Empirical Economics (2023) 65:2501–2520
https://doi.org/10.1007/s00181-023-02432-8

The oil price-macroeconomy dependence

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Received: 6 April 2022 / Accepted: 17 April 2023 / Published online: 29 May 2023

Abstract
This paper investigates the relationship between the price of oil and real output in the United States in the context of a Markov regime switching, identified, structural GARCH-in-Mean VAR model with copulas. We use the copula method to investigate the nonlinear dependence structure, as well as (upper and lower) tail dependence, between the price of oil and real output growth, and Markov regime switching to account for changing oil price dynamics over the sample period. We find an asymmetric negative dependence structure between oil price and output growth shocks and that oil price uncertainty has a negative and statistically significant effect on real output growth.

Keywords Oil price uncertainty · Markov regime-switching · Dependence · Copulae

JEL Classification C32 · E32 · G31

1 Introduction

This paper adds to the ongoing debate in macroeconomics about how oil price shocks and oil price uncertainty affect the level of economic activity. As Serletis and Xu (2019, p. 1045) recently put it, “ those of the view that positive oil price shocks have been the major cause of recessions in the United States (and other oil-importing countries) as, for example, Hamilton (1983, 1996, 2011), Hooker (1996), and Herrera et al.

We would like to thank the Editor and an anonymous referee for comments that greatly improved the paper.

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(2011), appeal to models that imply asymmetric responses of real output to oil price increases and decreases. These models are able to explain larger economic contractions in response to positive oil price shocks and smaller economic expansions in response to negative ones. On the other hand, those of the view that positive oil price shocks do not cause recessions as, for example, Kilian (2008), Edelstein and Kilian (2009), and Kilian and Vigfusson (2011a, b), appeal to theoretical models of the transmission of exogenous oil price shocks that imply symmetric responses of real output to oil price increases and decreases. These models cannot explain large declines in the level of economic activity in response to positive oil price shocks."

In this paper, we investigate the relationship between the real price of oil and real output, focusing on the role of oil price uncertainty. In doing so, we build on a series of recent papers by Elder and Serletis (2010, 2011), Bredin et al. (2011), Rahman and Serletis (2011, 2012), Pinno and Serletis (2013), Jo (2014), Elder (2018), and Serletis and Mehmandosti (2019) that appeal to the real options theory—see, for example, Bernanke (1983), Brennan and Schwartz (1985), Majd. and Pindyck (1987), Brennan (1990), Gibson and Schwartz (1990), and Dixit and Pindyck (1994). We use quarterly data for the United States, over the period from 1974:q1 to 2022:q4, and a different approach to the investigation of the relationship between the real oil price and real GDP. In particular, we use the copula approach to examine correlation and dependence structures between the real price of oil and real GDP, in the context of a Markov switching, bivariate identified structural GARCH-in-Mean VAR model. As in Serletis and Xu (2019), we take the Markov regime switching approach, associated with Hamilton (1988, 1989), and modify the Elder et al. (2010) model to account for instabilities in the relationship between the real oil price and real output. Moreover, we take the copula approach to investigate the nonlinear dependence structure, as well as tail dependence, between the real price of oil and real output.

The copula approach in this paper distinguishes it from the literature. Although the structural VAR model is similar to that in Serletis and Xu (2019), it is also significantly different. The present paper estimates the Markov switching, bivariate identified structural GARCH-in-Mean VAR model using the copula approach, thus capturing the dependence structure between oil prices and real output. In particular, the assumption in the standard VAR analysis is that the structural oil price and output shocks are not related. However, there can still be a dependence structure between the shocks. For example, a positive dependence between the oil price and output growth shocks implies that they are more likely to be large together or to be small together. This is a new approach to the investigation of the comovement between oil prices and real economic activity.

We estimate our internally-consistent simultaneous equations model by full information maximum likelihood, avoiding Pagan's (1984) generated regressor problems. We associate the oil price change VAR residual with exogenous oil price shocks, use the conditional standard deviation of the forecast error for the change in the real oil price as a measure of uncertainty about the impending real price of oil, and find that oil price uncertainty has had a negative and statistically significant effect on real GDP growth. We also find that asymmetric responses to oil price shocks; these results are consistent with the evidence in Elder et al. (2010); Elder and Serletis (2011), Bredin et al. (2011), Rahman and Serletis (2011, 2012), Pinno and Serletis (2013), Jo (2014), Elder (2018), and Serletis and Mehmandosti (2019). We find an asymmetric negative dependence structure between the oil price and output growth shocks, with the oil price shock more closely related to output growth dynamics when it is large and positive. However, such negative dependence could switch to symmetric relationship with changes in economic conditions. We present generalized impulse response functions that take into account the Markov regime switching in the Appendix. We show that our generalized impulse response functions are asymmetric, and this finding is consistent with the literature.

The rest of the paper is organized as follows. Section 2 discusses the data. Section 3 presents the Markov switching structural GARCH-in-Mean copula VAR model and a discussion of identification and estimation issues. Section 4 presents the empirical results. Section 5 investigates robustness of our results to alternative model specifications. The final section concludes the paper.

2 The data

We use quarterly data for the United States over the period from 1974:q1 to 2022:q4. For the real output series, Y_t, we use the real GDP series GDPC1 from the Federal Reserve Economic Database (FRED) maintained by the Federal Reserve Bank of St. Louis. For the real oil price, O_t, we use the WTI oil price WTISPLC divided by the consumer price index CPIAUCSL, with both series obtained from FRED.

We test for cointegration between $\ln Y_t$ and $\ln O_t$, using the Johansen (1988) maximum likelihood approach, and find that $\ln Y_t$ and $\ln O_t$ are not cointegrated. Based on this evidence, in what follows, we adopt the VAR model as the basic framework, as in Elder et al. (2010) and Serletis and Xu (2019), and use the first differences of $\ln Y_t$ and $\ln O_t$, $\Delta \ln Y_t$ and $\Delta \ln O_t$, denoted in what follows by y_t and o_t, respectively.

In the next section, we use the two variables in a near-VAR model, and analyze the preliminary results to gain statistical evidence for supporting the copula approach.

3 Methodology

We first discuss the basic near-VAR model that is estimated based on the conventional OLS approach. The results indicate a dependence structure for oil and output shocks. In Sect. 3.2, we show how to incorporate copulas in the estimation. We further improve the modeling by allowing a time-varying copula function that can capture the dynamic dependence structure.

