individual payoffs in the T games in the family $\theta : \Omega \rightarrow 2^W$ as:

$$\theta(\lambda, \mu, \tau, w, \mathbf{z}) = \prod_{t=1}^T \hat{C}(\lambda, \mathbf{z}^{-t})$$

This correspondence is also nonempty convex valued, u.h.c and takes values in the compact set W so it has a closed graph.

Finally we define for each $t \in T$ the correspondences $\xi_t : \Omega \rightarrow 2^{Z^t}$, which adjust net trades in each of the T games as follows:

$$\xi_t(\lambda, \mu, \tau, w, \mathbf{z}) = \arg \max \left\{ \sum_{h \in H} \lambda_h \cdot U_h^t(y_h) : y \in Z^t \right\}$$

Clearly each $\xi_t(\cdot)$ is nonempty valued. Furthermore, by concavity and continuity of $U_h^t(\cdot)$ respectively, it follows that each ξ_t is convex valued and has a closed graph. Therefore, the correspondence defined as $\xi = \prod_{t=1}^T \xi_t$ has the same properties.

Step II

Thus, the correspondence $\phi \times \psi \times \beta \times \theta \times \xi : \Omega \rightarrow 2^\Omega$ satisfies all the conditions of the Kakutani fixed point theorem, so it has a fixed point, i.e., there is a $(\bar{\lambda}, \bar{\mu}, \bar{\tau}, \bar{w}, \bar{\mathbf{z}}) \in \Omega$ such that the following hold:

$$\sum_{h \in H} \bar{\lambda}_h \left(\sum_{t=1}^T \bar{\mu}^t \bar{w}_h^t - U_h(\bar{\mathbf{z}}_h) \right) \geq \sum_{h \in H} p_h \left(\sum_{t=1}^T \bar{\mu}^t \bar{w}_h^t - U_h(\bar{\mathbf{z}}_h) \right), \quad \forall p \in \Delta_H \quad (4)$$

$$\sum_{t=1}^T \bar{\mu}^t \sum_{S \subseteq H} \bar{\tau}_S v_{\mathbf{z}^{-t}}^{\bar{\lambda}}(S) \geq \sum_{t=1}^T q^t \sum_{S \subseteq H} \bar{\tau}_S v_{\mathbf{z}^{-t}}^{\bar{\lambda}}(S), \quad \forall q \in \Delta_T \quad (5)$$

$$\sum_{S \subseteq H} \bar{\tau}_S \left[\bar{v}^{\bar{\lambda}}(S) - \sum_{h \in H} \bar{\lambda}_h U_h(\bar{\mathbf{z}}_h) \right] \geq \sum_{S \subseteq H} r_S \left[\bar{v}^{\bar{\lambda}}(S) - \sum_{h \in H} \bar{\lambda}_h U_h(\bar{\mathbf{z}}_h) \right] \quad (6)$$

$\forall r \in \Delta_{\mathcal{H}}$

$$\bar{w} \in \prod_{t=1}^T \hat{C}(\bar{\lambda}, \bar{\mathbf{z}}^{-t}) \quad (7)$$

$$\bar{z}^t \in \arg \max \left\{ \sum_{h \in H} \bar{\lambda}_h \cdot U_h^t(y_h) : y \in Z^t \right\}, \quad t = 1, 2, \dots, T \quad (8)$$

Step III

From (8) it follows that for $t = 1, 2, \dots, T$, $\sum_{h \in H} \bar{\lambda}_h \cdot U_h(\bar{z}_h) = v_{\bar{z}^t}^{\bar{\lambda}}(H)$. This fact along with (7) implies, by the definition of $\hat{C}(\bar{\lambda}, \bar{z}^{-t})$ that

$$\sum_{h \in H} \bar{\lambda}_h U_h(\bar{z}_h) \geq \sum_{h \in H} \bar{\lambda}_h \bar{w}_h^t$$

for each $t = 1, 2, \dots, T$. Therefore, we conclude that:

$$\sum_{h \in H} \bar{\lambda}_h \left(\sum_{t=1}^T \bar{\mu}^t \bar{w}_h^t - U_h(\bar{z}_h) \right) \leq 0 \quad (9)$$

