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Efficient equilibria in common interest voting games

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Abstract

We develop a unified derivation of asymmetric pure strategy equilibria and their optimality in the canonical common interest voting model of Austen-Smith and Banks (Am Polit Sci Rev 90(1):34–45, 1996). We also study the relationship between the most efficient equilibria, which have a remarkably simple and intuitive structure, and the symmetric mixed strategy equilibrium that has been commonly studied in the literature. In particular, while the efficiency in the symmetric mixed strategy equilibrium under unanimity rule is known to be decreasing in the number of voters, the efficiency does not depend on the number of voters above a threshold in the most efficient equilibria.

Keywords Committee decision making · Asymmetric equilibria · Public information · Private information · Strategic voting

JEL Classification C92 · D72 · D82

1 Introduction

Even for simple strategic voting games, it has been customary in the literature to focus on the most efficient *symmetric* strategy equilibrium, not least because of the large strategy space and the presence of multiple equilibria. While Feddersen and

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Pesendorfer (1998) and Wit (1998) derived the efficient *symmetric* equilibrium in the canonical binary common interest voting model of Austen-Smith and Banks (1996), it has been well known that the most efficient equilibrium is in asymmetric pure strategies.[1]

In the literature, Chakraborty and Ghosh (2003) showed that the efficient equilibria feature a certain number of agents always voting according to the private signal, and the rest always voting for one of the alternatives. Maug and Rydqvist (2008) explicitly derived asymmetric pure strategy equilibria in line with the strategy profiles, but they did not relate the equilibria to the earlier observation by Chakraborty and Ghosh (2003) and hence did not establish the optimality.[2] As the approaches adopted by Chakraborty and Ghosh (2003) and Maug and Rydqvist (2008) are very different from each other, the asymmetric equilibria and their optimality among all strategy profiles have not been studied systematically. As a result, little has been known about the implications of the efficient asymmetric equilibria with regards to results known in the strategic voting literature.

This paper offers a unified approach to the derivation of asymmetric equilibria and their optimality, so that the structure of the equilibria can be clearly understood. Specifically, we first pin down a class of pure strategy profiles the most efficient strategy profile must belong to, and subsequently derive the efficient asymmetric equilibria by searching for the equilibrium strategy profiles in the class. In deriving the equilibria we focus on each voter's incentive to deviate. Our derivation makes it clear that, in the most efficient equilibria, when a supermajority rule excessively favours an alternative that is ex ante more desirable, then some agents always vote *against* the alternative to offset the superfluous advantage imposed by the supermajority rule. Meanwhile, if the rule in place favours an ex ante more desirable alternative too little or even handicaps it, then some agents always vote for the alternative regardless of their private signals.[3]

The intuition behind the asymmetric pure strategy equilibria we derive in this paper is related to the symmetric mixed strategy equilibrium in Feddersen and Pesendorfer (1996). While their model focuses on majority rule in a large binary election where the voters may choose to abstain, the presence of partisan voters who always vote for one alternative or the other irrespective of the signals they receive amounts to supermajority rule from the view point of strategic voters whose preferred alternative depends on their signals (if they are informed). Moreover, their

[1] Related contributions to common-interest strategic voting models include Feddersen and Pesendorfer (1996), McLennan (1998), Coughlan (2000), Dekel and Piccione (2000), Gerardi and Yariv (2007), Ali and Kartik (2012) and Ellis (2016), among others. Experimental evidence is largely consistent with theoretical predictions (Guarnaschelli et al. 2000; Ali et al. 2008 etc.) while Kawamura and Vlaseros (2017) point to difficulty in playing an equilibrium even in a simple majority game. See Squintani (2019) for a recent survey.

[2] Maug and Rydqvist (2008) also assume away the non-generic cases that involve two sets of equilibria that differ in the number of agents voting according to the private signal. We complete the equilibrium characterization and show that the sets of equilibria feature the same efficiency.

[3] For example, a voting rule that handicaps an ex ante desirable outcome may be associated with situations where a supermajority is required to implement a promising reform, which is objectively more likely to be beneficial than an unsatisfactory status quo.

symmetric mixed strategy equilibrium partially "corrects" for the bias in the game created not only by the prior distribution of the state but also by partisan voters, through mixing. In our model, the nature of the bias is very similar to that of Feddersen and Pesendorfer (1996) but we explicitly focus on the rule in place, and derive the efficient equilibria in asymmetric pure strategies.

We also resort to the clear comparative statics of the efficient asymmetric equilibria to make comparison with the symmetric mixed equilibrium in terms of equilibrium strategies and efficiency. For example, it is well known that in a symmetric strategy equilibrium under unanimity, the probability of "convicting the innocent" ("acquitting the guilty) may become higher (lower) as the size of the committee/jury increases, and the expected payoff may be decreasing in the committee size. In contrast, the efficient asymmetric pure strategy equilibria feature a fixed number of agents always voting against the alternative the unanimity rule favours (i.e. they vote for "conviction" regardless of their private information), in such a way that the number of informative votes and the probability of convicting the innocent and the expected payoff is constant with respect to the committee size.

In what follows, we present the model in Sect. 2, and derive the most efficient equilibria in Sect. 3. We discuss some numerical examples of interest in Sect. 4 in order to gain intuition behind the efficient equilibria and compare it with the well-known symmetric mixed strategy equilibrium. We suggest an interpretation of our results in Sect. 5. Section 6 concludes.