3.1 The structural VAR

We modify the Elder et al. (2010) bivariate structural GARCH-in-Mean VAR model

$$\mathbf{B}\mathbf{z}_t = \mathbf{C} + \sum_{i=1}^{k} \mathbf{\Gamma}_i \mathbf{z}_{t-i} + \mathbf{\Psi}\sqrt{h_{o_t}} + \sum_{j=1}^{m} \tilde{\mathbf{\Gamma}}_j r_{t-j} + \sum_{l=1}^{n} \bar{\mathbf{\Gamma}}_l g_{t-l} + \boldsymbol{\epsilon}_t \qquad (1)$$

where

$$\mathbf{z}_t = \begin{bmatrix} o_t \\ y_t \end{bmatrix}; \quad \boldsymbol{\epsilon}_t = \begin{bmatrix} \epsilon_{o_t} \\ \epsilon_{y_t} \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}; \quad \boldsymbol{\Gamma}_i = \begin{bmatrix} \gamma_{i,11} & \gamma_{i,12} \\ \gamma_{i,21} & \gamma_{i,22} \end{bmatrix}; \quad \boldsymbol{\Psi} = \begin{bmatrix} 0 & 0 \\ \psi & 0 \end{bmatrix};$$
$$\tilde{\boldsymbol{\Gamma}}_j = \begin{bmatrix} \tilde{\gamma}_{j,11} \\ \tilde{\gamma}_{j,21} \end{bmatrix}; \quad \bar{\boldsymbol{\Gamma}}_l = \begin{bmatrix} \bar{\gamma}_{l,11} \\ \bar{\gamma}_{l,21} \end{bmatrix}.$$

The $\boldsymbol{B}$ matrix, which is lower triangular, identifies the model. Under such an identifying assumption, oil price shocks are treated as predetermined, and this assumption has been adopted by Edelstein and Kilian (2007), Kilian (2009), and Elder et al. (2010). In particular, Kilian (2009) states that this assumption is not testable, however, it can be well defended as detailed in Kilian (2009). Therefore, ϵ_{o_t} could be referred to as oil price shock and ϵ_{y_t} is the output growth shock. h_{o_t} is the time-varying variance of oil price growth, which is used to capture oil price uncertainty. The parameter ψ measures the effect of oil price uncertainty on economic growth.

It is to be noted that model (1) is a near-VAR since we also include two macroeconomic controls, which are crucial in the determination of oil prices. r is the federal funds rate (obtained from the FRED) capturing the stance of monetary policy and g is the index of global real economic activity in industrial commodity markets as proposed by Kilian (2009). It captures global oil demand, and was obtained from the Federal Reserve Bank of Dallas. To determine the lag lengths k, m, and n in equation (1), we allow each of k, m, and n to vary from 1 to 12 and by running 1728 regressions for each bivariate relationship we choose the specification that minimize the AIC value. The optimal lags are $k = 2$, $m = 3$, and $l = 2$.

We use the univariate GARCH(1,1) specification to model h_{o_t} as follows

$$h_{o_t} = d_1 + d_2\epsilon^2_{o_{t-1}} + d_3 h_{o_{t-1}}. \tag{2}$$

Conventional estimation of the model consisting of Eqs. (1) and (2) assumes

$$\boldsymbol{\epsilon}_t \sim (0, \mathbf{H}_t), \mathbf{H}_t = \begin{bmatrix} h_{o_t} & 0 \\ 0 & h_y \end{bmatrix}.$$

Note that we only specify a GARCH model for the disturbances of the oil price growth but not for output growth. Assuming a GARCH model for the output growth shock increases the complexity of the model and introduces estimation difficulties of the highly nonlinear model.

We verify the validity of the normality assumption by conducting a series of tests reported in Table 1. Panel A in Table 1 shows there is little evidence for a joint normal distribution of ϵ_{o_t} and ϵ_{y_t}. This result indicates that assuming a normal probability density for the maximum likelihood estimation may not be appropriate. On the other hand, Panel B suggests there is a negative dependence relationship between the two shocks. In this context, dependence is based on the concept of concordance. A positive dependence between ϵ_{o_t} and ϵ_{y_t} implies that they are more likely to be large together or to be small together—see Joe (1997).

Table 1 Bivariate normality tests and dependence measures between oil price and output growth shocks

A. Bivariate normality tests	
Mardia's test (Skewness)	176.880 (0.000)
Mardia's test (Kurtosis)	57.023 (0.000)
Henze-Zirkler's test	6.076 (0.000)
Royston's test	99.919 (0.000)
Doornik-Hansen's test	277.956 (0.000)
B. Dependence measures	
Spearman's ρ	−0.226
Kendall's τ	−0.155
C. Akaike information criterion	
Structural VAR	2112.231
Structural copula VAR	2047.898
Markov switching structural copula VAR	2001.325

Sample period, quarterly data: 1974:q1-2022:q4

Note that a dependence relationship in the context of the structural VAR model is not inconsistent with our identification strategy. For example, we use the lower triangular $\boldsymbol{B}$ matrix in Eq. (1) to identify the model. It follows that the structural shocks, ϵ_{o_t} and ϵ_{y_t}, are not related. However, they can still hold a dependence structure. In that sense, a correlation coefficient of zero, which is equivalent to a zero covariance between the two structural shocks, does not mean that there is no dependence. Therefore, we will estimate the model (1) to investigate the dependence between oil price shock and output growth shock, using the copula approach in the following section.

3.2 Dependence and copulas

Based on the evidence in Table 1, we estimate the structural VAR model using copulas. The copula is a multivariate distribution. Its univariate margins all follow the (0,1) uniform distribution. In our case, the copula C is defined by

$$F(\boldsymbol{\epsilon}_t) = C(F_1(\epsilon_{o_t}), F_2(\epsilon_{y_t})) \tag{3}$$

based on the Sklar (1959) theorem. In Eq. (3), $F(\boldsymbol{\epsilon}_t)$ is an unknown joint distribution function for ϵ_{o_t} and ϵ_{y_t}, and $F_1(\cdot)$ and $F_2(\cdot)$ are the two univariate margins corresponding to the structural shocks. The theorem permits the bivariate distribution $F(\cdot)$ to be made of the two margins with a dependence structure. In other words, we could piece together the joint distribution of ϵ_{o_t} and ϵ_{y_t} with the assumed margins and the dependence structure.

