Smoothing and predicting celestial pole offsets using a Kalman filter and smoother

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Abstract

It has been recognized since the early days of interplanetary spaceflight that accurate navigation requires taking into account changes in the Earth's rotation. In the 1960s, tracking anomalies during the Ranger VII and VIII lunar missions were traced to errors in the Earth orientation parameters. As a result, Earth orientation calibration methods were improved to support the Mariner IV and V planetary missions. Today, accurate Earth orientation parameters are used to track and navigate every interplanetary spaceflight mission. The approach taken at JPL (Jet Propulsion Laboratory) to provide the interplanetary spacecraft tracking and navigation teams with the UT1 and polar motion parameters that they need is based upon the use of a Kalman filter to combine past measurements of these parameters and predict their future evolution. A model was then used to provide the nutation/precession components of the Earth's orientation. As a result, variations caused by the free core nutation were not taken into account. But for the highest accuracy, these variations must be considered. So JPL recently developed an approach based upon the use of a Kalman filter and smoother to provide smoothed and predicted celestial pole offsets (CPOs) to the interplanetary spacecraft tracking and navigation teams. The approach used at JPL to do this and an evaluation of the accuracy of the predicted CPOs is given here. For assessing the quality of JPL's nutation predictions, we compare the time series of dX , dY provided by JPL with the predictions obtained from the IERS Rapid Service/Prediction Centre. Our results confirmed that the approach recently developed by JPL can be used for the successful nutation prediction. In particular, we show that after 90 days of prediction, the estimated errors are 43% lower for dX and 33% lower for dY than in the case of the official IERS products, and an average improvement is 19% and 22% for dX and dY , respectively.

Keywords Earth orientation parameters (EOP) · Celestial pole offset (CPO) · Free core nutation (FCN) · Predictions

1 Introduction

The Earth orientation parameters (EOPs)—the coordinates of the Earth's pole (x , y), Universal Time (UT) and the coordinates of the celestial pole (or celestial pole offsets—CPO)—describe irregularities of the Earth's rotation and are the necessary parameters for transforming coordinates between the celestial and terrestrial reference frames since they provide the rotation of the ITRF (International

Terrestrial Reference Frame) to the ICRF (International Celestial Reference Frame) as a function of time. These parameters are determined with the use of one or more space geodesy techniques such as GNSS (Global Navigation Satellite Systems), SLR (Satellite Laser Ranging), LLR (Lunar Laser Ranging), VLBI (Very Long Baseline Interferometry) or DORIS (Doppler Orbitography and Radiopositioning Integrated by Satellite). The International Earth Rotation and Reference Systems Service (IERS) provides long-term, monthly and daily Earth orientation data. These products are the effect of combining different space geodetic measurements (Bizouard and Gambis 2009). The newest daily EOP series, known as EOP 14 C04 (Bizouard et al. 2018), are consistent with the most recent International Terrestrial Reference Frame (ITRF2014; Altamimi et al. 2016) and are routinely delivered with a 30-day latency.

Knowledge of the EOPs is essential for numerous practical and scientific applications that include positioning

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and navigating in space and on Earth. Uncertainties in the changes of the Earth's orientation in space are the main source of error in tracking and navigating interplanetary spacecraft. Today, accurate Earth orientation parameters are used to track and navigate every interplanetary spaceflight mission. For this purpose, precise observations of the Earth's orientation should be delivered in real time. However, due to delays caused by data processing and complex computations, EOPs are available with some delays. Consequently, accurate predictions of the parameters of the Earth's orientation are needed.

The IERS provides rapid estimates and predictions of EOP in the frame of the Rapid Service/Prediction Centre (McCarthy and Luzum 1991). The available EOP time series contain forecasts for up to 90 days in future. However, there are other sources of EOP predictions and different forecast methods are applied by many scientific institutions. Auto-regression (Kosek et al. 2008), auto-covariance (Kosek et al. 2008), least-squares collocation (Hozakowski 1990), neural networks (Kalarus and Kosek 2004; Schuh et al. 2002), wavelets and fuzzy inference systems (Akyilmaz et al. 2011), Kalman filter (Freedman et al. 1994), spectral analysis and least-squares extrapolation (Akulenko et al. 2002), combination of least squares plus auto-regression (Kosek et al. 1998, 2008; Xu et al. 2012), combination of singular spectrum analysis and Copula-based analysis (Modiri et al. 2018), modelling and forecasting excitation functions (Chin et al. 2004) are the most common algorithms for EOP prediction.

In the years 2005–2009, the Earth Orientation Parameters Prediction Comparison Campaign (EOP PCC) performed an evaluation of different prediction techniques using the IERS EOP 05 C04 series as a reference (Kalarus et al. 2010). During the campaign, the solutions were assessed in terms of all EOPs: pole coordinates, Universal Time and length of day as well as precession–nutation. The analyses were conducted for three prediction intervals: medium term (< 500 days), short term (< 30 days) and ultra-short term (< 10 days). However, as shown in the summary of the EOP PCC, there is no particular prediction method that is best in terms of all orientation parameters as well as all considered prediction intervals.

The Jet Propulsion Laboratory (JPL), as part of the National Aeronautics and Space Administration (NASA), conducts many Earth orbiting and interplanetary missions. For safe and accurate tracking and navigating of spacecraft, JPL navigation teams need precise EOPs. These Earth orientation parameters are estimated by scientists from JPL with the use of a Kalman filter and smoother to combine past measurements of these parameters and predict their future evolution. In the past, a model was used to provide the nutation/precession components of the Earth's orientation separately. As a result, variations caused by the free core nutation (FCN) were not taken into account. However, for the highest

accuracy, these variations must also be considered. Consequently, JPL recently developed an approach based upon the use of a Kalman filter and smoother to provide smoothed and predicted celestial pole offsets (CPO) for the interplanetary spacecraft tracking and navigation teams.