2 Model

Consider a committee that consists of an odd number of strategic agents n. Each agent $i \in N \equiv \{1, 2, \dots, n\}$ simultaneously casts a costless binary vote, denoted by $x_i \in X = \{A, B\}$, for a collective decision with respect to the binary alternative $y \in Y = \{A, B\}$. The collective decision is determined by a k-majority rule favouring $y = A$, where $k \in \{0, 1, 2, \dots, \frac{n-1}{2}\}$ and $\frac{n+1}{2} - k$ or more votes are required for alternative A to be selected. Naturally, simple majority rule features $k = 0$, and unanimity rule features $k = \frac{n-1}{2}$.

The binary state of the world is denoted by $s \in S = \{A, B\}$. Ex ante each state is realized with equal probability 1/2. The agents have identical preferences $u_i : Y \times S \to \mathbb{R}$ and, specifically we denote the vNM payoff by $u_i(y, s)$ and assume $u_i(A, A) = u_i(B, B) = 1$ and $u_i(A, B) = u_i(B, A) = 0$ for any $i \in N$.[4] This implies that the payoff depends solely on whether the committee decision matches the state.

Before voting, each agent has two pieces of information. One is a private signal about the state $\sigma_i \in K = \{A, B\}$, for which the probability of the signal and the state being matched is given by $\Pr[\sigma_i = A \mid s = A] = \Pr[\sigma_i = B \mid s = B] = q$, where $q \in (1/2, 1]$. We also have $\Pr[\sigma_i = A \mid s = B] = \Pr[\sigma_i = B \mid s = A] = 1 - q$. In addition to the private signal, all agents in the committee have a public signal $\sigma_0 \in L = \{A, B\}$.

[4] As Austen-Smith and Banks (1996) noted, the assumption of symmetric payoffs with respect to the state is for expositional convenience only.

Specifically, we assume $\Pr[\sigma_0 = A \mid s = A] = \Pr[\sigma_0 = B \mid s = B] = \pi$ and $\Pr[\sigma_0 = A \mid s = B] = \Pr[\sigma_0 = B \mid s = A] = 1 - \pi$, where $\pi \in (1/2, 1]$.

The agents do not communicate before they vote and are not allowed to abstain. We say that a k-majority rule *favours* the public signal if $\sigma_0 = A$ and *handicaps* the public signal if $\sigma_0 = B$. Supermajority rules typically require more than a majority to overturn the status quo, which corresponds to A in our model.[5]

With respect to the common notation in the literature, the public signal in our model corresponds to the state more likely to be realized according to its common prior distribution (when there is no further public information available). For expositional convenience, we represent the shared information regarding the state by the combination of a uniform common prior and an additional public signal, rather than a potentially biased prior distribution of the state. This allows us to treat the two pieces of information (public and private) each agent receives in parallel and also makes it easier to clarify the relationship between the state that is ex ante more likely and the decision favoured by supermajority rule.

Before studying the equilibria of the game, let us make the following observation:

Fact *The voting game described above under a k-majority rule is equivalent to the voting game under simple majority rule with* $n + 2k$ *agents where* $2k$ *agents deterministically vote for A and the rest vote strategically.*

Clearly, this transformation into the simple majority voting game preserves the original feature of k-majority rule that there are n strategic agents and $\frac{n+1}{2} - k$ or more votes are required for alternative A to be selected. We resort to this observation throughout the paper to simplify the exposition.[6]

3 Efficient equilibria

In what follows, we investigate the transformed simple majority voting game with $n + 2k$ agents where $2k$ agents are non-strategic and vote for A regardless of the information they receive. When an agent votes for the public (private) signal with probability 1 irrespective of the other signal, we say that the agent votes *consistently* for the public signal (the private signal). In the main body of our analysis, we focus on a class of pure strategy profiles $M(n, k, c, d)$ for n strategic agents under k-majority rule, where all variables are non-negative integers, such that c agents consistently vote for the public signal (i.e. regardless of the private signal), d agents consistently

[5] If a rule favours the public signal, the voting rule is ex ante "good" in the sense that more votes are required to overturn the status quo which the public signal indicates is more likely to be desirable. However, as we will see later, the favour given by the rule to the status quo can be excessive, which may be partially mitigated by strategic agents' consistent votes against the status quo.

[6] The deterministic voters here play a similar role to partisan voters in Feddersen and Pesendorfer (1996), by effectively biasing majority rule.

vote against the public signal and $n - c - d$ agents vote consistently according to the private signal.[7]

We define an efficient equilibrium as a Bayesian Nash equilibrium where the probability that the committee decision matches the state is the highest among all equilibria. Since the game we study is a common interest game, an efficient equilibrium strategy profile is also a strategy profile that achieves the highest probability of the committee decision matching the state among all (equilibrium and non-equilibirum) strategy profiles (McLennan, 1998). Before identifying the efficient equilibria, we show that any efficient equilibria must belong to $M(n, k, c, d)$, by demonstrating that the probability of the committee decision matching the state can be improved if a strategy profile (whether it is in equilibrium or not) is outside of $M(n, k, c, d)$.

If a specific $M(n, k, c, d)$ is in equilibrium, there are multiple equilibria in which the probability that the committee decision matches the state is identical, such that each equilibrium strategy profile involves sets of c agents voting for the public signal, d agents voting against the private signals, and $n - c - d$ agents voting for the private signal.

Note that when a sufficient number of agents vote consistently for (against) the public signal, the committee decision is σ_0 ($\neq \sigma_0$) with probability 1, and we say such equilibria are *obedient* (*disobedient*). Both obedient and disobedient equilibria trivially exist, but we can rule out disobedient equilibria immediately as an efficient outcome because $\pi > 1/2$ and hence the obedient outcome that follows the public signal must is always better. In what follows, we will primarily focus on the efficient equilibria where the committee decision is not obedient, and in so doing find conditions under which obedient equilibria are efficient.