An appropriate copula to use is one which best captures dependence features of the outcome variables. In this paper, we use the BB1 copula in Joe (1997). The BB1 copula is given by

$$C(u, v) = \left(1 + [(u^{-\theta} - 1)^{\vartheta} + (v^{-\theta} - 1)^{\vartheta}]^{\frac{1}{\vartheta}}\right)^{-\frac{1}{\theta}} \tag{4}$$

for $\vartheta \geq 1$ and $\theta \geq 0$. In our case,

$$u = F_1(-\epsilon_{o_t}) \tag{5}$$

$$v = F_2(\epsilon_{y_t}). \tag{6}$$

Eq. (5) is a transformation that enables us to use the BB1 copula. In particular, the BB1 copula can only accommodate positive dependence. However, according to the evidence reported in Table 1, there is a negative dependence structure between the oil price growth shock and the output growth shock. Therefore, we define u using Eq. (5) to get a (constructed) positive dependence structure for estimation purposes.

To obtain a better understanding of the dependence structure between the oil price and output growth, let's define

$$\begin{aligned}\lambda_U &= \lim_{k \to 1} Pr[\epsilon_{o_t} > F_1^{-1}(k) | \epsilon_{y_t} > F_2^{-1}(k)] \\ &= \lim_{k \to 1} Pr[\epsilon_{y_t} > F_2^{-1}(k) | \epsilon_{o_t} > F_1^{-1}(k)].\end{aligned} \tag{7}$$

When λ_U is between 0 and 1, one would say the copula has upper tail dependence and no upper tail dependence if $\lambda_U = 0$; see Joe (1997). It is important to know that the concept of upper tail dependence is still built on the concept of dependence. Eq. (7) says that if λ_U is bigger than zero, there is a positive probability that one of ϵ_{o_t} and ϵ_{y_t} takes values greater than k given that the other is greater than k, for k is arbitrarily close to 1. In our case with the transformation in Eq. (5), λ_U quantifies the probability of a very large output growth shock since the oil price shock is very small.

On the other hand, let's define

$$\begin{aligned}\lambda_L &= \lim_{k \to 0} Pr[\epsilon_{o_t} < F_1^{-1}(k) | \epsilon_{y_t} < F_2^{-1}(k)] \\ &= \lim_{k \to 0} Pr[\epsilon_{y_t} < F_2^{-1}(k) | \epsilon_{o_t} < F_1^{-1}(k)].\end{aligned} \tag{8}$$

In a similar fashion to λ_U, λ_L quantifies the probability of having a smaller output growth shock, given that the oil price shock is larger. The BB1 copula accommodates both upper tail dependence and lower tail dependence. Notably, in the case of the BB1 copula, we have $\lambda_U = 2 - 2^{1/\vartheta}$ and $\lambda_L = 2^{-1/\vartheta\theta}$.

Note that the choice of the copula is important in our study. There are many copula functions used in the literature and some of them are asymmetric, such as for example the Clayton copula. However, the Clayton copula only allows for lower tail dependence, and adopting it would not capture upper tail dependence. For this reason, and because there is no clear prior about which type of tail dependence exists in our data, we use the relatively more flexible BB1 copula which allows for both lower tail and upper tail dependence.

3.3 Markov regime switching

Many studies have shown that the data correlations may not be constant [see, for example, Engle (2002)], suggesting that treating dependence as a constant is questionable. Patton (2006) is the first study that proposes a parametric model to describe the evolution of the copula function. In this regard, Manner and Reznikova (2012) provide a detailed survey of different time-varying copula approaches. According to their simulations, the Markov regime-switching copula is one of the superior time-varying copula approaches.

In what follows, we use the Markov switching approach, which allows us to study the dependence structure between oil prices and economic activity across different macroeconomic regimes. To support the validity of the time-varying approach, in panel C of Table 1 we report the AIC values of the near-VAR with the normality assumption, the near-VAR with the BB1 copula, and the Markov switching near-VAR with the BB1 copula. As can be seen, the time-varying approach is favored by the data.

The Markov regime switching, copula, bivariate structural VAR model can be written as

$$\mathbf{B}_{s_t}\mathbf{z}_t = \mathbf{C}_{s_t} + \sum_{i=1}^{k}\mathbf{\Gamma}_{i,s_t}\mathbf{z}_{t-i} + \mathbf{\Psi}_{s_t}\sqrt{h_{o_{s_t,t}}} + \sum_{j=1}^{m}\tilde{\mathbf{\Gamma}}_{j,s_t}r_{t-j} + \sum_{l=1}^{n}\bar{\mathbf{\Gamma}}_{l,s_t}g_{t-l} + \boldsymbol{\epsilon}_{s_t,t} \quad (9)$$

with

$$F(\boldsymbol{\epsilon}_{s_t,t}) = C(F_1(-\epsilon_{o_{s_t,t}}), F_2(\epsilon_{y_{s_t,t}})) \quad (10)$$

$$\epsilon_{o_{s_t,t}} \sim N(0, h_{o_{s_t,t}}) \quad (11)$$

$$\epsilon_{y_{s_t,t}} \sim N(0, h_{y_{s_t,t}}) \quad (12)$$

and

$$\mathbf{B}_{s_t} = \begin{bmatrix} 1 & 0 \\ b_{s_t} & 1 \end{bmatrix}; \quad \mathbf{\Gamma}_{i,s_t} = \begin{bmatrix} \gamma_{i,s_t,11} & \gamma_{i,s_t,12} \\ \gamma_{i,s_t,21} & \gamma_{i,s_t,22} \end{bmatrix}; \quad \mathbf{\Psi}_{s_t} = \begin{bmatrix} 0 & 0 \\ \psi_{s_t} & 0 \end{bmatrix};$$
$$\tilde{\mathbf{\Gamma}}_{j,s_t} = \begin{bmatrix} \tilde{\gamma}_{j,s_t,11} \\ \tilde{\gamma}_{j,s_t,21} \end{bmatrix}; \quad \bar{\mathbf{\Gamma}}_{l.s_t} = \begin{bmatrix} \bar{\gamma}_{l,s_t,11} \\ \bar{\gamma}_{l,s_t,21} \end{bmatrix}.$$

In Eqs. (9)–(12), s_t denotes the unobserved economic regime, and is assumed to follow a first order, homogeneous, two-state Markov chain governed by the transition matrix

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

where $p_{ij} = P(s_t = i \,|s_{t-1} = j), i, j = 1, 2$ and $p_{11} = 1 - p_{21}$ and $p_{12} = 1 - p_{22}$.

Equation (9) suggests that all the parameters in the $\mathbf{B}_{s_t}$, $\mathbf{C}_{s_t}$, $\mathbf{\Gamma}_{s_t}$, and $\mathbf{9}_{s_t}$ matrices are regime-dependent, taking different values across the two regimes (i and j can only take two values). The two assumed regimes will sufficiently describe the dynamic interactions between the oil price and economic growth; as suggested by Hamilton (1988, 1989), the two-regime model is sufficient for modeling recessions and expansions observed in many macroeconomic time series.

