Correction to: Asymptotic properties of Lee distance

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Published online: 20 February 2021

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Correction to: Metrika (2019) 82:385–408

<https://doi.org/10.1007/s00184-018-0687-7>

The statement of **Theorem 3** (p. 397) in Nikolov and Stoimenova (2019) should be corrected to:

Theorem 3 Let $M(\theta, N) = \{m_{ij}(\theta, N)\}_{i,j=1}^N$ be the Marginals matrix based on Lee distance. Then

$$m_{ij}(\theta_N, N) \frac{N}{\exp\left(\theta_N \mu + \frac{\theta_N^2 v^2}{2}\right)} \xrightarrow{N \rightarrow \infty} 1, \quad \text{for } i, j = 1, 2, \dots, N,$$

where

$$\mu = \frac{N c_N(i, j)}{N-1} - \frac{1}{N-1} \left\lfloor \frac{N+1}{2} \right\rfloor \left\lfloor \frac{N}{2} \right\rfloor,$$

$$v^2 = \begin{cases} \frac{2N^2 (c_N(i, j))^2 - N^3 c_N(i, j)}{2(N-2)(N-1)^2} - \frac{N^2 (N^2 - 2N + 4)}{48(N-1)^2}, & \text{for } N \text{ even} \\ \frac{2N^2 (c_N(i, j))^2 - N(N^2 - 1)c_N(i, j)}{2(N-2)(N-1)^2} - \frac{N(N+1)(N-3)}{48(N-2)}, & \text{for } N \text{ odd.} \end{cases}$$

and $\{\theta_N\}_{N=1}^\infty$ is a sequence of real numbers such that $\lim_{N \rightarrow \infty} \theta_N N^{\frac{3}{2}} = a$, for some real constant a .

The original article can be found online at <https://doi.org/10.1007/s00184-018-0687-7>.

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The quantity $m_{ij}(\theta, N)$ for fixed θ in the proof (p. 407) of the original statement of **Theorem 3** (p. 397) can be written as

$$m_{ij}(\theta, N) = \frac{1}{N} \exp \left(\theta (\tilde{\mu}(N) - \mu(N)) + \frac{\theta^2}{2} (\tilde{\sigma}^2(N) - \sigma^2(N)) \right) \frac{1 + \tilde{f}(\theta \tilde{\sigma}(N), N)}{1 + f(\theta \sigma(N), N)},$$

where $\mu(N) = \mathbf{E}(D_L)$, $\sigma^2(N) = \mathbf{Var}(D_L)$, $\tilde{\mu}(N) = \mathbf{E}(\tilde{D}_{N-1})$, $\tilde{\sigma}^2(N) = \mathbf{Var}(\tilde{D}_{N-1})$ and $f(\theta, N)$ and $\tilde{f}(\theta, N)$ are functions such that $\lim_{N \rightarrow \infty} f(\theta, N) = 0$ and $\lim_{N \rightarrow \infty} \tilde{f}(\theta, N) = 0$. Thus, the original statement is equivalent to

$$\lim_{N \rightarrow \infty} \frac{1 + \tilde{f}(\theta \tilde{\sigma}(N), N)}{1 + f(\theta \sigma(N), N)} = 1,$$

which is not trivial to be proved and even might not hold. However, if instead of fixed θ we consider a sequence of real numbers $\{\theta_N\}_{N=1}^{\infty}$ such that $\lim_{N \rightarrow \infty} \theta_N N^{\frac{3}{2}} = a$, for some real constant a , then

$$\lim_{N \rightarrow \infty} \theta_N \sigma(N) = c, \quad \lim_{N \rightarrow \infty} \theta_N \tilde{\sigma}(N) = \tilde{c},$$

where c and $\tilde{c}$ are some real constants, and $\lim_{N \rightarrow \infty} \frac{1 + \tilde{f}(\theta_N \tilde{\sigma}(N), N)}{1 + f(\theta_N \sigma(N), N)} = 1$. This completes the proof of the corrected formulation.

Furthermore, since the theorem holds for a sequence of real numbers $\{\theta_N\}_{N=1}^{\infty}$, it could be used to approximate the Marginals matrix $M(\theta, N)$ only when the value of θ is close to 0. However, our experience from simulation studies indicates that the original statement of **Theorem 3** (p. 397) might be true and the approximation could be applied even for large values of θ . As an illustration, see the example in **Section 4.2** (pp. 400–402).

Acknowledgements The authors would like to thank Prof. Mladen Savov for his careful review and constructive comments in relation to this work.

Reference

Nikolov N, Stoimenova J (2019) Asymptotic properties of Lee distance. *Metrika* 82:385–408

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