In order to derive the efficient equilibria, we first establish through the following Lemmas that in an efficient strategy profile (not necessarily an equilibrium strategy profile), i) no agent's strategy is to vote consistently against the private signal; and ii) the strategy profile must not involve both an agent whose strategy is to consistently vote for the public signal and an agent whose strategy is to consistently vote against the public signal. The former rules out strategy profiles such that an agent votes against the private signal irrespective of the realization of the public signal. The latter implies "no vote must be wasted" since a vote for and a vote against the public signal cancel each other and together they have no consequence on the final majority decision. Formally, as we will show that the efficient strategy profile has to feature $M(n, k, c, d)$ and moreover $(c \geq 0) \wedge (d = 0)$ or $(c = 0) \wedge (d > 0)$.

Lemma 1 *An efficient non-obedient strategy profile does not involve any strategy to vote consistently against the private signal.*

Proof Consider a strategy such that an agent votes against the private signal irrespective of the realization of the public signal. Without loss of generality, suppose the public signal σ_0 is A. According to the strategy, if $\sigma_i = A$ then the agent votes

[7] We will show shortly in Lemma 1 that voting consistently for the private signal cannot maximize the probability of the committee decision matching the state.

for B, and if $\sigma_i = B$ the agent votes for A. In what follows, we demonstrate that the strategy contradicts the maximization of the probability that the committee decision matches the state by showing that the probability is increased when the strategy of the agent is altered.

Let the probability that an arbitrary signal profile $\sigma_{-i} = \{\sigma_0, \sigma_1, \ldots, \sigma_{i-1}, \sigma_{i+1}, \ldots, \sigma_n\}$ is realized given the state is A be $r \equiv P(\sigma_{-i} \mid s = A)$. We have

$$P(s = A|\sigma_{-i}, \sigma_i = A) = \frac{rq}{rq + (1-r)(1-q)}, \tag{1}$$

$$P(s = A|\sigma_{-i}, \sigma_i = B) = \frac{r(1-q)}{r(1-q) + (1-r)q}. \tag{2}$$

Since $q \in (1/2, 1]$, the numerator of (1) is larger than that of (2). For the same reason, the denominator of (1) is smaller than that of (2). Thus we have

$$P(s = A|\sigma_{-i}, \sigma_i = A) > P(s = A|\sigma_{-i}, \sigma_i = B) \tag{3}$$

for any realization of σ_{-i}. Here (3) implies that given the same realization of all the other signals, $\sigma_i = A$ implies that the conditional probability of $s = A$ is higher than $\sigma_i = B$.

To illustrate the contradiction, consider a combination of a) a strategy profile (except for agent i's strategy) that leads to a positive probability that agent i is pivotal; and b) any realization of all signals such that agent i is pivotal given the strategy profile.[8] In order for agent i's consistent vote against the private signal to be consistent with the maximization of the probability that the committee decision matches the state, it has to be that, given $\sigma_0 = A$ and $\sigma_i = B$, the conditional probability of the committee decision matching the state when he is pivotal is higher if he votes for A than for B. However, if it is the case, (3) implies that if $\sigma_0 = A$ and $\sigma_i = A$, the conditional probability that the committee decision matches the state given he is pivotal must be higher when he votes for A than for B, since relative to the case where $\sigma_i = B$, there is one additional signal supporting $s = A$ given the same realization of all the other signals that makes him pivotal. Therefore, voting consistently against the private signal contradicts the maximization of the probability that the committee decision matches the state. □

The following Lemma further narrows down the set of strategy profiles we need to consider.

Lemma 2 *Any efficient non-obedient strategy profile must feature* $M(n, k, c, d)$, *and moreover we must have* $(c \geq 0) \wedge (d = 0)$ *or* $(c = 0) \wedge (d > 0)$.

[8] Note that we consider an arbitrary (equilibrium or non-equilibrium) non-obedient strategy profile of the other agents and thus they do not necessarily have to vote informatively.

Proof From Lemma 1, unless the efficient strategy profile is obedient to the public signal, there must be at least one agent whose strategy is vote for the private signal. Consider any such agent. If the agent is pivotal, the probability that the majority decision matches the state, conditional on the agent being pivotal, is q due to signal independence.

Let us add two extra agents who vote for the private signal to the committee, keeping the strategy profile of all the other agents fixed.[9] Those two agents change the outcome with positive probability and hence the conditional probability that the majority decision matches the state a) when the majority decision matches the state and both voters receive incorrect signals; and b) when the majority decision does not match the state and the two voters receive correct signals. The probability that a) occurs is $q(1-q)^2$. The probability that b) occurs is $(1-q)q^2$. Since $q > 1/2$, we have $(1-q)q^2 > q(1-q)^2$. Thus the two new voters increase the probability that the majority decision matches the state, which implies that a strategy profile that involves both a consistent vote for the public signal and a consistent vote against the public cannot be efficient. This is because, given each strategy profile of the other agents, those two votes cancel each other and do not affect the committee decision, while if the two votes are for their respective private signals, they strictly increase the probability that the committee decision matches the state. Thus we conclude that, in an efficient strategy profile, we must have $(c \geq 0) \wedge (d = 0)$ or $(d > 0) \wedge (c = 0)$. □

The preceding Lemmas do not directly address an equilibrium. However, as we noted earlier, in this common interest game an efficient equilibrium strategy profile is also a strategy profile that achieves the highest probability that the committee decision matches the state among all strategy profiles. Thus we use Lemma 2 and focus our analysis on $M(n, k, c, d)$ with $(c \geq 0) \wedge (d = 0)$ or $(c = 0) \wedge (d > 0)$ for the derivation of the efficient equilibria.