From (9) and (4) we have that

$$\sum_{h \in H} p_h \left(\sum_{t=1}^T \bar{\mu}^t \bar{w}_h^t - U_h(\bar{z}_h) \right) \leq 0, \quad \forall p \in \Delta_H \quad (10)$$

It follows that $\forall h \in H$, $U_h(\bar{z}_h) \geq \sum_{i=1}^L \bar{\mu}^i \bar{w}_h^i$. Hence, we conclude that $\forall S \subseteq H$,

$$\begin{aligned} \sum_{h \in S} \bar{\lambda}_h U_h(\bar{z}_h) &\geq \sum_{h \in S} \bar{\lambda}_h \sum_{t=1}^T \bar{\mu}^t \bar{w}_h^t \\ &= \sum_{t=1}^T \bar{\mu}^t \sum_{h \in S} \bar{\lambda}_h \bar{w}_h^t \\ &\geq \sum_{t=1}^T \bar{\mu}^t v_{\bar{z}^t}^{\bar{\lambda}}(S) \end{aligned} \quad (11)$$

It follows that

$$\sum_{S \subseteq H} \bar{\tau}_S \left[\sum_{t=1}^T \bar{\mu}^t v_{\bar{z}^t}^{\bar{\lambda}}(S) - \sum_{h \in S} \bar{\lambda}_h U_h(\bar{z}_h) \right] \leq 0 \quad (12)$$

Equation (5) implies that for each $t = 1, 2, \dots, T$ we have

$$\sum_{S \subseteq H} \bar{\tau}_S \sum_{t=1}^T \bar{\mu}^t v_{\bar{z}^t}^{\bar{\lambda}}(S) \geq \sum_{S \subseteq H} \bar{\tau}_S v_{\bar{z}^t}^{\bar{\lambda}}(S) \quad (13)$$

which implies that $\sum_{S \subseteq H} \bar{\tau}_S \sum_{t=1}^T \bar{\mu}^t v_{\bar{z}^t}^{\bar{\lambda}}(S) \geq \sum_{S \subseteq H} \bar{\tau}_S \bar{v}^{\bar{\lambda}}(S)$.

It follows that

$$\sum_{S \subseteq H} \bar{\tau}_S \left[\sum_{t=1}^T \bar{\mu}^t v_{\bar{z}^{-t}}^{\bar{\lambda}}(S) - \sum_{h \in S} \bar{\lambda}_h U_h(\bar{z}_h) \right] \geq \sum_{S \subseteq H} \bar{\tau}_S \left[\bar{v}^{\bar{\lambda}}(S) - \sum_{h \in S} \bar{\lambda}_h U_h(\bar{z}_h) \right] \quad (14)$$

From (14) and (12) we have that $\sum_{S \subseteq H} \bar{\tau}_S \left[\bar{v}^{\bar{\lambda}}(S) - \sum_{h \in S} \bar{\lambda}_h U_h(\bar{z}_h) \right] \leq 0$.

In this case equation (6) implies that $\forall S \subseteq H, \sum_{h \in S} \bar{\lambda}_h U_h(\bar{z}_h) \geq \bar{v}^{\bar{\lambda}}(S)$. Thus, we conclude that for each $t = 1, 2, \dots, T$ we have

$$\forall S \subseteq H, \sum_{h \in S} \bar{\lambda}_h U_h(\bar{z}_h) \geq v_{\bar{z}^{-t}}^{\bar{\lambda}}(S) \quad (15)$$

Therefore, $(U_h(\bar{z}_h))_{h \in H} \in \hat{C}(\bar{\lambda}, \bar{z}^{-t})$ for each $t = 1, 2, \dots, T$, and since $(U_h(\bar{z}_h))_{h \in H} \in V_{\bar{z}^{-t}}(H)$ for each $t \in T$, we have by fact (1) that indeed $(U_h(\bar{z}_h))_{h \in H} \in \bigcap_{t=1}^T \mathcal{C}(\bar{z}^{-t})$ as desired $\square$

The above result fits of course the particular structure of trades we consider here, but it can be potentially adjusted to capture alternative structures as well, which are represented by different mappings than those in (1). In some cases alternative assumptions may be required, but the strategy of the proof would still apply. For instance, if the span of market places overlap (have common dimensions), i.e., certain commodities that can be traded in many trading posts, then net trades in each spot may not be bounded. That may necessitate the use of constraints (such as 'cash in advance') to recover the elements of our proof.