Equation (10) tells us that the copula C has two different forms in the two unknown regimes. Equations (11) and (12) assume that the two structural shocks follow a univariate normal distribution in each regime. To make the model as flexible as possible, we allow the variance of GDP growth to be different across regimes, thus allowing homoscedasticity within each regime, but heteroscedasticity across regimes. We construct a Markov regime switching GARCH(1,1) specification regarding the regime-dependent variance of oil price growth, as follows

$$h_{o_{s_t},t} = d_{1,s_t} + d_{2,s_t}\epsilon^2_{o_{s_{t-1}},t-1} + d_{3,s_t}h_{o_{s_{t-1}},t-1}. \tag{13}$$

The estimation of our model is challenging under the Markov regime switching GARCH(1,1) specification in Eq. (13). In particular, Eq. (13) implies that $h_{o_{s_t},t}$ depends on s_t and also indirectly on $\{s_{t-1}, s_{t-2}, \ldots\}$. That is, $h_{o_{s_t},t}$ at time t depends on the entire sequence of regimes up to time t. One has to construct the likelihood function by integrating over all possible paths, and as it turns out the estimation is not tractable; this is a problem, called path dependence, that typically shows up in the estimation of regime-switching GARCH models. To address this problem, we need to use a collapsing procedure that could facilitate the evaluation of the likelihood function.

In this paper, we follow Gray (1996) and integrate out the regime-dependent error term ϵ_{o_t} and the regime-dependent variance $h_{o_{s_t},t}$ at time $t-1$ by taking the expectation so that the GARCH specification does not require the entire sequence of regimes up to time t . Therefore, we construct the regime-independent error term $\bar{\epsilon}_{o_t}$ and the regime-independent oil price volatility $\bar{h}_{o_t}$ by calculating

$$\bar{\epsilon}_{o_t} = p(s_t = 1|\Omega_{t-1})\epsilon_{o_{s_t=1},t} + p(s_t = 2|\Omega_{t-1})\epsilon_{o_{s_t=2},t}$$

and

$$\begin{aligned}\bar{h}_{o_t} &= p(s_t = 1|\Omega_{t-1})[(\Delta o_t - \epsilon_{o_{s_t},t=1})^2 + h_{o_{s_t=1},t}]\\ &+p(s_t = 2|\Omega_{t-1})[(\Delta o_t - \epsilon_{o_{s_t=2},t})^2 + h_{o_{s_t=2},t}]\\ &-[p(s_t = 1|\Omega_{t-1})(\Delta o_t - \epsilon_{o_{s_t=1},t}) + p(s_t = 2|\Omega_{t-1})(\Delta o_t - \epsilon_{o_{s_t=2},t})]^2\end{aligned}$$

where $p(s_t = 1|\Omega_{t-1})$ and $p(s_t = 2|\Omega_{t-1})$ are the prediction probabilities from the Hamilton (1989) filter. We then plug $\bar{\epsilon}_{o_t}$ and $\bar{h}_{o_t}$ into the GARCH specification (13) for the oil price so that it becomes

$$h_{o_{s_t},t} = d_{1,s_t} + d_{2,s_t}\bar{\epsilon}^2_{o_{t-1}} + d_{3,s_t}\bar{h}_{o_{t-1}}. \tag{14}$$

Thus, $h_{o_{s_t},t}$ depends only on the value of s_t and the likelihood function becomes tractable. Under the BB1 copula assumption, the density at time t conditional $s_t = i$

Table 2 Maximum likelihood parameter estimates

Parameter	Estimate Regime 1	Regime 2
ψ	−0.047 (0.000)	−0.027 (0.000)
δ	1.225 (0.000)	1.768 (0.000)
θ	0.629 (0.000)	0.805 (0.000)
λ_U	0.407	0.614
λ_L	0.239	0.520

Sample period, quarterly data: 1974:q1-2022:q4. Numbers in parentheses are p values

and lag information set Ω_{t-1} is

$$f(\boldsymbol{\epsilon}_t|s_t = i, \Omega_{t-1}) = F(\boldsymbol{\epsilon}_{s_t,t}) = C(F_1(-\epsilon_{o_{s_t},t}), F_2(\epsilon_{y_{s_t},t}))$$

The density at time t can be found by summing over all possible regimes:

$$f(\boldsymbol{\epsilon}_t|\Omega_{t-1}) = \sum_{i=1}^{2} f(\boldsymbol{\epsilon}_t|s_t = i, \Omega_{t-1}) p(s_t = i|\Omega_{t-1}). \tag{15}$$

The tractable log-likelihood is then

$$L = \sum_{t=1}^{T} \ln f(\boldsymbol{s}_t|\Omega_{t-1}) \tag{16}$$

with the conditional variance of $\epsilon_{o_{s_t},t}$ governed by equation (13). The corresponding estimation is performed in RATS using Maximum Likelihood and the BFGS (Broyden, Fletcher, Goldfarb & Shanno) algorithm combined with the derivative-free Simplex pre-estimation method.

4 Estimation results

We estimate the model using full information maximum likelihood. That is, all the parameter estimates are obtained simultaneously by maximizing the logged joint density built on the copula function and its density function. We report the ψ, δ, and θ parameter estimates, as well as the tail dependence parameters, λ_U and λ_L, in Table 2 for each of the two regimes, as they are defined by the copula function. We find that oil price uncertainty has a negative effect on economic growth in both regimes ($\psi = -0.047$ with a p-value of 0.000 in regime 1 and $\psi = -0.027$ with a p-value of 0.000 in regime 2), with the negative effect being larger in regime 1. The estimates of ψ highlight the important role of oil price uncertainty on the business cycle. Moreover, this effect is time-varying.

Table 2 shows that there is upper tail dependence in regime 1 ($\lambda_U = 0.407$), and to a larger extent in regime 2 ($\lambda_U = 0.614$), meaning that there is a tendency for output growth to significantly decline in the case of large oil price shocks. Moreover, a non-zero λ_L ($\lambda_L = 0.239$ in regime 1 and $\lambda_L = 0.520$ in regime 2) suggests lower tail dependence with that dependence being higher in regime 2. The interesting finding here is that upper tail dependence is larger than lower tail dependence in both regimes. To better understand the dependence structure between the oil price and economic growth, in Fig. 1 and 2 we provide the smoothed probabilities of each regime, $p(s_t = i|\Omega)$, for $i = 1, 2$, where Ω is the full sample information, as well as the distribution simulation (100000 draws) of the oil price and economic growth shocks, based on their joint distribution which is modeled by the copula function—see Joe (1997) for the algorithm.

