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Axiomatic characterizations of the core and the Shapley value of the broadcasting game

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Abstract
We study the cooperative game associated with a broadcasting problem (the allocation of revenues raised from the collective sale of broadcasting rights for a sports tournament). We show that the set of core allocations can be characterized with three axioms: *additivity*, *null team* and *monotonicity*. We also show that the Shapley value can be characterized with *additivity*, *equal treatment of equals* and *core selection*.

Keywords Broadcasting rights · Cooperative games · Core · Shapley value · Axioms

JEL Classification D63 · C71 · Z20

1 Introduction

Sports organizations (clubs) largely rely on the sale of broadcasting and media rights as their main source of revenue. Typically, such a sale is via collective bargaining (between the unionized clubs and the broadcasting companies). Thus, once an agreement is reached, the collected revenues have to be shared among the clubs. This is, by no means, an easy problem because the individual contributions (to those revenues) are not clearly identified.

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Bergantiños and Moreno-Ternero (2020a) introduce a formal model to analyze the problem of sharing the revenues from broadcasting sports leagues among participating clubs based on the audiences they generate. This model has already generated a sizable literature, dealing with several aspects of the problem.[1] A special emphasis within that literature has been made on the axiomatic approach. But a game-theoretical approach has also received attention. In particular, Bergantiños and Moreno-Ternero (2020a) associate with each broadcasting problem a natural cooperative game and study several aspects of it. More precisely, they characterize the allocations in the core of such a game and prove that its Shapley value coincides with the so called *equal-split* rule (a rule highlighted in the axiomatic approach to solve broadcasting problems directly).

In this paper, we study further the core and the Shapley value of the *broadcasting game* mentioned above. More precisely, we prove that the set of core allocations can be characterized with three axioms. On the one hand, *additivity* and *null team*.[2] The former says that revenues should be additive on audiences. The latter says that clubs with null audience should receive a null award. On the other hand, one of the following three *monotonicity* axioms.[3] *Weak club monotonicity* states that if the audiences of the games played by a certain club increase, and the rest of audiences remain the same, then this club cannot receive less. *Overall monotonicity* states that the rule should be monotonic on audiences. Finally, *pairwise monotonicity* states that if the aggregate audience of the games played by any pair of clubs increases, then no club can receive less.

We then formalize as an axiom that the rule (to solve broadcasting games) should only select allocations within the core (of the associated TU-game). This axiom has obvious normative appeal, as it guarantees the stability of the league being considered, preventing participating clubs to secede. We refer to the axiom as *core selection*. We obtain a new characterization of the *equal-split* rule (or the Shapley value of the associated TU-game) when combining such an axiom with *additivity* (already mentioned above) and *equal treatment of equals* (the standard notion of impartiality, which states that clubs generating the same audiences should get the same allocation).

We conclude this introduction mentioning that our work is obviously related to two important strands of the game-theory literature.

On the one hand, the strand of that literature that addresses various sharing problems by associating a transferable utility game to the problem, and constructing sharing rules by means of the standard values in that game. Classical instances are the so-called airport problems (e.g., Littlechild and Owen 1973; Littlechild 1974), in which the cost of a runway has to be shared among different types of airplanes, bankruptcy problems from the Talmud (e.g., O'Neill 1982; Aumann and Maschler

[1] See Bergantiños and Moreno-Ternero (2023e) for a recent survey of this literature.

[2] These are two standard axioms in cooperative game theory formalizing principles that can be traced back to Shapley (1953).

[3] Monotonicity axioms have a long tradition in axiomatic work. Thomson and Myerson (1980) is a seminal work. Bergantiños and Moreno-Ternero (2022b, 2022c) explore several alternative *monotonicity* axioms and their implications in this setting of broadcasting problems.

1985; Aumann 2010), minimum cost spanning tree problems (e.g., Bergantiños and Vidal-Puga 2007) or telecommunications problems (e.g., van den Nouweland et al. 1992, 1996).