Definition Let us define the following log odds ratio:

$$g \equiv \frac{ln(\pi) - ln(1-\pi)}{ln(q) - ln(1-q)}.$$

As we will see shortly, this ratio plays a key role in the optimal weight on the public signal, in terms of the number of agents who consistently vote for it. It is easy to see that g is positive under our assumptions on q and π, strictly increasing π and strictly decreasing in q. Also, if $\pi \geq q$ then $g \geq 1$. As g can be directly associated to a specific number of agents, it is useful to define $\lfloor g \rfloor$ as the largest integer that does not exceed g. We will discuss the interpretation and intuition later through simple examples in Sect. 4.

[9] As in the proof of Lemma 1, we consider an arbitrary (equilibrium or non-equilibrium) non-obedient strategy profile of the other agents.

Proposition *For the generic cases where g is not an integer, the efficient equilibria of the model are uniquely characterized by non-negative integers c and d as follows:*

1. *If the k-majority rule favours the public signal and* $g \geq 2k$, *the efficient equilibria feature* $c \in [g - 2k - 1, g - 2k]$ *and* $d = 0$;

$1'$. *If* $k = 0$ (*simple majority rule*), *the efficient equilibria feature* $c \in [g - 1, g]$ *and* $d = 0$;

2. *If the k-majority rule favours the public signal and* $g < 2k$, *the efficient equilibria feature* $c = 0$ *and* $d \in [2k - g - 1, 2k - g]$;

3. *If the k-majority rule handicaps the public signal, the efficient equilibria feature* $c \in [g + 2k - 1, g + 2k]$ *and* $d = 0$.

Under each k-majority rule above, if g above prescribes c or d such that the committee decision is σ_0 (*the public signal*) *with probability* 1, *then any c or d that does not change the outcome leads to a set of outcome equivalent equilibria that are efficient. If* $c > n$, *any equilibria where the committee decision is* σ_0 (*the public signal*) *with probability* 1 *are efficient.*

For the non-generic cases where g is an integer, there are two sets of equilibria that lead to the same expected payoff for each case above, where c or d corresponds to both endpoints of the respective interval.

Proof Let $m \equiv \frac{n+2k-1}{2}$ denote the the number of votes for each alternative that makes an agent pivotal in the committee with $n + 2k$ agents under simple majority rule. Throughout the proof, in order to derive the equilibrium under a supermajority rule, we follow the Fact we saw earlier and consider a committee with $n + 2k$ agents under simple majority rule, where $2k$ agents deterministically vote for A favoured by the supermajority rule.

Let $U_j^A(\sigma_i, \sigma_0)$ be agent i's expected payoff when he votes for A, conditional on his private signal σ_i and the public signal σ_0 in the event where he is pivotal, given a strategy profile of all the other agents. Likewise, let $U_j^B(\sigma_i, \sigma_0)$ be agent i's expected payoff when he votes for B, conditional on his private signal σ_i and the public signal σ_0 in the event where he is pivotal, given the same strategy profile of all the other agents as in $U_j^A(\sigma_i, \sigma_0)$. The index $j \in \{O, D, I\}$ represents the agent's pure strategy, such that O indicates that the agent is one of c agents to vote consistently for the public signal (*obedient* voting); D indicates that the agent is one of d agents to vote consistently against the public signal (*disobedient* voting); and I indicates that the agent is one of $n - c - d$ agents to vote for the private signal (*individually informative* voting).

Define $W_j(\sigma_i, \sigma_0) \equiv U_j^A(\sigma_i, \sigma_0) - U_j^B(\sigma_i, \sigma_0)$. If $W_j(\sigma_i, \sigma_0)$ is positive, agent i is better off voting for A than for B; and if $W_j(\sigma_i, \sigma_0)$ is negative, agent i is better off voting for B than for A. We will use $W_j(\sigma_i, \sigma_0)$ to examine each agent's incentive to deviate from his pure strategy j, given the signals σ_i, σ_0. Recall from Lemma 2 that, in deriving the efficient equilibria, we must have $(c \geq 0) \wedge (d = 0)$ or $(c = 0) \wedge (d > 0)$ so that we do not have to consider i) voting against the private signal or ii) the coexistence of agents who vote for and against the public signal.

Case 1: k-majority rule favours the public signal and $(c \geq 0) \wedge (d = 0)$

Let us first derive the equilibria in $M(n, k, c, d)$ for the case above. Suppose the public signal is $\sigma_0 = A$ so that k-majority rule favours the public signal. We will derive the equilibria by computing $c \geq 0$ such that neither the c obedient voters nor $n - c$ individually informative agents have incentive to deviate. Consider one of the c agents who follow the public signal and suppose that the public signal and his private signal disagree (i.e. $\sigma_i = B$). In order for him not to deviate from the obedient strategy, he must be weakly better off voting for A than B and thus we must have $W_O(B, A) \geq 0$. That is,

$$\begin{aligned} W_O(B,A) &= \pi(1-q)\binom{n-c}{m-2k-(c-1)} q^{m-2k-(c-1)}(1-q)^m \\ &\quad - q(1-\pi)\binom{n-c}{m} q^m (1-q)^{m-2k-(c-1)} \geq 0 \\ &\Rightarrow \pi(1-q)q^{-2k-(c-1)} \geq (1-\pi)q(1-q)^{-2k-(c-1)} \\ &\Rightarrow \frac{\pi}{1-\pi} \geq \left(\frac{q}{1-q}\right)^{c+2k} \\ &\Rightarrow c + 2k \leq g. \end{aligned} \tag{4}$$

If the public signal and the agent's private signal agree, (4) readily implies $W_O(A, A) \geq 0$.