The upper panel of Figure 1 shows that the U.S. economy has been in regime 1, which covers the major economic recessions, including the global financial crisis and the Covid-19 recession. More importantly, as can be seen in the lower panel of Fig. 1, there is negative dependence between the oil price and output growth shocks in that regime. It means that it is likely to observe a large positive (negative) oil price shock and a large negative (positive) output growth shock at the same time. The simulated distribution captures the strong upper tail dependence very well and also shows that lower tail dependence is relatively weak when the oil price shock takes large negative values. This feature is consistent with the reported tail dependence parameters in Table 2; the upper tail dependence parameter, λ_U, is larger than the lower tail dependence parameter, λ_L. We conclude that there is a higher chance for the negative output growth shock to become significantly large when the positive oil price shock is significantly large, compared with the case of having a significantly small and negative oil price shock and a significantly large and positive output growth shock. We conclude that the dependence structure between the price of oil and output growth is asymmetric, with the oil price shock more closely related to output growth dynamics when it is large and positive.

In the upper panel of Fig. 2, we see that regime 2 shows up in the non-recession periods. The lower panel of Fig. 2 also suggests a negative dependence between the oil price and economic growth shocks in regime 2. In fact, the upper tail dependence is slightly larger than the lower tail dependence, consistent with the reported dependence parameters in Table 1, since $\lambda_U = 0.614$ and $\lambda_L = 0.520$ in that regime.

5 Robustness

As noted by Kilian and Vigfusson (2011b), empirical results regarding the relationship between the price of oil and economic activity can be sensitive to the sample period and the choice of the oil price measure. They argue that the price of oil should be specified in real terms (which is what we have done), that the evidence using pre-1973 data should be viewed with caution, and that the refiners' acquisition cost (RAC) for imported crude oil should be used as the oil price series. In this section we investigate the robustness of our results to these model specifications. In particular, we use the refiners' acquisition

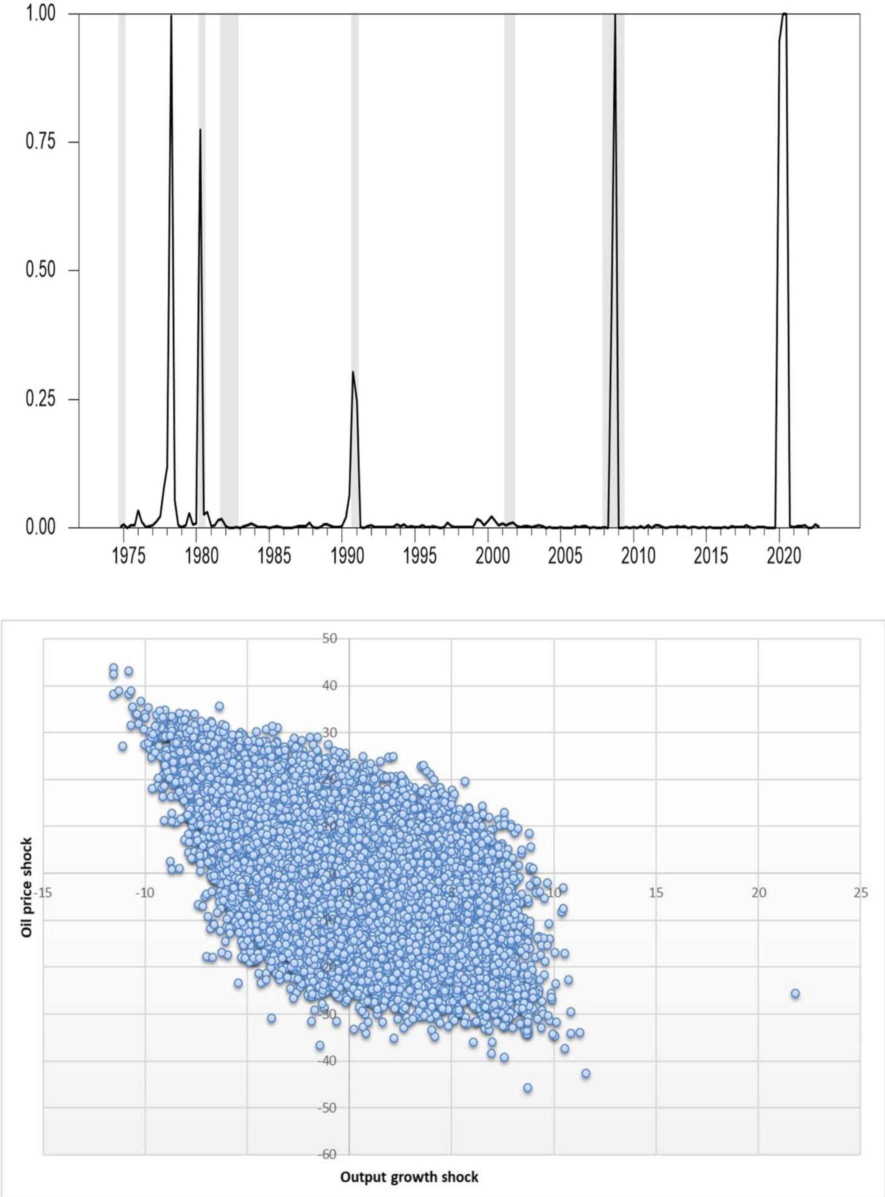


Fig. 1 Probability of regime 1 and simulated distribution

cost for imported crude oil to investigate the dependence relationship between the real price of oil and real GDP growth.