On the other hand, the strand of the literature that addresses various problems related to sports. Instances are the scoring or ranking of participants in tournaments or competitions (e.g., Slutzki and Volij 2005; Kondratev and Mazalov 2020; Kondratev et al. 2023), the prize allocation therein (e.g., Dietzenbacher and Kondratev 2023; Alcalde-Unzu et al. 2023) or, more generally, the design of stable and fair competitions or memberships (e.g., Le Breton et al. 2013; Anbarci et al. 2021). Finally, Palacios-Huerta (2014) gathers numerous intriguing connections between game theory and the most popular sport worldwide, which provide interesting lessons for research in mainstream economics.

The rest of the paper is organized as follows. In Sect. 2, we introduce the model (of broadcasting problems and games) and the notation. In Sect. 3, we study the core (of broadcasting games). In Sect. 4, we study the Shapley value (of broadcasting games). In Sect. 5, we conclude.

2 The model

We consider the model introduced in Bergantiños and Moreno-Ternero (2020a). Let N be a finite set of clubs. Its cardinality is denoted by n. We assume $n \geq 3$. For each pair of clubs $i, j \in N$, we denote by a_{ij} the broadcasting audience (number of viewers) for the game played by i and j at i's stadium. We use the notational convention that $a_{ii} = 0$, for each $i \in N$. Let $A \in \mathcal{A}_{n \times n}$ denote the resulting matrix of broadcasting audiences generated in the whole tournament involving the clubs within N.

A **problem** is $A \in \mathcal{A}_{n \times n}$ with zero entries in the diagonal. Let $\mathcal{P}$ denote the set of all problems.

Let $\alpha_i(A)$ denote the total audience achieved by club i, i.e.,

$$\alpha_i(A) = \sum_{j \in N} (a_{ij} + a_{ji}).$$

When no confusion arises, we write α_i instead of $\alpha_i(A)$.

For each $A \in \mathcal{A}_{n \times n}$, let $||A||$ denote the total audience of the tournament. Namely,

$$||A|| = \sum_{i,j \in N} a_{ij} = \frac{1}{2} \sum_{i \in N} \alpha_i.$$

A (sharing) **rule** (R) is a mapping that associates with each problem the list of the amounts that clubs get from the total revenue. Without loss of generality, we normalize the revenue generated from each viewer to 1 (to be interpreted as the "pay per view" fee). Thus, formally, $R : \mathcal{P} \to \mathbb{R}^N$ is such that, for each $A \in \mathcal{P}$,

$$\sum_{i \in N} R_i(A) = ||A||.$$

The *equal-split* rule divides the audience of each game equally among the two participating clubs. Formally,

Equal-split rule, ES: for each $A \in \mathcal{P}$, and each $i \in N$,

$$ES_i(A) = \frac{\alpha_i}{2}.$$

2.1 Axioms

We now introduce the axioms we shall use in this paper. Notice that the axioms are defined on the set of broadcasting problems and not on the set of cooperative games we shall associate to them.

The first axiom says that if two clubs have the same audiences (each time they play a third club), then they should receive the same amount.

Equal treatment of equals: For each $A \in \mathcal{P}$, and each pair $i, j \in N$ such that $a_{ik} = a_{jk}$, and $a_{ki} = a_{kj}$, for each $k \in N \backslash \{i, j\}$,

$$R_i(A) = R_j(A).$$

The second axiom says that revenues should be additive on A. Formally,

Additivity: For each pair A, $A' \in \mathcal{P}$,

$$R\left(A + A'\right) = R(A) + R\left(A'\right).$$

The third axiom says that if a club has a null audience, then such a club gets no revenue. Formally,

Null team: For each $A \in \mathcal{P}$, and each $i \in N$, such that $a_{ij} = 0 = a_{ji}$ for each $j \in N$,

$$R_i(A) = 0.$$

The three axioms defined above were introduced in Bergantiños and Moreno-Ternero (2020a) and used later elsewhere (e.g., Bergantiños and Moreno-Ternero 2021, 2022a, c, 2023a, b, c).