Next, let us consider one of the $n - c$ agents who vote for the private signal. If such an agent has received a private signal $\sigma_i = B$ and thus the two signals disagree, no deviation implies he is weakly better of voting for B and thus we must have $W_I(B, A) \leq 0$, which implies

$$c + 2k + 1 \geq g. \tag{5}$$

If the agent's private signal agrees with the the public signal, he must be weakly better off voting for A so that we must have $W_I(A, A) \geq 0$, which implies

$$c + 2k - 1 \leq g. \tag{6}$$

Note that (4) implies (6). Thus from (4) and (5), we conclude that for Case 1, we must have

$$c \in [g - 2k - 1, g - 2k]$$

for $g \geq 2k$ in equilibrium. For simple majority rule ($k = 0$) we have

$$c \in [g - 1, g]$$

as stated in the proposition.

Case 2: k-majority rule favours the public signal and $(c = 0) \wedge (d > 0)$

Let us complete the derivation of efficient equilibria under k-majority rule that favours the public signal is incomplete by examining the case where $(c = 0) \wedge (d > 0)$. Consider one of the d agents who vote consistently against the public signal (disobedient voting) and suppose the public signal and the private signal agree and $\sigma_0 = \sigma_i = A$. Given the signals, the no deviation condition for the disobedient agent is given by $W_D(A, A) \leq 0$ (i.e. voting for B is weakly is better), which implies

$$2k - d \geq g. \tag{7}$$

Now suppose $\sigma_i = B$ and thus the public signal and the private signal disagree. Then (7) implies $W_O(B, A) \leq 0$ and hence the agent is better off voting against the public signal (and thus voting for B).

Next, let us consider one of the $n - d$ agents who vote for the private signal. If the public signal and the private signal agree we must have $W_I(A, A) \geq 0$ and thus

$$2k - d - 1 \leq g. \tag{8}$$

If the public signal and the private signal disagree (i.e. $\sigma_i = B$), we must have $W_I(B, A) \leq 0$, which implies

$$2k - d + 1 \geq g. \tag{9}$$

It is easy to see that (7) implies (9). Therefore, from (7) and (8) we conclude that when $g < 2k$ the efficient equilibria are characterized by

$$c = 0 \text{ and } d \in [2k - g - 1, 2k - g]$$

as stated in the proposition.

Case 3: k-majority rule handicaps the public signal and $(c \geq 0) \wedge (d = 0)$

We will now derive equilibria in $M(n, k, c, d)$ for this case by computing $c \geq 0$ such that neither the c obedient voters nor $n - c$ individually informative agents have incentive to deviate. Suppose $\sigma_0 = B$ so that k-majority rule handicaps the public signal.Consider one of c agents who vote for the public signal and suppose the public signal and the private signal disagree (i.e. $\sigma_i = A$). The no deviation condition $W_O(A, B) \leq 0$ leads to

$$c - 2k \leq g. \tag{10}$$

For the case where the public signal and the private signal agree, (10) implies $W_O(B, B) \leq 0$.

Next, let us consider one of the $n - c$ agents who vote for the private signal. If such an agent has received a private signal A (and thus the public signal and the private signal disagree), no deviation implies we must have $W_I(A, B) \geq 0$ and hence

$$c - 2k + 1 \geq g. \tag{11}$$

If the agent's private signal agrees with the the public signal, we must have $W_I(B, B) \leq 0$ (i.e. voting for A is weakly worse) for no deviation, and thus

$$c - 2k - 1 \leq g. \quad (12)$$

Note that (10) implies (12). Therefore, combining (10) and (11) we conclude that for Case 3, we must have

$$c \in [g + 2k - 1, g + 2k] \text{ and } d = 0$$

in equilibrium as stated.

Case 4: k-majority rule handicaps the public signal and $(c = 0) \wedge (d > 0)$

For this case we show by contradiction that $(c = 0) \wedge (d > 0)$ does not hold in equilibrium since a no-deviation condition cannot be satisfied. Suppose $\sigma_0 = B$ so that k-majority rule handicaps the public signal. Consider one of the d disobedient agents who vote against the public signal, and suppose the public signal and the private signal agree. For him not to deviate, we must have $W_D(B, B) \geq 0$ (i.e. he is weakly better off voting for A), which implies

$$-d - 2k \geq g \Leftrightarrow d \leq -g - 2k. \quad (13)$$

That is, if the public signal and the private signal agree, the agent deviates and votes for the signals in agreement regardless of d. This contradicts $d > 0$ and we conclude that when the k-majority rule handicaps the public signal there is no equilibrium that features $(c = 0) \wedge (d > 0)$.

Optimally deterministic decision for the public signal

Our derivation for Cases 1–4 implies that the efficient equilibria may incorporate private signals and may contradict the public signal, or may lead to the committee decision that follows the public signal with probability 1. However, the equilibria where the committee decision follows the public signal with probability 1 (obedient equilibria) always exist irrespective of the parameter values. Let us show below for completion that such obedient equilibria cannot be efficient unless c and d stated in the proposition prescribe the obedient decision to the public signal.

For Cases 1 and 3 if we have $c \in \{0, 1, \dots, \frac{n-1}{2} - k\}$ in equilibrium, the probability that the committee's decision matches the state is higher than in any obedient equilibrium such that $c \geq \frac{n+1}{2} - k$ (i.e. there are $\frac{n+2k+1}{2}$ or more votes for the public signal). This is because from (4) and (10) an agent who votes for the public signal deviates if $c = \frac{n+1}{2} - k$ and hence the probability of that the committee's decision matches the state is higher, while the probability that the committee's decision matches the state is the same for any $c \geq \frac{n+1}{2} - k$ (i.e. the obedient outcome).