In this paper, the approach used at JPL to forecast CPO as well as an evaluation of the accuracy of the predicted values is given. As comparative data sets for our analyses, we used celestial pole offsets provided by the Rapid Service/Prediction Centre of IERS. The analyses presented here are based on the comparison of time series, spectra, differences between predicted and observed values and basic statistics. In Sect. 2, the fundamental information about celestial pole offsets is provided. Section 3 presents the algorithm used by the JPL team to determine CPO predictions. A detailed description of data formats as well as the transformation between older and newer precession–nutation parameters is provided in Sect. 4. Results are presented in Sects. 5 and 6 discusses and summarizes them.

2 Celestial pole offsets (CPO)

Celestial pole offsets (CPO) represent the difference between the observed position of the celestial pole and its position given by some precession–nutation model. The difference between the observed and modelled celestial pole positions is constantly monitored and regularly reported by the IERS. The CPOs are expressed in terms of ecliptic longitude and obliquity ($d\psi$ and $d\epsilon$, respectively). These values are the differential forms of the angles $\Delta\psi$ and $\Delta\epsilon$ that represent the celestial pole position according to the nutation theory (Kaplan 2005). Their accurate determination from VLBI measurements is provided since 1984. For the earlier conventional IAU (International Astronomical Union) 1980 precession–nutation model (IAU 1980 Theory of Nutation and the IAU 1976 Precession), most of the observed variations between the observed and modelled celestial pole can be explained by errors in a few terms of the model, mainly in the secular term and a few periodic terms (18.6 years, 1.0 years, 0.5 years and 14 days).

In 2000, the IAU adopted a new precession–nutation model for the Celestial Intermediate Pole in the celestial reference frame (Resolution B1.6, McCarthy and Capitaine 2003). With this new model, the observed CPO $d\psi$ and $d\epsilon$ are reduced to values below 1 mas. The IAU 2000 recommendations also gave a new parametrization of the celestial pole offsets based on the non-rotating origin of the Earth orientation matrix. The new CPO is defined as the corrections (dX , dY) to the Celestial Intermediate Pole (CIP) coordinates in the International Celestial Reference Frame (ICRF). At present, the IERS publishes celestial pole offsets referred to the IAU 2000A precession–nutation model in the form of dX

and dY . However, the traditional offsets given in longitude and obliquity and related to IAU 1980 theory of nutation and the IAU 1976 precession model can still be obtained from the IERS website. Nevertheless, in future, only dX and dY parameters related to the IAU 2000A precession–nutation model will be available to the users.

It is well known that the most accurate observations of the celestial pole offsets are provided by VLBI measurements. There are many time series of CPO estimated by scientific institutes. Among them are series computed at the analysis centres of the International VLBI Service for Geodesy and Astrometry (IVS) on the basis of VLBI observations. There are also combined solutions calculated from combinations of VLBI and other space geodesy techniques. While some models contain only celestial pole offsets determined from geodetic measurements, others also predict their future evolution. The most commonly used CPO models available to the public are: the United States Naval Observatory (USNO) combined CPO series produced by the IERS Rapid Service/Prediction Center (Dick and Thaller 2015; Wooden et al. 2010), the IVS combined CPO series produced by the IVS Combination Center (Böckmann et al. 2010) and the IERS C04 combined CPO series developed by the IERS Earth Orientation Product Center at the Paris Observatory (OPA) (Bizouard and Gambis 2009). Comparisons of the different CPO series were made by Malkin (2010a, b, 2013, 2014, 2017). It was shown that the considered series exhibit differences with each other as large as several tens of μas (Malkin 2017).

The observed celestial pole offsets contain several irregular oscillations which are not negligible at the submilliarc-second level of accuracy with the most significant one being the free core nutation (FCN) (Dehant et al. 2003). This is a pseudoharmonic signal with variable phase and amplitude ranging between 0.1 and 0.3 milliarcseconds having a retrograde period of 430 days (Brzezinski and Kosek 2004; McCarthy 2004). Because this is a free motion of the pole, the IAU 2000A nutation model does not include it. However, because of its non-negligible amplitude, the free core nutation is recommended by the IERS Conventions (McCarthy and Petit 2004) to be modelled and considered when transforming coordinates between the celestial and the terrestrial reference frame. Therefore, the FCN has been the subject of extensive investigation (e.g. Belda et al. 2016; Brzeziński 1994, 1996, 2000; Brzeziński and Petrov 1998; Brzeziński et al. 2002; Kalarus et al. 2006; Krásná et al. 2013; Lambert 2007; Malkin 2007, 2013, 2017; Mathews et al. 2002; Shirai and Fukushima 2001; Zhou et al. 2016). Many attempts also focused on predicting this oscillation (Brzezinski and Kosek 2004; Kalarus et al. 2006; Lambert 2007). In this paper, we concentrate on predicting the full celestial pole offsets that contain both FCN and other signals. We present the results of the method developed at JPL which is based on the

implementation of a Kalman filter to smooth and predict the nutation series. See, for example, Nahi (1969), Gelb (1974), and Bierman (1977) for a description of Kalman filters and smoothers.