Table 3 reports the key parameter estimates. We still find negative impacts of oil price uncertainty in both regimes. Moreover, the upper tail dependence and low tail dependence are evident. In Figs. 3 and 4, we present results with the real refiners'

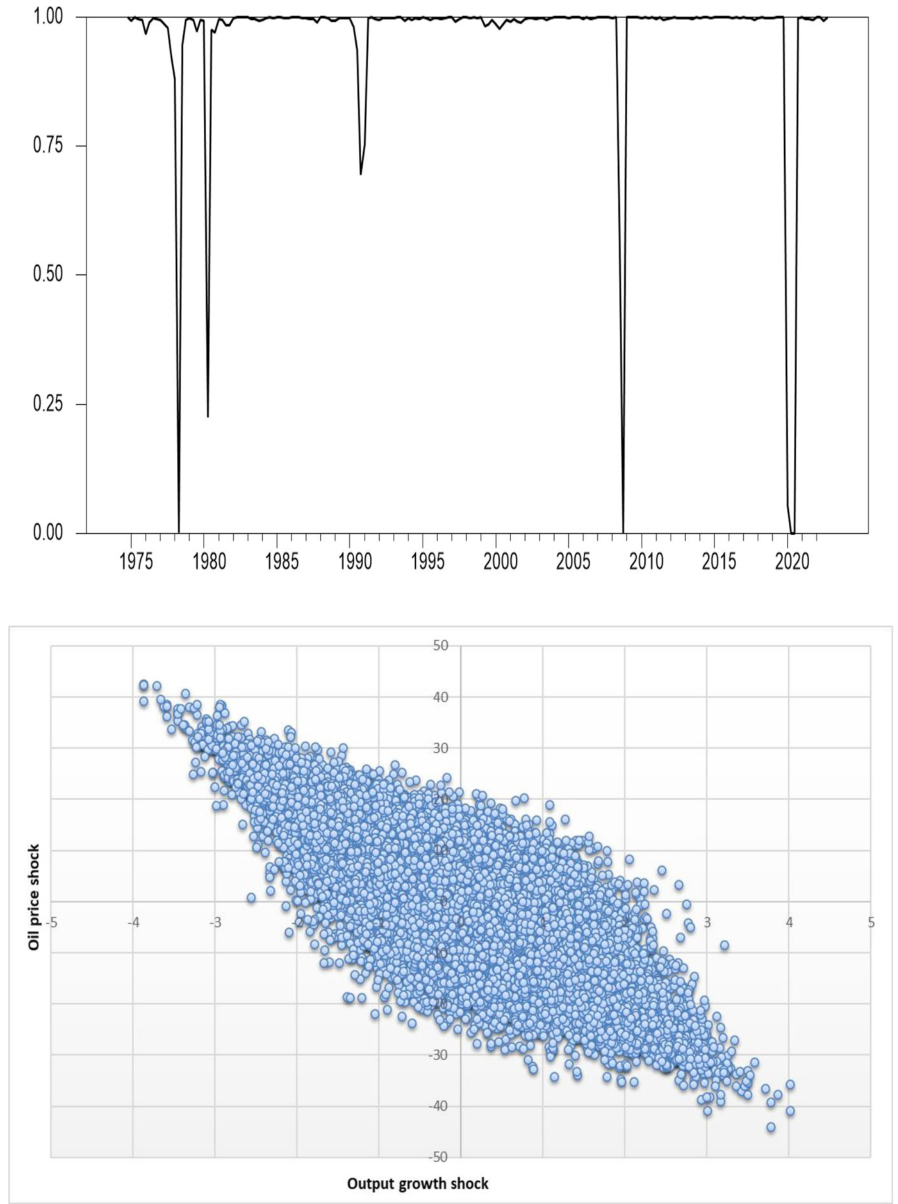


Fig. 2 Probability of regime 2 and simulated distribution

acquisition cost for imported crude oil, in the same fashion as in Figs. 1 and 2, respectively, with the real WTI oil price for the post-1973 era. As can be seen in Figs. 3 and 4, the regimes and simulated distributions are similar to those in Figs. 1 and 2. There is negative dependence between the oil price and output growth shocks in regime 1 (which covers economic recessions) as well as in regime 2 (which covers

Table 3 Maximum likelihood parameter estimates with the RAC

Parameter	Estimate	
	Regime 1	Regime 2
ψ	−0.045 (0.000)	−0.005 (0.000)
δ	1.268 (0.000)	2.388 (0.000)
θ	0.564 (0.000)	0.797 (0.000)
λ_U	0.380	0.695
λ_L	0.273	0.663

Sample period, quarterly data: 1974:q1-2022:q4.
Numbers in parentheses are p values

the non-recession periods). The simulated distributions capture the strong upper tail dependence in both regimes. In particular, $\lambda_U = 0.380$ and $\lambda_L = 0.273$ in regime 1. On the other hand, $\lambda_U = 0.695$ and $\lambda_L = 0.663$ in regime 2.

Overall, the asymmetric tail dependence is robust when considering two measures of oil prices. We especially show that the magnitude of the asymmetry is larger during economic recessions. The estimates indicate a large positive oil price shock will be more likely to accompany a large negative output shock when the economy experiences contractions. The policy implication is that oil reserves and stocks are important in mitigating oil price shocks, and they are particularly relevant during recessions.

Finally, in order to make our GARCH-in-Mean model comparable to the literature that studies oil price uncertainty, we present generalized impulse response functions that take into account the Markov regime switching in the Appendix (see Appendix Figures 5 and 6 based on the WTI and the RAC, respectively). The degree of asymmetry is stronger based on the RAC. Therefore, our generalized impulse response functions track the changing economic environment and are in line with the literature which shows that the output impulse responses are not generally symmetric to positive and negative oil price shocks—see, for example, Elder et al. (2010) and Serletis and Xu (2019).

6 Conclusion

We investigate the dependence structure between the price of oil and real output in the context of a bivariate, Markov regime switching, identified structural GARCH-in-Mean VAR model with copulas. We use quarterly data for the United States (over the period from 1974:q1 to 2022:q4) and find that oil price uncertainty has a negative and statistically significant effect on economic growth, consistent with evidence in Elder et al. (2010) and Serletis and Xu (2019). We also present clear evidence of an asymmetric negative dependence structure between oil price and output growth shocks, and provide a new explanation of asymmetric responses of output growth to oil price shocks characterized by upper tail dependence.

A possible extension of the current study will be the investigation of dependence structures between oil prices and macroeconomic activity across countries, potentially making a distinction between oil producers and oil importers. Given that the scope