Likewise for Case 2, if $d \in \{0, 1, \dots, \frac{n+2k-1}{2}\}$ in equilibrium, a higher payoff is achieved than any equilibrium where the committee decision is against the public signal with probability 1, since the expected payoff therein is smaller than 1/2.

Also from our discussion on Cases 1 and 3 above, it is clear that, if $c > g$, any equilibrium in $M(n, k, c, d)$ must lead to the committee decision that follows the public signal with probability 1.

Equal payoff for the two equilibria when g is an integer[10]

Let us focus on Case 1 above. The statement is proved similarly for the other Cases. From (4) the no deviation condition for the obedient agent whose private signal disagrees with the public signal is $c \geq g - 2k$, while from (5) the no deviation condition for the individually informative agent whose private signal disagrees with the public signal is $c \leq g - 2k - 1$. Thus for an integer g, there are two sets of equilibria, namely one set with $c = g - 2k$ and the other set with $c = g - 2k - 1$, while $d = 0$ in both.

Suppose $c = g - 2k$. Then (4) implies that the obedient agent whose private signal disagrees with the public signal is indifferent between voting for the private signal and voting for the public signal, so that he is indifferent between the two sets of equilibrium strategy profiles with $c = g - 2k$ and $c = g - 2k - 1$.[11] Suppose $c = g - 2k - 1$. Then (5) implies that the individually informative agent whose private signal disagrees with the public signal is indifferent between voting for the private signal and voting for the public signal, so that he is indifferent between the two sets of equilibrium strategy profiles. Thus conditional upon receiving a private signal that disagrees with the public signal, the agents' expected payoffs are the same the same in both sets of equilibria.

Consider an obedient agent who has received a private signal that agrees with the public signal in one set of the equilibria. His conditional expected payoff is given by $(q/(1-q))L_O$, where L_O is the expected payoff of the obedient agent when the signals disagree in the equilibrium. Similarly, the conditional expected payoff of an individually informative agent in the set of equilibria is given by $(q/(1-q))L_I$, where L_I is the expected payoff of the individually informative agent when the signals disagree in the equilibrium. Clearly, L_O and L_I are unchanged between the two sets equilibria, namely those with $c = g - 2k$ and those with $c = g - 2k - 1$, and thus $(q/(1-q))L_O$ and $(q/(1-q))L_I$ are also the same. Since the conditional payoffs are unchanged between the two sets equilibria whether the private signal agrees or disagrees with the public signal, we conclude that the ex ante payoff is also the same in both sets of equilibria. □

4 Examples

Let us look at a few simple examples to gain intuition behind the asymmetric equilibria presented in the Proposition. We first discuss the efficient equilibria under simple majority and supermajority rules where the CJT holds. For unanimity rule, we make an explicit comparison between the symmetric mixed strategy equilibrium studied by Feddersen and Pesendorfer (1998) and the efficient equilibria we have derived above, in order to highlight some qualitative differences.

[10] Maug and Rydqvist (2008) assume away integer g.

[11] Note that (4) is for the pivotal event but the conditional expected payoff for the non-pivotal events remains unchanged as we fix the other agents' strategies.

4.1 Majority and supermajority rules

As briefly mentioned earlier, the log odds ratio $g \equiv \frac{ln(\pi)-ln(1-\pi)}{ln(q)-ln(1-q)}$ specifies the "weight" the public signal (, which matches the state with probability π) has with respect to private signals (, each of which matches the state with probability q) in the efficient asymmetric equilibria, independently of the committee size n.[12] Suppose $q = 0.6$ and $\pi = 0.8$, in which case we have $g \approx 3.419$. This means the public signal is "worth" three private signals. Consequently, the efficient pure strategy equilibria prescribed for $k = 0$ in the Proposition feature three agents voting for the public signal ($c = 3$) and the rest voting for their private signals.[13]

Figure 1 compares the efficiency of the efficient equilibria and that of the symmetric mixed strategy equilibrium for $k = 0$. We can see that for $n = 3$ and $n = 5$, both lead to the same expected payoff since the best is for the committee decision to follow the public signal with probability 1, where the probability that the committee decision matches the state is $\pi = 0.8$. While the number of agents who actually vote for the public signal may differ, the equilibria are outcome equivalent. Since $g \approx 3.419$, three votes must be cast for the public signal independently of the realization of the private signals, but they already constitute a majority for $n = 3$ and $n = 5$. At $n = 7$ the probabilities that the committee decision matches the state diverge, but they both converge to 1 as n becomes larger.

A supermajority rule may either favour or handicap the public signal. Suppose as earlier that $q = 0.6$, $\pi = 0.8$, and in addition let $k \geq 1$ and $\sigma_0 = A$ so that a k-majority rule favours the public signal. According to the Proposition, for $g > 2k$, if $k = 1$ then one agent votes for the public signal and the rest vote according to their private signals in the efficient pure strategy equilibria. Recall our observation that the voting game under a k-majority rule is equivalent to the voting game under simple majority rule with $n + 2k$ agents where $2k$ agents deterministically vote for A. This implies the rule $k = 1$ itself gives the public signal a weight of two votes, and thus there must be one additional vote for the public signal in the efficient pure strategy equilibria, since the public signal is worth three private signals.