3 Prediction method developed at JPL

The celestial pole offset (CPO) time series ($dX(t)$, $dY(t)$), is empirically modelled as:

$$z(t) = \text{FS}(t) + \text{LT}(t) + \text{RM}(t) + v_z(t) \quad (1)$$

where:

- $z(t)$ is a component of CPO, either $z(t) = dX(t)$ or $z(t) = dY(t)$ (instead of $z = dX + i dY$). Each component is fit to the data and predicted in time separately.
- $\text{FS}(t)$ is a Fourier series term given by:

$$\text{FS}(t) = \sum_{k=1}^k C_k \cos\left(\frac{2\pi t}{T_k}\right) + S_k \sin\left(\frac{2\pi t}{T_k}\right) \quad (2)$$

Two periods ($k=2$) representing the annual and free core nutation (FCN) frequencies are used following Mathews et al. (2002). Since FCN is a broadband process, a best-fit period (along with the annual period of $T_1 = 365.25$ days) is searched in the interval of 400–460 days. The optimal T_2 obtained this way is typically 443–449 days.

- $\text{LT}(t)$ is a “linear” or “low-frequency” trend. While non-linear secular variation can be seen for multidecadal time scales, a linear function may be used for a shorter-scale fit and prediction
- $\text{RM}(t)$ is a “residual memory” process, representing short-term offset (local mean) and modelled by a first-order autoregressive (AR-1) process. The AR-1 parameters used here are: 100 days for the decay time constant and $0.001225 \text{ (mas)}^2/\text{day}$ for the driving noise power.
- $v_z(t)$ is the observation white noise (no memory) whose variance is given with each data sample to be fit.

3.1 Prediction procedure

The near-daily nutation series data from NASA Goddard Space Flight Center (GSFC) are used. The dX and dY series for the interval from 1 January 1998 to the present are fit by weighted least squares to the model in Eq. (1).

The fit is performed each time the solution is updated. Data spanning 365 days are used for the fit. And during each fit a search is conducted for the best-fitting FCN period. So a different FCN period is used in each fit. The parameters being fit are those describing a linear trend and two periodic components (annual and FCN). The residual of the fit is

then smoothed with the Kalman filter. For the prediction, time series are extended by 90 days from the last fitted dates by using functional extensions for the FS (sinusoidal) and LT (linear) components and the Kalman filter for the RM component.

4 Data description

4.1 Predictions of CPO provided by JPL

JPL's predictions of celestial pole offsets (CPO), made by using a Kalman filter and smoother, use two types of files: a long file from NASA Goddard Space Flight Center (GSFC) and 453 short files each being approximately 15 months long.

The first file contains smoothed Earth orientation parameters covering the time period from 20 January 1962 to 28 November 2017. It contains smoothed nutation observations based on VLBI measurements (covering the time period from 1 January 1998 to 18 August 2017) and predictions of these values (covering the time period from 19 August 2017 to 28 November 2017). The smoothed nutation observations from this data set are our reference for comparing the predicted nutation time series and computing the prediction errors.

All short data files contain 365 days of smoothed nutation observations followed by about 3 months (from 90 to 112 days) of its predictions. The successive short files have first dates (first days of observation) that sequentially increase by 1 day and so do last dates in the files (that are also the last date of prediction). However, since VLBI observations are not done every day, gaps in the dX , dY observations exist. As a result, the date of the first predicted value does not increase daily but can be the same for several consecutive files. This is the reason for different number of predictions in successive files. Nevertheless, in our analyses we used 90 predictions from each file.

The long reference file and the short files are both created from the same input VLBI solution. The only difference between them is that the smoothed nutation observations in the long file start on 1 January 1998, whereas in the short files they are always 1 year long. Where they overlap, the values in the long and short files (those that are created from the same solution) are identical.

4.2 Predictions of CPO from IERS Rapid Service/Prediction Centre

For assessing the quality of JPL's nutation predictions, we compare the time series of dX , dY provided by JPL with the predictions obtained from the IERS Rapid Service/Prediction Centre (USNO combined CPO series, Dick and Thaller

2015; Wooden et al. 2010). This service provides Earth Orientation Parameters (EOP)—coordinates of the pole, Universal Time as well as celestial pole offsets (CPO)—on a rapid turnaround basis. These solutions are especially needed by real-time users and other researchers who need high-quality EOP parameters earlier than their availability in the final series published by the IERS Earth Orientation Center.

The main products of the IERS Rapid Service/Prediction Centre are rapid determinations (both data and predictions) of EOP, including celestial pole offsets, from the IERS Bulletin A (IERS 2018). The time series are available directly through the IERS website or at the IERS FTP Server (<ftp://ftp.iers.org>). There are two basic types of data: standard EOP data files and daily EOP data files.

Among standard EOP data files, two kinds of time series are available: *finals.all* and *finals.data*. Both of them contain daily observations of pole coordinates (x , y), UT1-UTC and nutation components (dX and dY or $d\psi$ and $d\epsilon$) followed by their predictions. The nutation predictions always start about 3 weeks before the predictions of x and y or Universal Time (e.g. the predictions of nutation start on 28 January 2018, whereas the predictions of x and y and Universal Time—on February 16). The time series are updated daily—the 1-day prediction is replaced by an observation and the last prediction is added to the end of the file. The length of (x , y) and UT1-UTC prediction is approximately 1 year, whereas for CPO there is a maximum 90-day prediction. The values of EOP observations and predictions as well as the dates of the first and last prediction are the same for *finals.all* and *finals.data*. The only difference between these two files is the length of the file—for *finals.all* the time series starts on 2 January 1973, while the *finals.data* start on 1 January 1992. The files are available with respect to the IAU 1980 model ($d\psi$ and $d\epsilon$ values) as well as the IAU 2000 model (dX and dY values).

Daily EOP data files, named *finals.daily*, correspond to the short files described in Sect. 4.1. They contain about 70–80 days of nutation observations followed by 90 days of predictions. The updated daily EOP data files (with successive predictions) are added every day. The nutation data are available with respect to the former precession–nutation model (IAU 1980) as well as the present precession–nutation model IAU 2000A.

The nutation observations from the *finals.all* file were used as a reference for comparing the predicted values from the daily files.