Bergantiños and Moreno-Ternero (2022b) introduced several monotonicity axioms. We consider three of them here. The first one says that if the audiences of the games played by a certain club increase and the rest of audiences remain the same, then this club cannot receive less. Formally,

Weak club monotonicity: For each pair A, $A' \in \mathcal{P}$ and each $i \in N$,

$$\left.\begin{array}{l} a_{ij} \leq a'_{ij} \text{ for all } j \in N \backslash \{i\} \text{ and} \\ a_{ji} \leq a'_{ji} \text{ for all } j \in N \backslash \{i\} \\ a_{jk} = a'_{jk} \text{ when } i \notin \{j, k\} \end{array}\right\} \quad \Rightarrow \quad R_i(A) \leq R_i\left(A'\right).$$

The second one says that the rule should be monotonic on A. Formally,

Overall monotonicity: For each pair $A, A' \in \mathcal{P}$ and each $i \in N$,

$$a_{jk} \leq a'_{jk} \text{ for each } j,k \in N \quad \Rightarrow \quad R_i(A) \leq R_i\left(A'\right).$$

The third one says that if the aggregate audience of the games played by any pair of clubs increases, then no club can be worse off. Formally,

Pairwise monotonicity: For each pair $A, A' \in \mathcal{P}$ and each $i \in N$,

$$a_{kj} + a_{jk} \leq a'_{kj} + a'_{jk} \text{ for each } j,k \in N \quad \Rightarrow \quad R_i(A) \leq R_i(A').$$

Bergantiños and Moreno-Ternero (2022b) proved the following relationship among the previous axioms.

Remark 1 The following statements hold:

(a) If a rule R satisfies pairwise monotonicity, then R also satisfies overall monotonicity.
(b) If a rule R satisfies overall monotonicity, then R also satisfies weak club monotonicity.

2.2 Cooperative games

A **cooperative game with transferable utility**, briefly a **TU game**, is a pair (N, v), where N denotes a set of agents and $v : 2^N \to \mathbb{R}$ satisfies $v(\emptyset) = 0$.

The **core** is defined as the set of feasible payoff vectors, upon which no coalition can improve. As the population N will remain fixed, we avoid its use in the notation. Formally,

$$Core(v) = \left\{ x \in \mathbb{R}^N \text{ such that } \sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \geq v(S), \text{ for each } S \subset N \right\}.$$

The **Shapley value** (Shapley 1953) is defined for each player as the average of his contributions across orders of agents.

Bergantiños and Moreno-Ternero (2020a) associate with each broadcasting problem $A \in \mathcal{P}$ a *TU* game (N, v_A). To so so, they take an optimistic approach, noting that the highest possible revenue that a game between teams i and j in the former's stadium may generate is a_{ij}. So, by breaking away from the league, the most optimistic scenario for any coalition of teams is to generate the same revenue they generated before leaving the league. Formally, given a broadcasting problem $A \in \mathcal{P}$, for each $S \subset N$, $v_A(S)$ is defined as the total audience of the games played by the clubs in S. Namely,

$$v_A(S) = \sum_{i,j \in S, i \neq j} a_{ij} = \sum_{i,j \in S, i<j} \left(a_{ij} + a_{ji}\right).$$

The game v_A is equivalent to the game defined by van den Nouweland et al. (1996) to the so-called Terrestial Flight Telephone System. They prove that such a game is convex and the Shapley value coincides with the Nucleolus and the τ-value. Also, we can report that, based on Lemma 1 below, the core of this game is quite large.

3 The core of the broadcasting game

We prove in this section that the allocations within the core can be obtained as the allocations induced by the set of rules satisfying *additivity*, *null team* and one of the three *monotonicity* axioms listed above. To do so, we first consider the following lemma, which provides the characterization of the core obtained in Bergantiños and Moreno-Ternero (2020a).

Lemma 1 *Let $A \in \mathcal{P}$. Then, $x = \left(x_i\right)_{i\in N} \in Core(v_A)$ if and only if, for each $i \in N$, there exist $\left(x_i^j\right)_{j\in N\setminus\{i\}}$ satisfying three conditions*:

(i) $x_i^j \geq 0,$ *for each* $j \in N\setminus\{i\}$;

(ii) $\sum_{j\in N\setminus\{i\}} x_i^j = x_i$;

(iii) $x_i^j + x_j^i = a_{ij} + a_{ji},$ *for each* $j \in N\setminus\{i\}$;

Let $\mathcal{D}$ be the set of rules satisfying *additivity*, *null team* and *weak club monotonicity*. Given a problem A, let $\mathcal{D}(A)$ be the set of allocations induced by the rules in $\mathcal{D}$ for that problem. Namely,

$$\mathcal{D}(A) = \left\{x \in \mathbb{R}^N \text{ such that } R(A) = x \text{ for some } R \in \mathcal{D}\right\}.$$

We can now state our result.