Meanwhile, if $k = 3$ the rule gives a weight of six private votes to the public signal, which is more than the public signal is worth. In this case ($g < 2k$), the Proposition states that there must be two votes *against* the public signal and the rest vote according to the private signals. The votes against the public signal are to offset the excessive weight the rule puts on the public signal.

Now suppose $k = 1$ and $\sigma_0 = B$ so that the supermajority rule handicaps the public signal. The Proposition indicates that the efficient pure strategy equilibria in this case feature five agents voting for the public signal and the rest vote for the private signals. Intuitively, two ($= 2k$) of the five votes for the public signal are to "correct" for the bias against the public signal imposed by the

[12] Recall that g is increasing in π and decreasing in q.

[13] Note that c is independent of n but is decreasing in k. That is, given g, as supermajority rule favours the public signal more, the number of obedient votes for the public signal decreases.

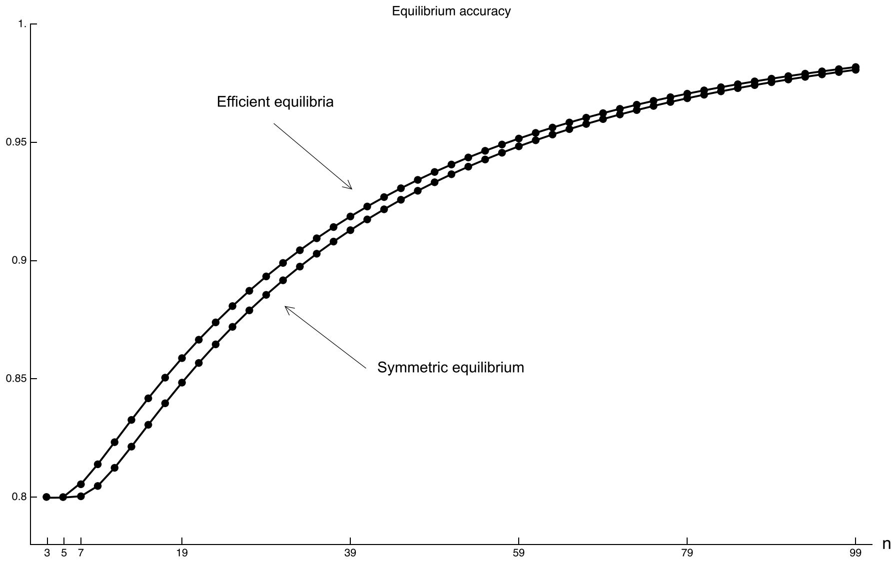

Fig. 1 Efficiency comparison under simple majority

rule, and the remaining three votes are to give the appropriate weight to the public signal, after the correction to simple majority rule.

4.2 Unanimity rule and "convicting the innocent"

Let us consider some interesting implications of the efficient pure strategy equilibria on voting behaviour and efficiency under unanimity rule. It has been well known since Feddersen and Pesendorfer (1998) that information aggregation fails and in addition there are some peculiar properties in the symmetric mixed strategy equilibrium. For example, under unanimity rule the probability of "convicting the innocent" becomes higher, and the probability of "acquitting the guilty" becomes lower as the committee size increases. Here we demonstrate that, while the CJT fails in both (Feddersen and Pesendorfer 1998) and the equilibria derived in this paper, the comparative statics in the efficient pure strategy equilibria with respect to committee size is different, and the equilibrium strategy profiles have a simple and intuitive structure.

For comparison, let $q = 0.7$ and $\pi = 0.5$ following the leading example in Feddersen and Pesendorfer (1998), so that the strategic environment is identical. In our terms the public signal is uninformative, and in their terms the "standard of reasonable doubt" is 0.5 for conviction. Also, $y = A$, which is the alternative unanimity rule favours in our model, corresponds to "acquittal" and

$y = B$ corresponds to "conviction" in Feddersen and Pesendorfer (1998),[14] and states $s = A$ and $s = B$ correspond to the defendant being innocent and guilty, respectively.[15]

Under unanimity our calculation does not suffer from indivisibility so that we let n be any positive integer larger than or equal to 2. Since $g = 0$ (i.e. in our terms, the public signal is worthless) and unanimity implies $k = \frac{n-1}{2}$, according to the Proposition, the efficient pure strategy equilibria feature $d = n - 1$. This means that in the equilibria all but one agent vote for conviction, so that the committee decision is determined solely by one agent's vote for his private signal. As a result the probability of convicting the innocent is the same as the probability of acquitting the guilty, which is $1 - q = 0.3$ regardless of the committee size. This is in contrast to the error probabilities in Feddersen and Pesendorfer (1998) where, as shown in Fig. 2, each error probability converges monotonically to a different value as the committee size increases. Note that, even though the probability of convicting the innocent is indeed lower in the symmetric mixed strategy equilibria, the probability of acquitting the guilty is so much higher that the overall efficiency is still higher in the asymmetric pure strategy equilibria.

The right-hand side of Fig. 2 compares the efficiency. We can see that in the symmetric equilibrium, the expected payoff is decreasing in n and weakly lower than in the efficient equilibria, while the expected payoff in the efficient equilibria is constant.

Generally, the efficient equilibria under unanimity for given q and π are characterized by the fixed effective number of agents who vote according to the private signals, which is independent of n. This independence leads to the failure of CJT in the efficient equilibria as the information from the private signals is not aggregated. We have seen from the Proposition that when $q > \pi$ and thus $g < 1$, there is one voter who solely determines the committee decision by voting for the private signal. If $q = 0.6$, $\pi = 0.8$, and thus $g \approx 3.419$ as seen in Sect. 4.1, the Proposition implies that $2 \times \frac{n-1}{2} - 3 = n - 4$ agents must vote against A (hence for B) regardless of their private signals, which leaves four agents who vote according to their private signals. In other words, under unanimity, $\lfloor g \rfloor + 1$ agents vote for their private signals and the rest vote against A in the efficient equilibria.[16] Thus the feature that the probability of "convicting the innocent", the probability of "acquitting the guilty", and the expected payoff are all independent of sufficiently large n holds true beyond the example we have looked at in Fig. 2.