The main differences between the prediction files provided by JPL and those obtained from the IERS Rapid Service/Prediction Centre are the length of the prediction intervals and the length of the observation data that were used to determine the predictions. All of JPL's predictions were determined based on smoothed nutation observations

spanning 365 days. However, because of gaps in the observations, the length of JPL's CPO predictions ranges from 90 to 112 days. The files available at IERS Rapid Service/Prediction Centre website always contain 90 days of predictions that were calculated from 70 to 80 days of observations.

For the consistency of these two solutions, in our analyses we use the nutation predictions from the period since 4 December 2015 (1-day prediction in first short file) to 12 June 2017 (90-day prediction in the last short file). For the comparisons, we used 90 predictions from each file. For the considered time period, we have 425 daily data files both from IERS Rapid Service/Prediction Centre and from JPL.

4.3 Conversion of CPO from IAU 1980 to IAU 2000

The celestial pole offsets show differences between the observed celestial motion of the pole with the corresponding one predicted by the conventional IAU precession and nutation models. The classical CPO definition gives the offsets in longitude $d\psi$ and in obliquity $d\epsilon$. However, the new parametrization introduced dX and dY as the corrections to the Celestial Intermediate Pole (CIP) coordinates in the International Celestial Reference Frame. The changes also concern the conventional precession–nutation model; the former IAU 1980 Theory of Nutation and the IAU 1976 Precession have been replaced with the IAU 2000 precession–nutation model (see Sect. 2). The nutation predictions calculated by JPL with the use of a Kalman filter and smoother are given as dX , dY corrections referred to the IAU 2000 precession–nutation model.

The IERS Rapid Service/Prediction Centre now provides the celestial pole offsets with reference to IAU 2000 (dX , dY values) and still IAU 1980 ($d\psi$, $d\epsilon$ values) convention. However, former daily files were given only as offsets in longitude $d\psi$ and in obliquity $d\epsilon$ determined from IAU 1980. Consequently, the daily nutation files covering the considered time period in this paper (4 December 2015–12 June 2017) are only available as ($d\psi$, $d\epsilon$) in the old precession–nutation model.

The conversion between ($d\psi$, $d\epsilon$) and (dX , dY) that are referred to the same precession–nutation model can be done following the formulas given in Chapter 5 of the IERS 2000 conventions (IERS 2010). However, the conversion of the IAU 1980 celestial pole offsets ($d\psi$, $d\epsilon$) into the IAU 2000 celestial pole offsets ($d\psi$, $d\epsilon$ or dX , dY) requires more calculations. This can be done with the use of the package of subroutines, `uai2000.package`, available at the Earth Orientation Center of Paris Observatory (<https://hpiers.obspm.fr>) (IERS 2003). The programs, originally written in Fortran, are based upon SOFA (Standards of Fundamental Astronomy) matrix transformations. The International Astronomical Union's SOFA service provides astronomical software packages (Wallace 1998). The packages contain sets of algorithms

and procedures for implementing standard models used in fundamental astronomy. The full package of the newest release of subroutines written in Fortran 77 is available at SOFA's website (<http://www.iausofa.org/>). In our computations, for the transformation of CPO from IAU 1980 to IAU 2000 model, we used previously mentioned `uai2000.package` written by Ch. Bizouard from Systèmes de Référence Temps-Espace (SYRTE) and available at <http://hpiers.obspm.fr/eop-pc/models/models.html#software>.

5 Results and discussion

Figure 1 shows an example of: the reference nutation series, an example of prediction series, and the observed nutation series, which were the basis for the predictions. One can see that the predictions made by JPL are smoother compared to the predictions provided by the IERS. This is the result of the smoothing procedure applied in JPL during the prediction computation (see Sect. 3). As can be also seen from Fig. 1, the IERS observation series used for the predictions are several times shorter than the series used for the predictions made at JPL.

In Fig. 2, the amplitude spectra for reference series and example prediction series for IERS and JPL are shown. The spectra were computed using Fourier Transform Band Pass Filter (FTBPF; Kosek 1995). Amplitude spectra confirm the significant smoothing of predictions produced by JPL. In general, the spectra of the predicted series have smaller amplitudes than the spectra of the reference series of observed nutation. However, in the case of the IERS predictions, a good spectral correspondence with a reference nutation series can be observed in the prograde part of the spectrum for periods 5–20 days. Nevertheless, it should be kept in mind that a good spectral agreement between the observed series and its prediction does not imply a good prediction. For example, both reference and evaluated series may be characterized by similar power of a particular oscillation, but at the same time, they can be shifted in phase relative to each other. As a consequence, the predicted values can be poorly correlated with the observed series and characterized by high prediction errors.

A common method of assessing the quality of predictions is to determine the root mean square (RMS) of the differences between the predicted and reference series, also known as RMS prediction error (e.g. Kalarus et al. 2006; Malkin 2010a). Figure 3a, b shows the RMS of the differences between the predictions and the reference nutation values for various prediction lengths, considering the period September 2015–February 2017. The RMS errors for each day of prediction are also given in Table 1 in “Appendix 1”.