4 Application to economies with asymmetric information

Consider an economy as above this time enriched with uncertainty which is summarized by a finite set of states of nature Ω . The *economy with uncertainty* is now $\mathcal{E} = \{(X_h, u_h, e_h) : h \in H\}$ where for each $h \in H$:

- (i) $X_h \subset \mathfrak{R}_+^{L\Omega}$ is the random commodity set
- (ii) $u_h : \mathfrak{R}_+^{L\Omega} \rightarrow \mathfrak{R}$ is the utility function
- (iii) $e_h \in \mathfrak{R}_+^{L\Omega}, e_h(\omega) \in X_h(\omega)$ is the random initial endowment.

Following Koutsougeras and Yannelis (1993), the *private information* of each individual $h \in H$ is represented by a partition $\mathcal{F}_h$ of Ω , which captures all the events which are verifiable by an individual. The interpretation is that upon occurrence of a state of nature $\omega \in \Omega$ each individual $h \in H$ perceives that the event $\mathcal{F}_h(\omega) \in \mathcal{F}_h$ has occurred. When $\mathcal{F}_h(\omega) = \mathcal{F}_h(\omega')$ for some pair $\omega \neq \omega'$ information is *incomplete* and when $\mathcal{F}_h(\omega) \neq \mathcal{F}_k(\omega)$ for some pair $h \neq k$ it is *asymmetric* among individuals.

We can now define an *ex ante economy with private information* as $\mathcal{E}^a = \{(X_h, u_h, e_h, \mathcal{F}_h) : h \in H\}$.

Let us establish some notation that will be necessary in the sequel. As usual for such an economy the common knowledge events are represented by the partition

$\mathcal{F} = \bigwedge_{h \in H} \mathcal{F}_h$ (the finest common coarsening of $(\mathcal{F}_h)_{h \in H}$). Let $\#\mathcal{F} = T + 1$ so that $\mathcal{F} = \{A_1, \dots, A_t, \dots, A_T, \emptyset\}$, where $A_t \cap A_{t'} = \emptyset$ for $t, t' \in T$ and $\bigcup_{t \in T} A_t = \Omega$. A useful fact following from the definition of $\mathcal{F}$ is that, given $A_t \in \mathcal{F}$, for each $\mathcal{F}_h$ there exist states of nature $\Omega_h^{A_t} \subset \Omega$ such that $\mathcal{F}_h(\omega) \cap \mathcal{F}_h(\omega') = \emptyset$ for $\omega, \omega' \in \Omega_h^{A_t}$ and $\bigcup_{\omega \in \Omega_h^{A_t}} \mathcal{F}_h(\omega) = A_t$.

We endow this economy with a structure of T ‘clearing houses’ which correspond to the elements of $\mathcal{F}$ as follows: for each event $A_t \in \mathcal{F}$, where $t = 1, 2, \dots, T$, we associate a ‘clearing house’ t which distributes trades for the contingent commodities corresponding to this event. Specifically the admissible trades of each individual $h \in H$ in the clearing house t are:

$$Z_h^t = \left\{ z \in \Re^{L\Omega} : z = w \chi_{A_t}, \text{ for } e_h + w \in \prod_{h \in H} X_h \right\} \quad (16)$$

In this case a given profile $\mathbf{z}_h = (z^t)_{t \in T} \in \prod_{t \in T} Z_h^t$ gives rise to an overall net trade $z_h = \sum_{t \in T} z_h^t \in \Re^{L\Omega}$. As before preferences over such profiles of net trades $\mathbf{z}_h \in Z_h^t$ for each $h \in H$ are induced from the utility functions by defining $U_h : \Re_+^{L\Omega T} \rightarrow \Re$ as $U_h(\mathbf{z}_h) = u_h(e_h + \sum_{t \in T} z_h^t)$.