Theorem 1 *For each $A \in \mathcal{P}$,*

$$Core(v_A) = \mathcal{D}(A).$$

Proof We first prove that $Core(v_A) \subset \mathcal{D}(A)$. Let $A \in \mathcal{P}$. Let $x = \left(x_i\right)_{i\in N} \in Core(v_A)$. By Lemma 1, for each $i \in N$ we can find $\left(x_i^j\right)_{j\in N\setminus\{i\}}$ satisfying conditions (i), (ii), and (iii).

Given $A' \in \mathcal{P}$ and $i, j \in N$ we define

$$y_i^j\left(A'\right) = \begin{cases} \frac{x_i^j}{a_{ij}+a_{ji}}\left(a'_{ij} + a'_{ji}\right) & \text{if } a_{ij} + a_{ji} \neq 0 \\ \frac{a'_{ij}+a'_{ji}}{2} & \text{otherwise.} \end{cases}$$

Given $A' \in \mathcal{P}$ and $i \in N$ we now define the rule R^y as

$$R_i^y(A') = \sum_{j \in N \setminus \{i\}} y_i^j(A').$$

It is obvious that R^y satisfies *additivity*, *null team* and *weak club monotonicity*. Now, for each $i \in N$,

$$R_i^y(A) = \sum_{j \in N \setminus \{i\}} x_i^j = x_i.$$

We now prove that $Core(v_A) \supset \mathcal{D}(A)$. Let R be a rule satisfying *additivity*, *null team* and *weak club monotonicity*.

For each pair $i, j \in N$, with $i \neq j$, let $\mathbf{1}^{ij}$ denote the matrix with the following entries:

$$\mathbf{1}_{kl}^{ij} = \begin{cases} 1 & \text{if } (k,l) = (i,j) \\ 0 & \text{otherwise.} \end{cases}$$

Notice that $\mathbf{1}_{ji}^{ij}$ is the zero matrix (which we denote as 0), *i.e.*, the matrix with only zero entries.

Let $A \in \mathcal{P}$ and $i \in N$. As R satisfies *additivity*,

$$R_i(A) = \sum_{j,k \in N : j \neq k} a_{jk} R_i\left(\mathbf{1}^{jk}\right). \tag{1}$$

As R satisfies *null team*,

$$R_i(A) = \sum_{j \in N \setminus \{i\}} \left(a_{ij} R_i\left(\mathbf{1}^{ij}\right) + a_{ji} R_i\left(\mathbf{1}^{ji}\right)\right). \tag{2}$$

For each pair $i, j \in N$ with $i \neq j$ we define

$$x_i^j = a_{ij} R_i\left(\mathbf{1}^{ij}\right) + a_{ji} R_i\left(\mathbf{1}^{ji}\right).$$

By Lemma 1, it is enough to show that $\left(x_i^j\right)_{j \in N \setminus \{i\}}$ satisfies conditions (*i*), (*ii*), and (*iii*).

By *null team*, $R_i(0) = 0$. By *weak club monotonicity*, $R_i\left(\mathbf{1}^{ij}\right) \geq R_i(0) = 0$ and $R_i\left(\mathbf{1}^{ji}\right) \geq R_i(0) = 0$. Then, $x_i^j \geq 0$. Thus, condition (*i*) holds.

By (2), $R_i(A) = \sum_{j \in N \setminus \{i\}} x_i^j$. Thus, condition (*ii*) holds.

Also, for each pair $i, j \in N$, with $i \neq j$,

$$\begin{aligned} a_{ij} + a_{ji} &= a_{ij} \left\|\mathbf{1}^{ij}\right\| + a_{ji} \left\|\mathbf{1}^{ji}\right\| \\ &= a_{ij} \sum_{k \in N} R_k\left(\mathbf{1}^{ij}\right) + a_{ji} \sum_{k \in N} R_k\left(\mathbf{1}^{ji}\right). \end{aligned}$$

By *null team*, for each $k \in N \setminus \{i, j\}$, $R_k\left(\mathbf{1}^{ij}\right) = R_k\left(\mathbf{1}^{ji}\right) = 0$. Thus,

$$\begin{aligned} a_{ij} + a_{ji} =& a_{ij}\big(R_i\big(\mathbf{1}^{ij}\big) + R_j\big(\mathbf{1}^{ij}\big)\big) + a_{ji}\big(R_i\big(\mathbf{1}^{ji}\big) + R_j\big(\mathbf{1}^{ji}\big)\big) \\ =& x_i^j + x_j^i, \end{aligned}$$

which implies that condition (*iii*) also holds. □

Remark 2 We now prove that if we consider rules that fail one of the three axioms associated with the rules given by $\mathcal{D}(A)$, then we can have allocations outside the core.