It is not surprising that the RMS prediction error depends on which reference series was used to determine the RMS of

Fig. 1 Example of the data series: **a** from IERS: blue line—observation series which is the basis for the IERS prediction, red line—the nutation prediction series produced by the IERS, yellow line—the JPL nutation reference file, green line—the IERS nutation reference

file. **b** From JPL: blue line—observation series which is the basis for the JPL prediction, red line—the nutation prediction series produced by JPL, yellow line—the JPL nutation reference file, green line—the IERS nutation reference file

the difference between the predicted and observed nutations. Generally, the RMS prediction error is smaller when the consistent reference series is used to determine this value. By “consistent reference”, we mean the JPL reference series used

to evaluate JPL predictions and the IERS reference series for assessing the quality of IERS prediction. The only exception to this is a dX nutation component for IERS prediction where we observe lower error in comparison with IERS reference

Fig. 2 Comparison of amplitude spectra for reference series and example prediction series for IERS (red lines) and JPL (blue lines). The spectra were computed for the nutation predictions and reference predictions for the time period MJD 57,386–57,475. The spectra were computed using Fourier Transform Band Pass Filter (FTBPF) with parabolic transfer function. All dX and dY series which were filtered had daily sampling and the length of 90 days. The oscillations that were passed through the filter had periods between 1 and 50 days that corresponded to the frequencies of 11.57 μHz and 0.23 μHz , respectively

for predictions up to 50 days and higher error for this reference in terms of predictions over the 50th day (Fig. 3a). The values of RMS error increase with increasing prediction days and starting from about 25-day prediction are significantly smaller for the JPL case (Fig. 3, Table 1). If we analyse the RMS errors for the “consistent reference”, for the JPL predictions these values are changing from 0.06 mas (for dX) and 0.05 mas (for dY) for 1-day prediction through 0.10 mas (for dX) and 0.09 mas (for dY) for 45-day prediction up to 0.10 mas (for dX) and 0.13 mas (for dY) for 90-day prediction (Fig. 3, Table 1). For the predictions provided by the IERS, these values are as follows: 0.06 mas for both dX and dY for 1-day prediction, 0.12 mas for both dX and dY for 45-day prediction and finally 0.18 mas for dX and 0.20 mas for dY in the case of 90-day prediction. Generally, we observe a faster increase in error with increasing day of prediction for the predictions provided by the IERS. This occurs when using either the IERS or JPL reference series. An interesting fact is that starting from 40th day of prediction, the RMS error related to the IERS reference nutation series is lower for JPL predictions (Table 1). Nevertheless, these errors are still higher than errors of JPL predictions compared to JPL reference. The reason of better quality of JPL predictions compared to IERS ones is the smoothing of these series. As it can be seen from Fig. 1, the predictions produced by IERS have significantly higher amplitudes than JPL ones, but sometimes they are

in opposite phases with the reference nutation series. The smoothing of the JPL series results in the fact that the differences between observed and predicted nutations for all days of prediction are much closer to each other comparing to the differences obtained for the IERS series.

In order to quantify the improvement of the accuracy of JPL predictions with respect to the official IERS product, we computed the percentage of JPL RMS reduction, taking RMS of the IERS forecasts as a reference:

$$\text{RMS reduction} = \frac{\text{RMS}_{\text{JPL}} - \text{RMS}_{\text{IERS}}}{\text{RMS}_{\text{IERS}}} \cdot 100\% \quad (3)$$

A negative value for the RMS reduction means that the JPL predictions are more accurate than the IERS predictions. Figure 4 shows the resulting RMS reduction. As it can be seen from this figure, the RMS in JPL predictions decreases with increasing day of prediction compared to the corresponding values provided by the IERS. For the last day of prediction, this reduction achieved 43% for dX and 33% for dY. However, up to about 25 days of prediction for dX, and up to 10 days of prediction for dY, we observe that JPL RMS prediction error is similar to the corresponding values obtained for the IERS series or even higher. We can therefore assume that the prediction method developed by JPL is particularly effective for forecasts longer than 20 days. In general, for dX the JPL RMS change ranges from +7 to

Fig. 3 RMS of the differences between the nutation prediction series and reference values for the period September 2015–February 2017 for various prediction lengths. **a** IERS predictions in comparison with

IERS and JPL reference series. **b** JPL predictions in comparison with IERS and JPL reference series

–43% with an average of –19%, and for dY ranges from +10 to –36% with an average of –22%.

An interesting issue is to check to see whether the quality of the prediction depends on not only the day of prediction but also on the date of this prediction. To do this, we show plots presenting the absolute values of differences

between predicted and reference nutation series depending on the number of prediction files, for 1-day and 90-day predictions (Fig. 5). This file number corresponds to the date of the prediction beginning and end. For both IERS and JPL predictions, we have 425 files with 90 predictions in each of them (see Sect. 4 with data description). For example, the

Fig. 4 Reduction in the RMS errors, expressed in percentage with respect to the RMS of IERS predictions for dX and dY . A negative value for the per cent RMS reduction means that the JPL predictions are more accurate than the IERS predictions

time span between first and last files is 4 December 2015–15 March 2017 for 1-day predictions and 2 March 2016–12 June 2017 for 90-day predictions. One can see that the differences for 90-day prediction consist of both short-term and long-term changes. This is visible, especially in the case of the IERS series where we observe a noticeable oscillation in the differences with a period of about 200–250 days. The amplitudes of these oscillations are much lower in the case of predictions made by JPL. The origin of the pseudoharmonic behaviour of the IERS prediction residuals should be studied more thoroughly, taking into consideration longer time span of analysis as well as details of the IERS prediction algorithm. However, due to insufficient data length, this is out of the scope of this manuscript. In contrast, the JPL CPO prediction residuals show rather random dependence on the date of prediction, and this should also be analysed in detail.

To check whether some kind of periodicity in prediction residuals also occurs for other than 90-day predictions, we draw the same plots as in Fig. 5 but for all days of prediction (Figs. 6, 7). As can be seen from these diagrams, a periodicity in the maxima of differences between predicted and reference nutation series appears for almost all days of predictions (except the first several days). With the increasing day of prediction, the length of the period with increased error also rises. This effect is much more visible for IERS predictions. Smoothing of JPL series effectively reduces this phenomenon. Probably, some periodic effect has its impact on the measured nutations and it is not predicted correctly, resulting in higher prediction errors and sometimes in opposite phases of predictions in comparison with observations (Fig. 1). This problem almost vanishes when the predictions are smoothed.