Each clearing house distributes to each coalition $S \subseteq H$ the trades:

$$Z_S^t = \left\{ z \in \prod_{h \in S} Z_h^t : \sum_{h \in S} z_h = 0 \right\} \quad (17)$$

i.e., each clearing house $t \in T$ distributes only the contingent commodities corresponding to the event $A_t \in \mathcal{F}$.

The interpretation of this organization of trade is that allocations of contingent commodities are formed through proposed trades in blocks of *contingent commodities corresponding to each common knowledge event*. In line with our interpretation we postulate that coalitions can only block the trades corresponding to a common knowledge event each time, i.e., the enforcement powers of coalitions (or alternatively the commitment of coalition members) are limited to contingent trades corresponding to a common knowledge event.

Definition 4 Given a profile $\mathbf{z} \in \prod_{t \in T} Z_H^t$ we say that a coalition $S \subseteq H$ blocks the proposal for the event A_t , $z^t \in Z_H^t$, if there is $\hat{z}^t \in Z_S^t$ so that $U_h(\mathbf{z}_h^{-t}, \hat{z}_h^t) > U_h(\mathbf{z})$ for all $h \in S$, where $\mathbf{z}_h^{-t} = (z_h^r)_{r \neq t}$.

Definition 5 Given a profile $\mathbf{z} \in \prod_{t \in T} Z_H^t$ the core in the event A_t , denoted as $C(\mathbf{z}^{-t})$, is the set of feasible contingent trades on the common knowledge event A_t that are not blocked by any coalition.

We denote by $\mathcal{C}(\mathbf{z}^{-t})$ the set of payoffs corresponding to the elements of $C(\mathbf{z}^{-t})$

Definition 6 The profile $\mathbf{z} \in \prod_{t \in T} Z_H^t$ is a **coarse core-equilibrium** of the economy $\mathcal{E}^a$, denoted $C^T(\mathcal{E}^a)$, if and only if it is individually rational and

$$(U_h(\mathbf{z}_h))_{h \in H} \in \bigcap_{t \in T} C(\mathbf{z}^{-t})$$

The setup of the economy with asymmetric information corresponds *mutatis-mutandis* to the one in the last section. Hence we may assert that the main result of the last section applies, so $C^T(\mathcal{E}^a) \neq \emptyset$. Such allocations in an economy with asymmetric information have a very interesting property which can be formalized as follows. Given a profile of net trades $\mathbf{z} = \prod_{t \in T} Z_H^t$, the event that a given coalition $S \subseteq H$ blocks is $B_S \subseteq \Omega$ where:

$$\begin{aligned} & \exists \hat{\mathbf{z}} \in \mathfrak{R}^{LB_S S} \text{ such that} \\ & (i) \forall \omega \in B_S, \sum_{h \in S} \hat{z}_h(\omega) = 0 \\ & (ii) \forall h \in S, U_h(\mathbf{z}_h^{\Omega \setminus B_S}, \hat{z}_h^{B_S}) > U_h(\mathbf{z}) \end{aligned} \quad (18)$$

Proposition 1 Let $\mathbf{z} \in C^T(\mathcal{E}^a)$. Then for each coalition $S \subseteq H$, we have: $B_S \in \bigwedge_{h \in H} \mathcal{F}_h \Rightarrow B_S = \emptyset$.

Proof If $B_S \neq \emptyset$ and $B_S \in \bigwedge_{h \in H} \mathcal{F}_h$ it must be $B_S = A_{\bar{t}}$ for some $\bar{t} \in T$. In that case $\mathbf{z}^{\bar{t}} \notin C(\mathbf{z}^{-\bar{t}})$, which contradicts the hypothesis $\square$

This proposition asserts that an equilibrium in the sense defined above has the property that at the *ex ante* stage the event that a coalition blocks is not common knowledge, i.e., the event that the coalition S can block is not one that everyone agrees on.