Let R^1 be the rule in which, for each game $(i,j) \in N \times N$, the revenue goes to the club with the highest audience and ties are divided equally among both clubs. Namely, for each $A \in \mathcal{P}$ and $i \in N$,

$$R_i^2(A) = \sum_{j \in N:\alpha_i > \alpha_j} (a_{ij} + a_{ji}) + \sum_{j \in N:\alpha_i = \alpha_j} \frac{a_{ij} + a_{ji}}{2}.$$

R^2 satisfies *null team and weak club monotonicity*, but not *additivity*.

The uniform rule, which divides $||A||$ equally among all clubs, satisfies additivity and *weak club monotonicity,* but not *null team*.

Let R^2 be such that for each $A \in \mathcal{P}$, and each $i \in N$,

$$R_i^2(A) = \sum_{j \in N} (2a_{ij} - a_{ji}).$$

R^2 satisfies *additivity and null team*, but not *weak club monotonicity*.

In the next proposition we obtain a similar characterization to Theorem 1 with *overall monotonicity* or *pairwise monotonicity* instead of *weak club monotonicity*.

Let $\mathcal{D}^1$ (respectively $\mathcal{D}^2$) be the set of rules satisfying *additivity*, *null team* and *overall monotonicity* (respectively *pairwise monotonicity*). Given a problem A, let $\mathcal{D}^1(A)$ (respectively $\mathcal{D}^2(A)$) be the set of allocations induced by the rules within $\mathcal{D}^1$ (respectively $\mathcal{D}^2$) on A. Namely,

$$\begin{aligned} \mathcal{D}^1(A) =& \left\{x \in \mathbb{R}^N \text{ such that } R(A) = x \text{ for some } R \in \mathcal{D}^1\right\} \text{ and} \\ \mathcal{D}^2(A) =& \left\{x \in \mathbb{R}^N \text{ such that } R(A) = x \text{ for some } R \in \mathcal{D}^2\right\}. \end{aligned}$$

Proposition 1 *For each* $A \in \mathcal{P}$,

$$Core(v_A) = \mathcal{D}^1(A) = \mathcal{D}^2(A).$$

Proof As mentioned in Remark 1, if a rule satisfies *overall monotonicity* or *pairwise monotonicity*, then it also satisfies *weak club monotonicity*. Thus,

$$\begin{aligned} \mathcal{D}^1(A) \subset& \mathcal{D}(A) = Core(v_A) \text{ and} \\ \mathcal{D}^2(A) \subset& \mathcal{D}(A) = Core(v_A). \end{aligned}$$

As for the converse inclusions, it suffices to notice that the rule R^y defined in the proof of Theorem 1 satisfies *overall monotonicity* and *pairwise monotonicity*. Thus, similarly to the proof of Theorem 1, we can prove that $Core(v_A) \subset \mathcal{D}^1(A)$ and $Core(v_A) \subset \mathcal{D}^2(A)$. □

4 The Shapley value of the broadcasting game

Bergantiños and Moreno-Ternero (2020a) show that the *equal-split* rule coincides with the Shapley value of the associated cooperative game. That is, for each $A \in \mathcal{P}$,

$$ES(A) = Sh(N, v_A).$$

The *equal-split* rule was further studied in subsequent work (e.g., Bergantiños and Moreno-Ternero 2020b, 2021, 2022b, 2023d). We now provide a new axiomatic characterization in which we use the following new axiom,

Core selection: For each $A \in \mathcal{P}$ and each $S \subset N$,

$$\sum_{i \in S} R_i(A) \geq \sum_{i,j \in S, i<j} \left(a_{ij} + a_{ji}\right).$$

Notice that if R satisfies core selection then the rule should select allocations within the core of the associated cooperative game. Namely, $R(A) \in Core\left(v_A\right)$.