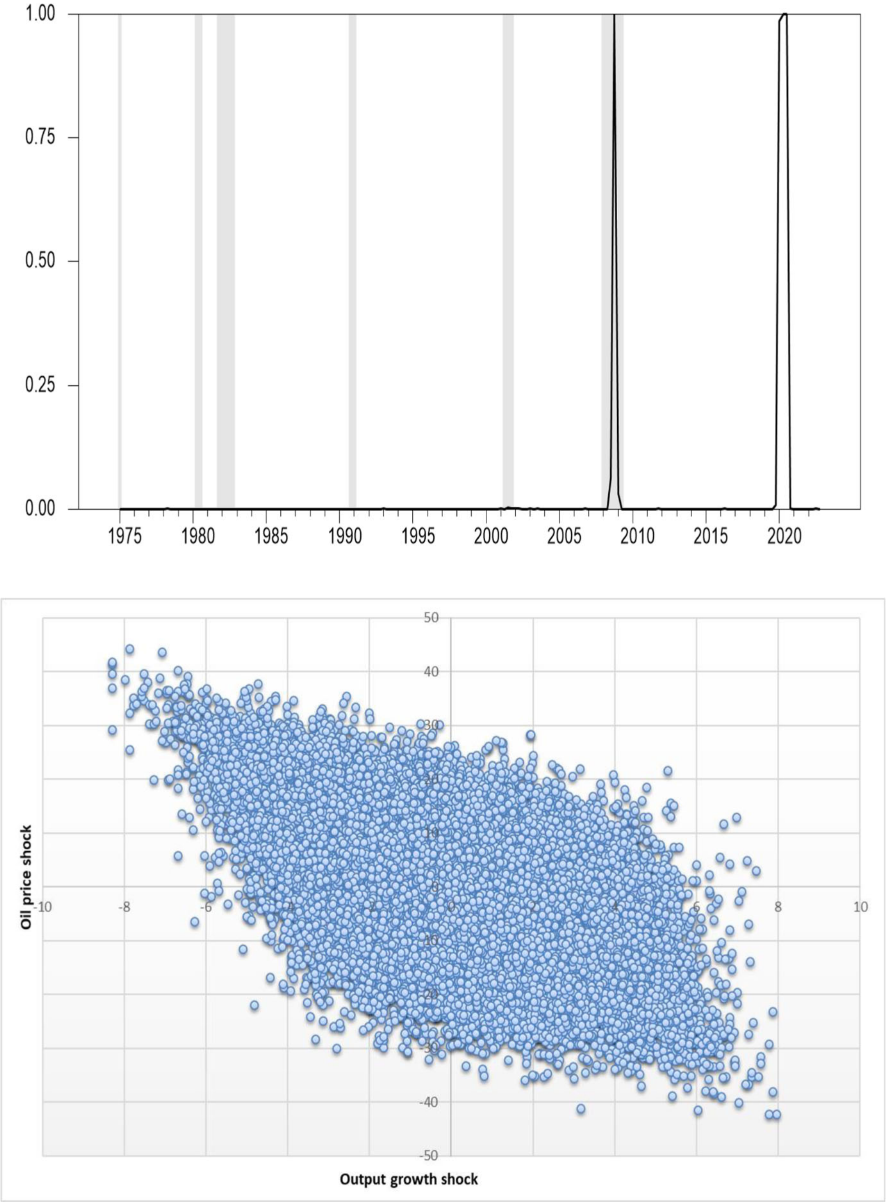
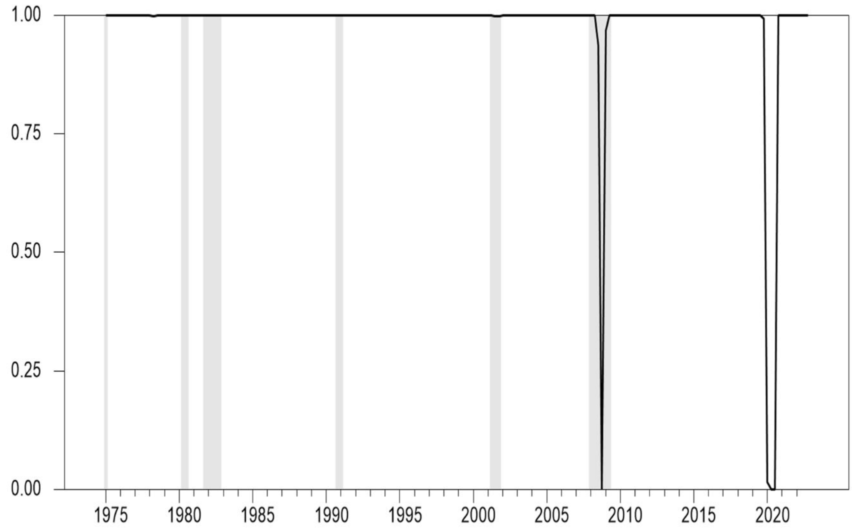


Fig. 3 Probability of regime 1 and simulated distribution: Refiners' acquisition cost

of the present paper is the development of the Markov switching near-VAR copula model for the US economy, we leave this as an area for potentially productive future research.



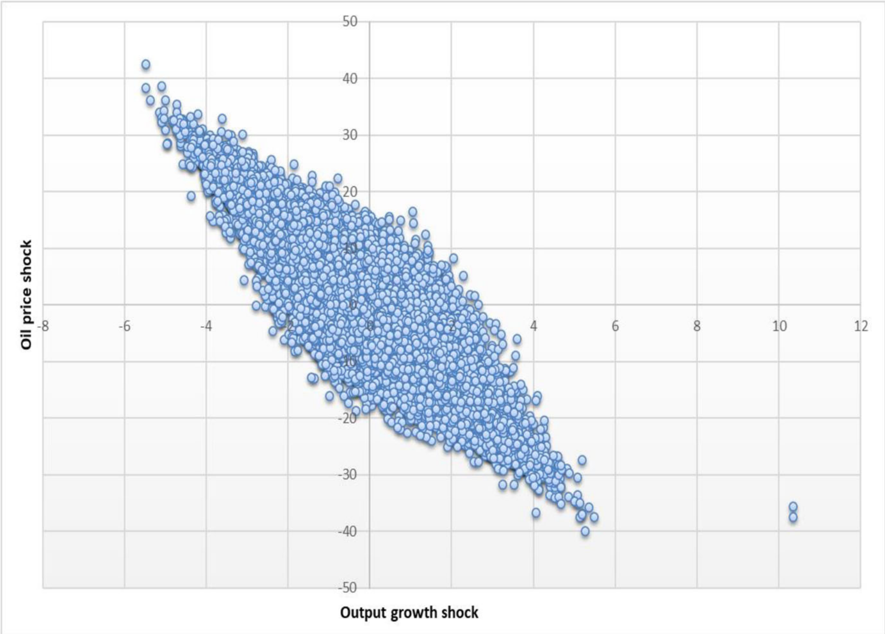


Fig. 4 Probability of regime 2 and simulated distribution: Refiners' acquisition cost

Conflicts of Interest We have no conflicts of interest.