This property can be further developed in an expected utility context in order to strengthen results along the line of Milgrom and Stokey (1982) (see also a more elaborate version in Koutsougeras and Yannelis (2017)). To this end suppose that preferences over contingent trades adhere to expected utility with a prior π defined on Ω :

$$U_h(\mathbf{z}) = u_h \left(e_h + \sum_{t \in T} z^t \right) = \sum_{\omega \in \Omega} \pi(\omega) v_h \left(e_h(\omega) + \sum_{t \in T} z^t(\omega) \right)$$

We will also need the concept of *interim* expected utility, which encapsulates the way that individuals evaluate contingent plans after they become aware of the occurrence of an event. Given $\omega \in \Omega$ the *interim* expected utility of an individual $i \in H$ is given by:

$$\forall z \in \mathfrak{R}^{L\Omega}, v_h^\omega(z) = \sum_{\omega' \in \Omega} q_h(\omega') v_h(e_h(\omega') + z(\omega')).$$

where

$$q_h(\omega') = \begin{cases} \frac{\pi(\omega')}{\sum_{\omega'' \in \mathcal{F}_h(\omega)} \pi(\omega'')} & \text{if } \omega' \in \mathcal{F}_h(\omega) \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

The interpretation is that once individuals receive a signal that a state of nature $\omega \in \Omega$ has occurred, they perform a Bayesian update of their priors according to the event $\mathcal{F}_h(\omega)$ that they have perceived.

Notice that combining the definition of *interim* expected utility with our notation we can express preferences over net trades in each common knowledge event A_t as follows:

$$\begin{aligned} U_h(\mathbf{z}^{-t}, z^t) &= u_h \left(e_h + \sum_{s \neq t} z^s + z^t \right) \\ &= \sum_{\omega \in \Omega} \pi(\omega) v_h \left(e_h(\omega) + \sum_{s \neq t} z^s(\omega) + z^t(\omega) \right) \\ &= \sum_{t \in T} \sum_{\omega \in A_t} \pi(\omega) v_h \left(e_h(\omega) + \sum_{s \neq t} z^s(\omega) + z^t(\omega) \right) \\ &= \sum_{t \in T} \sum_{\omega \in \bigcup_{\omega \in \Omega_h^{A_t}} \mathcal{F}_h(\omega)} \pi(\omega) v_h \left(e_h(\omega) + \sum_{s \neq t} z^s(\omega) + z^t(\omega) \right) \\ &= \sum_{t \in T} \sum_{\omega' \in \Omega_h^{A_t}} \sum_{\omega \in \mathcal{F}_h(\omega')} \pi(\omega) v_h \left(e_h(\omega) + \sum_{s \neq t} z^s(\omega) + z^t(\omega) \right) \\ &= \sum_{t \in T} \sum_{\omega' \in \Omega_h^{A_t}} \sum_{\omega \in \mathcal{F}_h(\omega')} \left(\sum_{\omega'' \in \mathcal{F}_h(\omega)} \pi(\omega'') \right) q_h(\omega) v_h \left(e_h(\omega) + \sum_{s \neq t} z^s(\omega) + z^t(\omega) \right) \\ &= \sum_{t \in T} \sum_{\omega' \in \Omega_h^{A_t}} \sum_{\omega \in \mathcal{F}_h(\omega')} \left(\sum_{\omega'' \in \mathcal{F}_h(\omega)} \pi(\omega'') \right) v_h^\omega \left(e_h + \sum_{s \neq t} z^s + z^t \right) \end{aligned}$$

Let $\mathbf{z} \in C^T(\mathcal{E}^a)$. An event that a given coalition $S \subseteq H$ blocks *in the interim* is $D_S \subseteq \Omega$ where:

$$\begin{aligned} &\exists \hat{\mathbf{z}} \in \mathfrak{N}^{LD_S S} \text{ such that } \forall \omega \in D_S : \\ &(i) \sum_{h \in S} \hat{z}_h(\omega) = 0 \\ &(ii) v_h^\omega \left(e_h + \left[(z^s)_{s \in \Omega \setminus D_S}, (\hat{z}^s)_{s \in D_S} \right] \right) > v_h^\omega \left(e_h + \sum_{t \in T} z_h^t \right), \quad \forall h \in S. \quad (20) \end{aligned}$$

A result along the line of Milgrom and Stokey (1982) (Theorem 1, p. 21) is as follows

Proposition 2 Let $\mathbf{z} \in C^T(\mathcal{E}^a)$. Then for each coalition $S \subseteq H$, we have: $D_S \in \bigwedge_{h \in H} \mathcal{F}_h \Rightarrow D_S = \emptyset$

Proof Suppose that $D_S \neq \emptyset$ and $D_S \in \bigwedge_{h \in H} \mathcal{F}_h$, i.e., there exists some $\bar{t} \in T$ so that $D_S = A_{\bar{t}}$.