The problem described above is also reflected in histograms of the distribution of the absolute values of the differences

presented in Fig. 8 for IERS predictions and in Fig. 9 for JPL predictions. Histograms for 1-day predictions have similar character for both IERS and JPL predictions. However, they differ significantly in the case of 90-day predictions; for the IERS the absolute differences between the predicted and reference nutation series are more diverse; and we observe more higher values. Finally, the histograms show that the most common absolute values of differences between the predicted and reference nutation series for JPL predictions are 0.07–0.08 for dX and 0.02 for dY for 1-day predictions and 0.02 for dX and 0.02–0.17 for dY for 90-day predictions (Fig. 9).

The use of a Kalman filter and smoother provides CPO predictions that are as accurate as those achieved by other authors. For example, in the newest work of Belda et al. (2018), the authors proved that the use of an accurate FCN prediction can reduce significantly the errors of CPO predictions. They developed a simple method of CPO prediction that used the conventional IERS EOP 08 C04 CPO series (Bizouard and Gambis 2009) and an empirical FCN model fitted to VLBI data (Belda et al. 2016), as input data sets. Daily CPO series were forecasted by extrapolating the FCN model with the amplitude taken from the 08 C04 CPO series of the 400 days preceding the prediction epoch. The CPO predictions developed by Belda et al. (2018) revealed visibly lower RMS prediction errors than official IERS products, reaching about 0.090 mas after 25 days, 0.100 mas after 50 days and about 0.105 mas after 90 days (Fig. 5 in Belda et al. 2018). The average RMS reduction compared to the IERS Rapid Service forecasts was equal to 39.5% for dX and 33.8% for dY . Another advantage of using FCN predictions in forecasting CPO is that the errors are almost the same size starting from around 100th day of prediction. In comparison, the RMS prediction error obtained for CPO forecasts

Fig. 5 Example of absolute value of differences between predicted and reference values of nutation series for two chosen cases: 1-day prediction and 90-day prediction. **a** IERS predictions in comparison with IERS reference. **b** JPL predictions in comparison with JPL reference

developed by JPL reached about 0.080 after 25 days, 0.100 mas after 50 days and about 0.117 mas after 90 days (see Fig. 4 and Table 1 in Appendix 1). Therefore, the increase in RMS for JPL predictions is less steady. However, when going from ultra-short to short-term and mid-term predictions, the increase in JPL prediction errors is less rapid than

for the case of Belda et al. (2018) predictions. The average RMS reduction compared to the IERS Rapid Service forecasts is lower than for the CPO predictions developed by Belda et al. (2018) (19% and 22% for dX and dY, respectively). However, it should be kept in mind that in contrast to the length of JPL predictions which is 90 days, the length

Fig. 6 Absolute value of differences between predicted and reference values of nutation series: IERS predictions in comparison with IERS reference nutation series. The units are mas

Fig. 7 Absolute value of differences between predicted and observed values of nutation series: JPL predictions in comparison with JPL reference nutation series. The units are mas

of the series developed using IERS EOP 08 C04 CPO and FCN model is 1 year, which visibly affects the size of average RMS reduction. Moreover, the predictions of Belda et al. (2018) include the period 2000–2016, while JPL series evaluated here cover the period between December 2015 and June 2017.

A different method of CPO prediction was proposed by Malkin (2010b). In his method, simply called ZM2, the author computed a Gaussian smoothing of each CPO series derived from the CPO data provided by International VLBI Service for Geodesy and Astrometry (IVS CPO series, Malkin 2007), depending on a certain smoothing parameter. After smoothing the CPO observations with Gaussian filter, the ZM2 CPO forecasts were developed using an autoregression fitting. The resulting series included two-year CPO predictions and were developed using two different values of smoothing parameter (a), $a=2$ and $a=16$, the

second of which indicated stronger smoothing. The resulting CPO predictions demonstrated visible increase in accuracy as compared to the IERS forecasts. The RMS prediction error reached about 0.080 mas after 25 days, 0.090 mas after 50 days and about 0.100 mas after 90 days for $a=16$, and about 0.100 mas after 25 days, 0.110 mas after 50 days and about 0.120 mas after 90 days for $a=2$ (Fig. 3 in Malkin 2010b). Therefore, the accuracy of CPO prediction obtained from method proposed by JPL is between ZM2 accuracy for $a=16$ and ZM2 accuracy for $a=2$. In his work, Malkin (2010b) proved that the smoothing of the input CPO series can provide decreased prediction errors. Similarly to the method developed by Belda et al. (2018), for ZM2 forecasts the error growth is almost uniform along the 100-day period considered.

In contrast to the methods developed by JPL, Belda et al. (2018) and Malkin (2010b), the model proposed by Lambert

Fig. 8 Distribution of the absolute value of the differences between predicted and reference nutation series for 1-day and 90-day predictions, IERS predictions compared to IERS reference nutation series. The red curve indicates normal distribution fit

Fig. 9 Distribution of the absolute value of the differences between predicted and reference nutation series for 1-day and 90-day predictions, JPL predictions compared JPL reference nutation series. The red curve indicates normal distribution fit

(2007) does not predict CPO but FCN and does not contain trends. However, it is recommended by the IERS Conventions as a CPO model. The basis for the FCN computation was the daily combined IERS EOP 05 C04 series computed at the Paris Observatory, and the analysis of its deviation from the theory of precession–nutation. In order to calculate FCN parameters, the least-squares fit in sliding two-year intervals, and with the mean CPO being removed in each interval, was used. The FCN parameters obtained for the last two-year interval were used for developing a 1-year prediction. In contrast to other prediction methods described here, the series provided by Lambert (2007) are updated once per year. These FCN predictions are characterized by almost linear error growth, $0.1325 \mu\text{mas/day}$, as reported in Lambert (2007). The RMS prediction error reaches about 0.060 mas after 25 days, 0.100 mas after 50 days and about 0.130 mas after 90 days (Fig. 4 in Lambert 2007). Therefore, the CPO prediction developed by JPL reveals to give slightly smaller errors than predictions of Lambert (2007) starting from about 25th day of prediction.