Theorem 2 *A rule satisfies additivity, equal treatment of equals and core selection if and only if it is the equal-split rule.*

Proof We already know that the *equal-split* rule satisfies *additivity* and *equal treatment of equals* (e.g., Bergantiños and Moreno-Ternero 2020a). By Lemma 1, we deduce that it also satisfies *core selection*.

Conversely, let R be a rule satisfying the three axioms. Let $A \in \mathcal{P}$ and $i \in N$. By (1),

$$R_i(A) = \sum_{j,k \in N, j \neq k} a_{jk} R_i\left(\mathbf{1}^{jk}\right).$$

As R satisfies *additivity* and *equal treatment of equals*, using similar arguments to those used in the proof of Theorem 1 in Bergantiños and Moreno-Ternero (2021), we can prove that there exists $x \in \mathbb{R}$ such that for each pair $i, j \in N$,

$$\begin{aligned} R_i\left(\mathbf{1}^{ij}\right) &= R_j\left(\mathbf{1}^{ij}\right) = x, \text{ and} \\ R_l\left(\mathbf{1}^{ij}\right) &= \frac{1-2x}{n-2} \text{ for each } l \in N \backslash \{i, j\}. \end{aligned}$$

As R satisfies *core selection*, it follows from Lemma 1 that $R_l\left(\mathbf{1}^{ij}\right) = 0$ for each $l \in N \backslash \{i, j\}$. Then, $x = \frac{1}{2}$ and hence $R\left(\mathbf{1}^{ij}\right) = ES\left(\mathbf{1}^{ij}\right)$. By (1), we conclude that $R(A) = ES(A)$. □

Remark 3 The axioms used in Theorem 2 are independent.

Let R^3 be the rule in which, for each game $(i,j) \in N \times N$ the revenue goes to the club with the lowest number of the two. Namely, for each $A \in \mathcal{P}$, and each $i \in N$,

$$R_i^3(A) = \sum_{j \in N : j > i} (a_{ij} + a_{ji}).$$

R^3 satisfies *additivity and core selection*, but not *equal treatment of equals*.

The uniform rule, defined above, satisfies additivity and *equal treatment of equals,* but not *core selection*.

The rule R^1, defined above, satisfies *equal treatment of equals* and *core selection*, but not *additivity*.

We now discuss other existing axiomatic characterizations of the *equal-split rule* (i.e., the Shapley value of the associated TU-game). Bergantiños and Moreno-Ternero (2020a) characterize it with *additivity*, *equal treatment of equals* and *null team* (instead of *core selection*). Bergantiños and Moreno-Ternero (2020b) characterize it with *equal treatment of equals* and another axiom (*half sharing of additional club viewers*), as well as with *null team* and another axiom (*equal benefits from additional viewers*). Bergantiños and Moreno-Ternero (2021) characterize it with *additivity* and three other axioms (*symmetry*, *maximum aspirations* and *non-negativity*). Bergantiños and Moreno-Ternero (2022b) characterize it with *equal treatment of equals* and another monotonicity axiom (*club monotonicity*).[4] Finally, Bergantiños and Moreno-Ternero (2023d) characterize it with two other axioms (*reallocation proofness* and *single-conference*), as well as with *null team* and two other axioms (*reallocation proofness* and *multi-conference*).

5 Conclusion

We have studied the cooperative game that is naturally associated with the so-called broadcasting problem (i.e., the problem that arises when participating clubs in a competition have to share the collectively raised revenues from selling the broadcasting rights for the competition). We have characterized the allocations in the core of such a game with the combination of three axioms: *additivity*, *null team*, and a *monotonicity* axiom coming from a trio (*weak club monotonicity*, *overall monotonicity*, or *pairwise monotonicity*). The Shapley value of such a game, with coincides with the so-called *equal-split* rule for broadcasting problems, always lies within the core of the game. In other words, it satisfies the axiom of *core selection*. As a matter of fact, we have also provided a new characterization of this rule with the combination of three axioms: *additivity*, *equal treatment of equals* and *core selection*.

[4] This is, essentially, a counterpart result to Young's famous characterization of the Shapley value (e.g., Young 1985).

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