In this case there exists $\hat{\mathbf{z}} \in \mathfrak{N}^{LA_{\bar{t}}S}$ such that $\forall \omega \in A_{\bar{t}}: \sum_{h \in S} \hat{z}_h(\omega) = 0$, and $v_h^\omega(e_h + \sum_{t \neq \bar{t}} z_h^t + \hat{z}_h^{\bar{t}}) > v_h^\omega(e_h + \sum_{t \in T} z_h^t)$ for each $h \in S$. Therefore, we have:

$$\begin{aligned} U_h(\mathbf{z}_h^{-\bar{t}}, \hat{z}_h^{\bar{t}}) &= u_h \left(e_h + \sum_{t \neq \bar{t}} z_h^t + \hat{z}_h^{\bar{t}} \right) \\ &= \sum_{t \in T} \sum_{\omega' \in \Omega_h^{A_t}} \sum_{\omega \in \mathcal{F}_h(\omega')} \left(\sum_{\omega'' \in \mathcal{F}_h(\omega)} \pi(\omega'') \right) v_h^\omega \left(e_h + \sum_{t \neq \bar{t}} z_h^t + \hat{z}_h^{\bar{t}} \right) \\ &> \sum_{t \in T} \sum_{\omega' \in \Omega_h^{A_t}} \sum_{\omega \in \mathcal{F}_h(\omega')} \left(\sum_{\omega'' \in \mathcal{F}_h(\omega)} \pi(\omega'') \right) v_h^\omega \left(e_h + \sum_{t \neq \bar{t}} z_h^t + z_h^{\bar{t}} \right) \\ &= U_h(\mathbf{z}_h^{-\bar{t}}, z_h^{\bar{t}}) \end{aligned}$$

It follows that $z^{\bar{t}} \notin C(\mathbf{z}^{-\bar{t}})$, which contradicts the hypothesis $\square$

The last proposition uncovers a nice stability property of equilibrium allocations: once an equilibrium allocation has been contracted at the ex ante stage, after a state of nature materializes and individuals update their information the event that a coalition blocks *at the interim stage* is not common knowledge, i.e., not everyone agrees that the coalition can block at the interim stage. Of course this conclusion is true for the grand coalition as well.

5 Conclusion

We have obtained an equilibrium for a family of conditional cooperative games which are derived from restrictions in the coordination of the members of coalitions. We used here a standard deterministic model in order to expose our ideas and proofs, so the enforcement constraints of coalitions were imposed in an *ad hoc* way. In particular we assumed that trading proposals were restricted to trades of subsets of commodities and then coalitions were restricted to have enforcement powers only for a specific subset of commodities each time.

Such a setup can be construed as representing some kind of market ‘incompleteness’, in the sense of a lack of means to conduct trades across the whole spectrum of commodities, i.e., available contracts regard exchange only of certain subsets of commodities. Such contracts can be negotiated in corresponding market places, hence

our identification of certain subsets of commodities with a corresponding ‘market place’. From this viewpoint the standard notion of the core can be viewed as the case where there are contracts available for the exchange of any commodity against any other, so coordination within coalitions is not limited. Notice that consistently with our interpretation the standard core notion for an exchange economy (as well as its existence proof!) is derived from the equilibrium notion we defined in this paper if we consider the ‘trivial’ partition of the set of commodities $\{L, \emptyset\}$.