Taking into consideration above findings, we can conclude that the method proposed by JPL to predict CPO, although the forecasted series are visibly more smoothed than for other solutions, can provide an accuracy comparable with the newest results obtained by other authors. The errors obtained are clearly smaller than the uncertainty of current operational CPO predictions.

6 Summary and conclusions

In this paper, we presented the method of predicting nutation series recently developed by JPL that uses a Kalman filter and smoother. We compared JPL's predictions with predictions provided by the IERS Rapid Service/Prediction Centre.

Our results confirm that the approach recently developed by JPL can be used to successfully predict nutation. Based on more than 400 sample predictions, we demonstrate the evolution of the prediction error as a function of a number of prediction days. We show that the (dX, dY) RMS of the difference between predicted and reference nutation series changes from (0.06, 0.05) mas for 1-day predictions to (0.10, 0.13) mas for 90-day predictions. These errors for the case of the IERS are equal to (0.06, 0.06) mas for 1-day prediction and (0.18, 0.20) mas for 90-day predictions. We also demonstrated that the prediction accuracy depends on the chosen reference series.

Starting from the 25th day of prediction, the smoothed series provided by JPL appears to have lower errors than predictions provided by the IERS. Therefore, this method can be especially advantageous for longer predictions. Another advantage of JPL's approach is that the growth of the RMS

prediction error with increasing day of prediction is much less rapid than the case of IERS predictions.

We reported that an average percentage of RMS decreasing for JPL predictions with respect to the official IERS predictions is equal to 19% for dX and 22% for dY, and this improvement increases with increasing day of prediction. For the 90th day of a forecast, the smoothed predictions developed by JPL represent 43% improvement for dX and 33% improvement for dY.

The results showed that the use of a Kalman filter and smoother provides predictions that are as accurate as those achieved by other authors.

We also noted that a good spectral agreement between the observed series and its prediction does not entail an accurate prediction. In particular, we observed that spectral agreement with reference series was visibly better for the official IERS predictions. However, the smoothed JPL predictions revealed to produce visibly lower RMS prediction errors and higher correlations with observed series.

One big disadvantage of IERS predictions is a periodic increase in differences between observed and predicted nutation values. With the increasing day of prediction, the length of the period with increased error also rises. For smoothed JPL predictions, this problem is almost invisible. The pseudoharmonic behaviour of the IERS prediction residuals should be analysed in detail in future, taking into consideration longer time span of analysis as well as details of the IERS prediction algorithm.

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Author contributions RG and TMC developed the CPO prediction approach and provided the JPL and IERS data; JN, TMC, RG, JS and MW designed the research; JN, JS and MW analysed the data, performed computations and drew figures; and JN, TMC, RG, JS and MW wrote the paper.

Data availability All data sets are available upon request from the authors.

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Appendix 1

See Table 1.

Table 1 RMS prediction errors for all days of predictions

Day of prediction	IERS vs. IERS		JPL vs. JPL		IERS vs. JPL		JPL vs. IERS	
	dX	dY	dX	dY	dX	dY	dX	dY
1	0.065	0.056	0.059	0.049	0.080	0.080	0.073	0.076
2	0.071	0.058	0.074	0.061	0.088	0.082	0.081	0.083
3	0.077	0.061	0.082	0.068	0.095	0.090	0.088	0.091
4	0.080	0.064	0.081	0.067	0.093	0.091	0.093	0.095
5	0.081	0.066	0.079	0.065	0.089	0.089	0.094	0.095
6	0.081	0.069	0.079	0.066	0.085	0.089	0.095	0.094
7	0.082	0.072	0.082	0.067	0.085	0.095	0.096	0.095
8	0.084	0.074	0.082	0.068	0.088	0.096	0.097	0.099
9	0.085	0.073	0.085	0.070	0.094	0.093	0.095	0.103
10	0.086	0.071	0.086	0.071	0.099	0.094	0.095	0.105
11	0.087	0.072	0.082	0.071	0.098	0.094	0.095	0.105
12	0.087	0.073	0.078	0.070	0.095	0.094	0.096	0.101
13	0.087	0.076	0.078	0.070	0.092	0.094	0.097	0.097
14	0.087	0.078	0.079	0.072	0.090	0.096	0.097	0.096
15	0.088	0.078	0.078	0.072	0.090	0.097	0.099	0.099
16	0.089	0.077	0.083	0.073	0.092	0.096	0.101	0.103
17	0.091	0.077	0.087	0.074	0.097	0.097	0.101	0.105
18	0.089	0.077	0.085	0.072	0.095	0.097	0.099	0.104
19	0.086	0.078	0.083	0.069	0.094	0.099	0.096	0.102
20	0.085	0.082	0.083	0.068	0.093	0.101	0.095	0.102
21	0.086	0.086	0.086	0.069	0.093	0.103	0.097	0.100
22	0.089	0.086	0.092	0.068	0.094	0.101	0.103	0.101
23	0.092	0.085	0.093	0.068	0.096	0.103	0.107	0.103
24	0.093	0.085	0.090	0.071	0.098	0.108	0.108	0.106
25	0.093	0.086	0.086	0.074	0.098	0.113	0.106	0.105
26	0.092	0.088	0.082	0.073	0.098	0.114	0.103	0.100
27	0.091	0.092	0.080	0.072	0.096	0.115	0.101	0.097
28	0.091	0.095	0.081	0.074	0.096	0.115	0.100	0.099
29	0.092	0.097	0.082	0.075	0.097	0.117	0.100	0.105
30	0.093	0.098	0.083	0.073	0.099	0.118	0.102	0.109
31	0.097	0.099	0.086	0.077	0.104	0.122	0.103	0.112
32	0.100	0.100	0.086	0.080	0.105	0.124	0.103	0.112
33	0.102	0.102	0.086	0.080	0.105	0.125	0.102	0.111
34	0.104	0.105	0.086	0.081	0.104	0.126	0.104	0.111
35	0.107	0.108	0.089	0.084	0.105	0.124	0.107	0.111
36	0.108	0.109	0.091	0.086	0.104	0.127	0.112	0.114
37	0.110	0.110	0.092	0.086	0.106	0.130	0.115	0.117
38	0.111	0.111	0.093	0.085	0.108	0.135	0.115	0.117
39	0.111	0.113	0.091	0.087	0.110	0.138	0.113	0.116
40	0.112	0.116	0.088	0.088	0.110	0.138	0.112	0.113
41	0.113	0.119	0.088	0.090	0.110	0.141	0.111	0.110
42	0.115	0.122	0.091	0.091	0.110	0.142	0.113	0.110
43	0.116	0.123	0.094	0.091	0.113	0.144	0.115	0.115
44	0.117	0.124	0.101	0.090	0.118	0.146	0.119	0.120
45	0.118	0.123	0.105	0.091	0.122	0.148	0.121	0.122

