# Erratum to: On weighted total least-squares with linear and quadratic constraints 

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1 Erratum to: J Geod (2013) 87: 279-286
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In the afore-mentioned publication, Eqs. (19) and (30) contain some typos, which need to be corrected for. Eq. (19) should read

$$
\begin{align*}
& \left.\frac{\partial \Phi}{\partial \xi} \right\rvert\, \tilde{\mathrm{e}}_{\mathrm{y}}, \tilde{\mathrm{e}}_{\mathrm{A}}, \hat{\lambda}_{1}, \hat{\lambda}_{2}, \hat{\lambda}_{3}, \hat{\xi} \\
& \quad=-2\left(-A^{T} \hat{\lambda}_{1}+\tilde{\mathrm{E}}_{\mathrm{A}}^{\mathrm{T}} \hat{\lambda}_{1}+K^{T} \hat{\lambda}_{2}+M \hat{\xi} \hat{\lambda}_{3}\right)=0 \tag{19}
\end{align*}
$$

and Eq. (30) should read

$$
\begin{align*}
\hat{\lambda}_{2}= & {\left[K \mathrm{R}_{3} \mathrm{M} \hat{\xi} \hat{\xi}^{T} M^{T} \mathrm{R}_{3} \mathrm{~K}^{\mathrm{T}} / \hat{\xi}^{T} M \mathrm{R}_{3} M \hat{\xi}+\mathrm{KR}_{3} \mathrm{~K}^{\mathrm{T}}\right]^{-1} } \\
& \times\left[-K \mathrm{R}_{3} \mathrm{M} \hat{\xi} \hat{\xi}^{T} M^{T} \mathrm{R}_{3}\left(A^{\mathrm{T}} R_{1}+R_{2}\right) \mathrm{y} / \hat{\xi}^{T} M \mathrm{R}_{3} M \hat{\xi}\right. \\
& +a_{0}^{2} K \mathrm{R}_{3} \mathrm{M} \hat{\xi} / \hat{\xi}^{T} M \mathrm{R}_{3} M \hat{\xi} \\
& \left.+\mathrm{K}_{0}-\mathrm{KR}_{3}\left(A^{\mathrm{T}} R_{1}+R_{2}\right) \mathrm{y}\right] \tag{30}
\end{align*}
$$

When taking these corrections into account, the proposed WTLS algorithm is as follows:

1st step: $[N, c]=A^{T} P_{y}[A, y], \xi^{(0)}=N^{-1} c$.
2nd step: For $i \in N$ compute:

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$$
\begin{align*}
R_{1}^{(\mathrm{i})}= & {\left[Q_{y}+\left(\hat{\xi}^{(i-1)^{\mathrm{T}}} \otimes I_{n}\right) Q_{A}\left(\hat{\xi}^{(i-1)} \otimes I_{n}\right)\right]^{-1} }  \tag{31}\\
\hat{\lambda}_{1}^{(i)}= & R_{1}^{(\mathrm{i})}\left(y-A \hat{\xi}^{(i-1)}\right)  \tag{32}\\
R_{2}^{(\mathrm{i})}= & \left(I_{\mathrm{m}} \otimes \hat{\lambda}^{(i)^{T}}\right) Q_{A}\left(\hat{\xi}^{(i-1)} \otimes I_{n}\right) R_{1}^{(\mathrm{i})}  \tag{33}\\
R_{3}^{(\mathrm{i})}= & \left(A^{\mathrm{T}} R_{1}^{(\mathrm{i})} A+R_{2}^{(\mathrm{i})} A\right)^{-1}  \tag{34}\\
\hat{\lambda}_{2}^{(i)}= & {\left[K R_{3}^{(\mathrm{i})} \mathrm{M} \hat{\xi}^{(i-1)} \hat{\xi}^{(i-1)^{T}} M^{T} R_{3}^{(\mathrm{i})} \mathrm{K}^{\mathrm{T}} / \hat{\xi}^{(i-1)^{T}} M R_{3}^{(\mathrm{i})} M \hat{\xi}^{(i-1)}\right.} \\
& \left.+\mathrm{K} R_{3}^{(\mathrm{i})} \mathrm{K}^{\mathrm{T}}\right]^{-1} \times\left[-K R_{3}^{(\mathrm{i})} \mathrm{M} \hat{\xi}^{(i-1)} \hat{\xi}^{(i-1)^{T}} M^{T} R_{3}^{(\mathrm{i})}\left(A^{\mathrm{T}} R_{1}^{(\mathrm{i})}\right.\right. \\
+ & \left.R_{2}^{(\mathrm{i})}\right) \mathrm{y} / \hat{\xi}^{(i-1)^{T}} M R_{3}^{(\mathrm{i})} M \hat{\xi}^{(i-1)} \\
+ & a_{0}^{2} K R_{3}^{(\mathrm{i})} M \hat{\xi}^{(i-1)} / \hat{\xi}^{(i-1)^{T}} M R_{3}^{(\mathrm{i})} M \hat{\xi}^{(i-1)} \\
+ & \left.\mathrm{K}_{0}-\mathrm{K} R_{3}^{(\mathrm{i})}\left(A^{\mathrm{T}} R_{1}^{(\mathrm{i})}+R_{2}^{(\mathrm{i})}\right) \mathrm{y}\right]  \tag{35}\\
\hat{\lambda}_{3}^{(i)}= & {\left[\hat { \xi } ^ { ( i - 1 ) } M R _ { 3 } ^ { ( \mathrm { i } ) } \left(\left(A^{\mathrm{T}} R_{1}^{(\mathrm{i})}+R_{2}^{(\mathrm{i})}\right) \mathrm{y}\right.\right.} \\
& \left.\left.+K^{T} \hat{\lambda}_{2}^{(i)}\right)-a_{0}^{2}\right] / \hat{\xi}^{(i-1)^{T}} M R_{3}^{(\mathrm{i})} M \hat{\xi}^{(i-1)} \tag{36}
\end{align*}
$$

$$
\begin{equation*}
\hat{\xi}^{(i)}=R_{3}^{(\mathrm{i})}\left(\left(A^{\mathrm{T}} R_{1}^{(\mathrm{i})}+R_{2}^{(\mathrm{i})}\right) \mathrm{y}+K^{T} \hat{\lambda}_{2}^{(i)}+M \hat{\xi}^{(i-1)} \hat{\lambda}_{3}^{(i)}\right) \tag{37}
\end{equation*}
$$

3rd step: Repeat 2nd step until

$$
\begin{equation*}
\left\|\hat{\xi}^{(i)}-\hat{\xi}^{(i-1)}\right\|<\delta \tag{38}
\end{equation*}
$$

for some chosen threshold $\delta$.
Unfortunately, a general proof of convergence of this algorithm cannot be given. Therefore, the statement made before Eq. (26) need to be modified. Instead of saying
"... and due to the invertible property of $\left(A^{\mathrm{T}} R_{1} A+R_{2} A\right)$ (since ( $\left.A^{\mathrm{T}} R_{1} A\right)^{-1}$ exists) ...",
it should read:
" $\ldots$ and when assuming that the inverse of $\left(A^{\mathrm{T}} R_{1} A+R_{2} A\right)$ exists ...".

In example 3: rapid satellite positioning using the constrained WTLS algorithm as a regularized approach, we found that the unconstrained solution suffers from ill-conditioning; adding constraints has a regularizing effect.