A partition of commodities into subsets arises naturally in asymmetric information environments. The presence of asymmetric information where the enforcement powers of coalitions are constrained by informational or incentive considerations, lends itself as an interesting context for the application of our equilibrium notion. We demonstrated this here by offering an application to an asymmetric information environment. The study in this direction however is far from complete. We believe that future development of our approach along the lines of recent advances in the area, such as de Castro et al. (2019) or Moreno-Garcia and Torres-Martínez (2020) or Sun et al. (2013), will provide valuable contributions and insights to issues pertaining to incentives and efficiency under asymmetric information, while applications of our setup along the lines of Qin and Yang (2020) seems a promising direction towards asymptotic convergence.

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6 Appendix

Proof of Lemma 1 Non emptiness follows from the fact that the game is balanced. Convexity and closedness follow from the fact that the core is defined by a set of linear inequalities. By the maximum theorem $v_{\mathbf{z}^{-t}}^{\lambda_{-t}}(\cdot)$ is a continuous function of $\mathbf{z}^{-t}$ so the mapping $\mathbf{z}^{-t} \mapsto \hat{C}(\lambda, \mathbf{z}^{-t})$ is u.h.c. $\square$

Proof of lemma 2 Clearly if $\mathbf{y} \in \prod_{t=1}^T F_H^t$ is a solution to (3), then $\mathbf{y} = \sum_{t=1}^T \mathbf{y}^t \in \Re^{LH}$ satisfies $\sum_{h \in H} \mathbf{y}_h = 0$. Therefore, it must be

$$\begin{aligned} \sum_{h \in H} \lambda_h u_h(z_h) &\geq \sum_{h \in H} \lambda_h u_h(y_h) \\ &= \sum_{h \in H} \lambda_h U_h(\mathbf{y}_h) \end{aligned} \quad (21)$$

Suppose that $\sum_{h \in H} \lambda_h u_h(z_h) > \sum_{h \in H} \lambda_h U_h(\mathbf{y}_h)$.

Define $V(y) = \{x \in \Re^{LH} : \sum_{h \in H} \lambda_h u_h(e_h + x_h) > \sum_{h \in H} \lambda_h u_h(e_h + y_h)\}$.

By the concavity of $u_h(\cdot)$, there exists a supporting hyperplane with normal $p \in \mathfrak{N}^{LH}$, so that $x \in V(y) \Rightarrow px > py$. The differentiability of $u_h(\cdot)$ implies that the hyperplane with this property is unique. That is

$$\forall q \neq p, \exists x \in \{w \in \mathfrak{N}^{LH} : qw = qy\} \cap \{w \in \mathfrak{N}^{LH} : pw > qy\} \text{ s.t. } x \in V(y) \quad (22)$$

By hypothesis $z \in V(y)$, so we conclude that $pz > py$. Denoting $y^t = y\chi_{A^t}$, $z^t = z\chi_{A^t}$ and $p^t = p\chi_{A^t}$ we have that

$$\begin{aligned} \sum_{t \in T} p^t z^t &= pz \\ &> py \\ &= \sum_{t \in T} p^t y^t \end{aligned} \quad (23)$$

Hence, it must be $p^t z^t > p^t y^t$ for some $t \in T$.

For every $\mu \in (0, 1)$, consider the vector $x(\mu) = \sum_{s \neq t} y^s + y^t + \mu(z^t - y^t)$. We have

$$\begin{aligned} px(\mu) &= \sum_{s \neq t} p^s y^s + p^t [y^t + \mu(z^t - y^t)] \\ &= \sum_{t \in T} p^t y^t + \mu p^t (z^t - y^t) \\ &> \sum_{t \in T} p^t y^t \\ &= py \end{aligned} \quad (24)$$

Let $q^t \in \mathfrak{N}^{A^t H}$ be a vector perpendicular to $(z^t - y^t)$. Define the vector $q = (q^t + \sum_{s \neq t} p^s) \in \mathfrak{N}^{LH}$.