Table 1 (continued)

Day of pre- diction	IERS vs. IERS		JPL vs. JPL		IERS vs. JPL		JPL vs. IERS	
	dX	dY	dX	dY	dX	dY	dX	dY
46	0.118	0.124	0.102	0.094	0.121	0.148	0.119	0.121
47	0.119	0.127	0.098	0.095	0.121	0.149	0.114	0.118
48	0.119	0.130	0.096	0.097	0.119	0.151	0.112	0.119
49	0.121	0.133	0.096	0.099	0.118	0.149	0.113	0.122
50	0.122	0.133	0.094	0.103	0.116	0.147	0.115	0.125
51	0.124	0.133	0.093	0.102	0.118	0.147	0.117	0.127
52	0.127	0.133	0.094	0.096	0.121	0.153	0.119	0.129
53	0.129	0.135	0.092	0.096	0.122	0.158	0.118	0.129
54	0.129	0.138	0.091	0.099	0.124	0.161	0.114	0.126
55	0.128	0.142	0.091	0.099	0.124	0.163	0.111	0.125
56	0.127	0.146	0.092	0.099	0.123	0.163	0.110	0.128
57	0.126	0.148	0.095	0.100	0.125	0.162	0.112	0.132
58	0.127	0.149	0.098	0.102	0.127	0.162	0.116	0.134
59	0.128	0.150	0.101	0.106	0.128	0.166	0.119	0.136
60	0.129	0.151	0.099	0.108	0.127	0.169	0.119	0.137
61	0.130	0.152	0.098	0.111	0.126	0.172	0.117	0.136
62	0.131	0.154	0.098	0.115	0.126	0.174	0.117	0.136
63	0.134	0.156	0.099	0.113	0.125	0.172	0.119	0.139
64	0.138	0.156	0.099	0.114	0.127	0.172	0.123	0.143
65	0.144	0.156	0.101	0.114	0.131	0.173	0.127	0.147
66	0.150	0.157	0.105	0.112	0.139	0.175	0.131	0.149
67	0.154	0.159	0.105	0.111	0.141	0.176	0.133	0.149
68	0.156	0.161	0.105	0.114	0.143	0.177	0.131	0.147
69	0.155	0.163	0.106	0.116	0.144	0.178	0.127	0.145
70	0.153	0.165	0.108	0.115	0.144	0.179	0.125	0.144
71	0.151	0.166	0.106	0.113	0.138	0.179	0.125	0.145
72	0.149	0.167	0.103	0.112	0.136	0.181	0.125	0.149
73	0.148	0.168	0.104	0.117	0.137	0.185	0.125	0.152
74	0.147	0.170	0.101	0.120	0.137	0.187	0.122	0.151
75	0.147	0.172	0.099	0.124	0.138	0.189	0.117	0.148
76	0.149	0.174	0.098	0.127	0.138	0.192	0.115	0.148
77	0.153	0.175	0.100	0.127	0.141	0.193	0.118	0.149
78	0.158	0.174	0.102	0.126	0.143	0.190	0.123	0.149
79	0.165	0.174	0.111	0.121	0.153	0.190	0.129	0.150
80	0.170	0.173	0.117	0.119	0.161	0.193	0.134	0.151
81	0.172	0.174	0.114	0.119	0.160	0.196	0.135	0.150
82	0.171	0.174	0.109	0.121	0.160	0.198	0.131	0.148
83	0.169	0.176	0.106	0.123	0.159	0.198	0.126	0.148
84	0.168	0.179	0.106	0.123	0.156	0.195	0.125	0.153
85	0.169	0.182	0.102	0.123	0.153	0.197	0.128	0.156
86	0.172	0.185	0.102	0.121	0.152	0.200	0.131	0.156
87	0.175	0.189	0.105	0.122	0.156	0.205	0.133	0.158
88	0.178	0.193	0.104	0.124	0.156	0.209	0.134	0.161
89	0.179	0.196	0.103	0.128	0.158	0.211	0.132	0.162
90	0.179	0.198	0.102	0.132	0.158	0.213	0.130	0.162

The units are mas

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