[14] That is, all agents have to vote for B (conviction) for B to be selected, while A (acquittal) is chosen as long as one or more agents vote for A.

[15] The payoff scale differs but it has no bearing on the characterization of equilibrium strategies. There is no difference in the comparative statics with respect to efficiency between the model of Feddersen and Pesendorfer (1998) and ours.

[16] Recall that $\lfloor g \rfloor$ denotes the largest integer that does not exceed g.

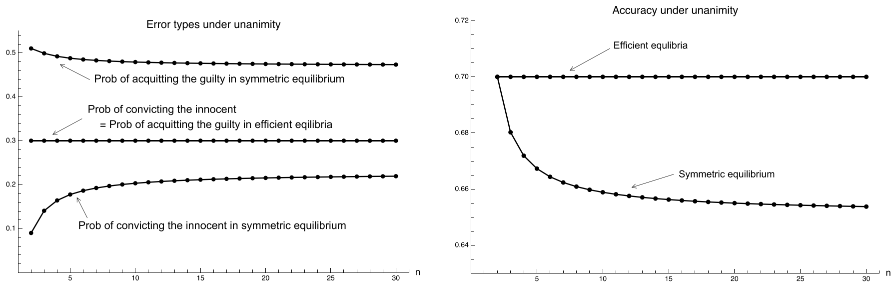

Fig. 2 Comparison between efficient equilibria and symmetric equilibrium under unanimity

5 Discussion

The asymmetry of the strategies that maximize the efficiency and a potentially large number of voters would call for some coordination before they vote. In particular, even if all agents agree to play an efficient equilibrium, they still face a coordination problem as to which agents are to be included in the set of agents who vote for the public signal (the number of which is denoted by c), for example. This is in contrast to the symmetric mixed strategy equilibrium, where playing an identical strategy could be considered focal.

We argue that the asymmetric equilibria derived in this paper points to a type of pre-voting deliberation, where the agents endogenously "correct" for the exogenously given voting rule and public information before the arrival of private signals.[17] Specifically, deliberation in this context is *not* for the exchange of private information, which may be limited due to time, institutional or other constraints as implicitly assumed in the strategic voting models with common interests, but for the coordination on one of the efficient equilibria. The deliberation would lead to a non-binding agreement as to which specific agents are to vote or against the ex-ante desirable alternative and which agents are to vote according to their private information, given g and the voting rule k in place. According to this interpretation, such non-binding "role assignment" should be the focus of this type of deliberation ahead of the arrival of private signals and voting. Once the role assignment is done in accordance with an efficient strategy profile, it is self-enforcing since no agent has incentive to deviate.[18]

[17] Note that the public signal in our model can be interpreted as representing a potentially biased prior distribution of the state. Thus it would not be unnatural to postulate that the public information may be shared earlier than the arrival of private signals.

[18] If $\pi > q$, the public signal in our model can be thought of as information from a third-party exert who has superior information about the state than each individual agent and provides expertise to the committee. The role assignment in this interpretation may be such that a limited number of committee members ($c = \lfloor g \rfloor$ under simple majority rule) listen to the expert opinion and vote accordingly, while the other members do not listen to the expert and vote according to their own private private information. Also in this interpretation, the public signal/expert information may arrive later than the private signals.

6 Conclusion

We have presented a unified approach to the asymmetric pure strategy equilibria and their optimality in the voting model of Austen-Smith and Banks (1996). While the difference in the efficiency between the equilibria and the symmetric mixed strategy equilibrium often studied in the strategic voting literature may not be large, we have observed that the efficient equilibria have a remarkably simple and intuitive structure. For example, the equilibria demonstrate the peculiarity of unanimity rule in a striking way, such that the effective number of informative voters and thus the efficiency remain unchanged irrespective of the committee size.

Appendix

The expected payoff for the efficient equilibria in Fig. 1 is derived from the following expression

$$\pi \sum_{i=\frac{n+1}{2}-k-c}^{n-c-d} \binom{n-c-d}{i} q^i (1-q)^{n-c-d-i}$$
$$+ (1-\pi) \sum_{i=\frac{n+1}{2}+k-d}^{n-c-d} \binom{n-c-d}{i} q^i (1-q)^{n-c-d-i},$$

where $k = 0$ (simple majority rule) and $d = 0$ (no agent votes against the public signal). The expected payoff in the symmetric mixed strategy equilibrium is given by

$$\pi \sum_{i=\frac{n+1}{2}-k}^{n} \binom{n}{i} r_A^i (1-r_A)^{n-i} + (1-\pi) \sum_{i=\frac{n+1}{2}+k}^{n} \binom{n}{i} r_B^i (1-r_B^i)^{n-i},$$

where $k = 0$, $r_A = q + (1-q)(1-\beta)$ and $r_A = q\beta$, such that β is the equilibrium probability that the agents vote according to the private signal when their private signal and the public signal disagree (and they vote according to the signals when they agree). The equilibrium strategy is explicitly derived in Wit (1998).

The expected payoff in the symmetric mixed equilibrium under unanimity rule in Fig. 2 can be found in Feddersen and Pesendorfer (1998).

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