We have:

$$\begin{aligned} qx(\mu) &= \sum_{s \neq t} q^s y^s + q^t [y^t + \mu(z^t - y^t)] \\ &= \sum_{t \in T} q^t y^t + \mu q^t (z^t - y^t) \\ &= \sum_{t \in T} q^t y^t \\ &= qy \end{aligned} \quad (25)$$

According to (22) there is some μ so that $x(\mu) \in V(y)$. Hence

$$\sum_{h \in H} \lambda_h U_h(y_h^t + \mu(z_h^t - y_h^t), \mathbf{y}_h^{-t}) = \sum_{h \in H} \lambda_h u_h \left(e_h + y_h^t + \mu(z_h^t - y_h^t) + \sum_{s \neq t} y_h^s \right)$$

$$\begin{aligned}
&= \sum_{h \in H} \lambda_h u_h (e_h + x_h(\mu)) \\
&> \sum_{h \in H} \lambda_h u_h \left(e_h + \sum_{t \in T} y_h^t \right) \\
&= \sum_{h \in H} \lambda_h U_h (y_h)
\end{aligned} \tag{26}$$

However, notice that $\sum_{h \in H} [y_h^t + \mu(z_h^t - y_h^t)] = 0$ so $y^t + \mu(z^t - y^t) \in F_H^t$. But this fact along with (26) implies that

$$y^t \notin \arg \max \left\{ \sum_{h \in H} \lambda_h U_h (w, y_h^{-t}) : w \in F_H^t \right\}$$

which is a contradicts the hypothesis of the lemma.

Therefore, from (21) we conclude that it must be $\sum_{h \in H} \lambda_h u_h(z_h) = \sum_{h \in H} \lambda_h U_h(y_h)$ $\square$

References

- Debreu, G., Scarf, H.: A limit theorem on the core of an economy. *Int. Econ. Rev.* **4**, 236–246 (1963)
- de Castro, L., Pesce, M., Yannelis, N.C.: A new approach to the rational expectations equilibrium: existence, optimality and incentive compatibility. *Ann. Finance* (2019). <https://doi.org/10.1007/s10436-019-00349-w>
- Gale, D.: The core of a monetary economy without trust. *J. Econ. Theory* **19**, 456–491 (1978)
- Herings, J.J., Peters, H., Predtetchinski, A.: The strong sequential core in a dynamic exchange economy. *Econ. Theory* **24**(1), 147–162 (2002)
- Koutsougeras, L.C.: A two stage core with applications to asset markets and differential information economies. *Econ. Theory* **11**, 563–584 (1998)
- Koutsougeras, L.C.: On an Edgeworth characterization of rational expectations equilibria in atomless asset markets. In: Aliprantis, C., Alkan, A., Yannelis, N.C. (eds.) *Current Trends in Economic Theory*, pp. 255–264. Springer, Berlin (1999)
- Koutsougeras, L.C., Shafer, W.J.: The rational expectations core. Unpublished manuscript, University of Illinois at Champaign-Urbana (1993)
- Koutsougeras, L., Yannelis, N.C.: Incentive compatibility and information superiority of the core of an economy with differential information. *Econ. Theory* **3**, 195–216 (1993)
- Koutsougeras, L., Yannelis, N.C.: A note on information, trade and common knowledge, in sunspots and non-linear dynamics. Springer, pp 403–409 (2017)
- Milgrom, P., Stokey, N.: Information, trade and common knowledge. *J. Econ. Theory* **26**(1), 17–27 (1982)
- Moreno-Garcia, E., Torres-Martínez, J.P.: Information within coalitions: risk and ambiguity. *Econ. Theory* **69**, 125–147 (2020)
- Qin, C., Yang, X.: On the equivalence of rational expectations equilibrium with perfect Bayesian equilibrium. *Econ. Theory* **69**, 1127–1146 (2020)
- Repullo, R.: The core of an economy with transaction costs. *Rev. Econ. Stud.* **LV**, 447–458 (1988)
- Sun, Y., Wu, L., Yannelis, N.C.: Incentive compatibility of rational expectations equilibrium in large economies: a counterexample. *Econ. Theory Bull.* **1**(1), 3–10 (2013)
- Vind, K.: Equilibrium with coordination. *J. Math. Econ.* **12**, 275–285 (1983)
- Vind, K.: Perfect competition or the core. *Eur. Econ. Rev.* **39**, 1733–1745 (1995)
- Zhao, J.: The hybrid solutions of an N-person game. *Games Econ. Behav.* **4**, 145–160 (1992)

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