Appendix

Generalized Impulse Response Functions

To get a comprehensive understanding of the impact of oil price shocks on output growth, we generalize the method in Gray (1996) and calculate generalized impulse response functions that take into account the switching between regimes. In doing so, we also consider the responses of output growth to positive and negative oil price shocks in the environment where the economy could switch between the two regimes. We define the unconditional generalized impulse response function by

$$E(z_{t+k}|\bar{\Omega}_{t+k-1}, \delta) - E(z_{t+k}|\bar{\Omega}_{t+k-1}) \tag{17}$$

where $E(z_{t+k}|\bar{\Omega}_{t+k-1}, \delta)$ is the predicted value of z_{t+k} based on a simulated information set $\bar{\Omega}_{t+k-1}$, the change in the regimes over time, and an oil shock δ at time t, where δ is the standard deviation of the growth rate of the price of oil. Equation (17) gives the response of output growth to a change in the oil price considering the history of the regimes. Because the path-dependence problem makes the calculation intractable, we adopted the collapsing procedure in Gray (1996) here to address this issue. For example, we have

$$\begin{aligned} E(z_{t+k}|\bar{\Omega}_{t+k-1}, \delta) = {} & p(s_{t+k=1}|\bar{\Omega}_{t+k-1}, \delta)(z_{t+k}|\bar{\Omega}_{t+k-1}, s_{t+k=1}, \delta) \\ & + p(s_{t+k=2}|\bar{\Omega}_{t+k-1}, \delta)(z_{t+k}|\bar{\Omega}_{t+k-1}, s_{t+k=2}, \delta) \end{aligned} \tag{18}$$

where

$$\bar{\Omega}_{t+k-1} = \left\{E(z_{t+k-1}|\bar{\Omega}_{t+k-2}, \delta), ..., E(z_{t+1}|\bar{\Omega}_{t}, \delta), E(z_{t}|\Omega_{t-1}, \delta)\right\} \cup \Omega_{t-1}.$$

The calculations are as follows:

- *Step 1* We start from time period t in our data. We then calculate $h_{o_{s_t},t}$ for each regime, using the information set $\bar{\Omega}_{t-1}$.
- *Step 2* We draw $\epsilon_{o_{s_t},t}$ from the univariate normal distribution with zero mean and variance $h_{o_{s_t},t}$ which is obtained from *Step 1* for each regime. We then draw $\epsilon_{y_{s_t},t}$ using the corresponding BB1 copula conditional on $\epsilon_{o_{s_t},t}$ for each regime.
- *Step 3* Repeat *Step 1* and *Step 2* recursively for time period i where $i \in [t+1, \ldots, t+k]$ for all $i \in [t+1, ..., t+k]$. In doing so, we construct the time-varying variance of the oil price shock using Eq. (14) and the collapsing procedure proposed by Gray (1996).
- *Step 4* $E(z_i|\bar{\Omega}_{i-1}, s_i)$, $i \in [t, \cdots, t+k]$ is constructed based on the recursive VAR system with Eq. (18) given $\boldsymbol{\epsilon}_{i,s_t}$ from the previous three steps. To take into account the switching from t to $t+k$, we update the predicted probabilities $p(s_i = 1|\bar{\Omega}_{i-1})$ and $p(s_i = 2|\bar{\Omega}_{i-1})$ where $i \in [t, \ldots, t+k]$ by adopting the Hamilton (1994) filter.

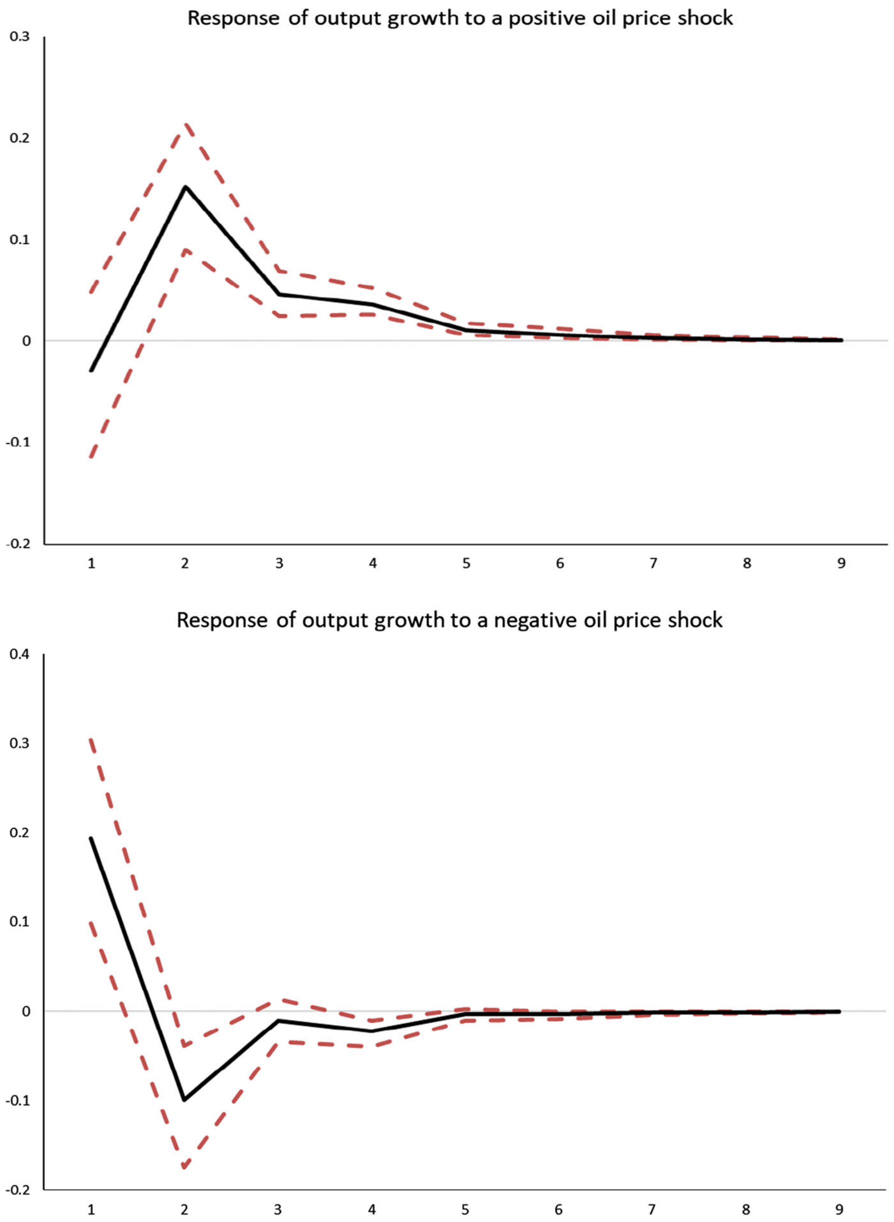


Fig. 5 Impulse response functions for the Markov switching structural GARCH-in-mean copula VAR

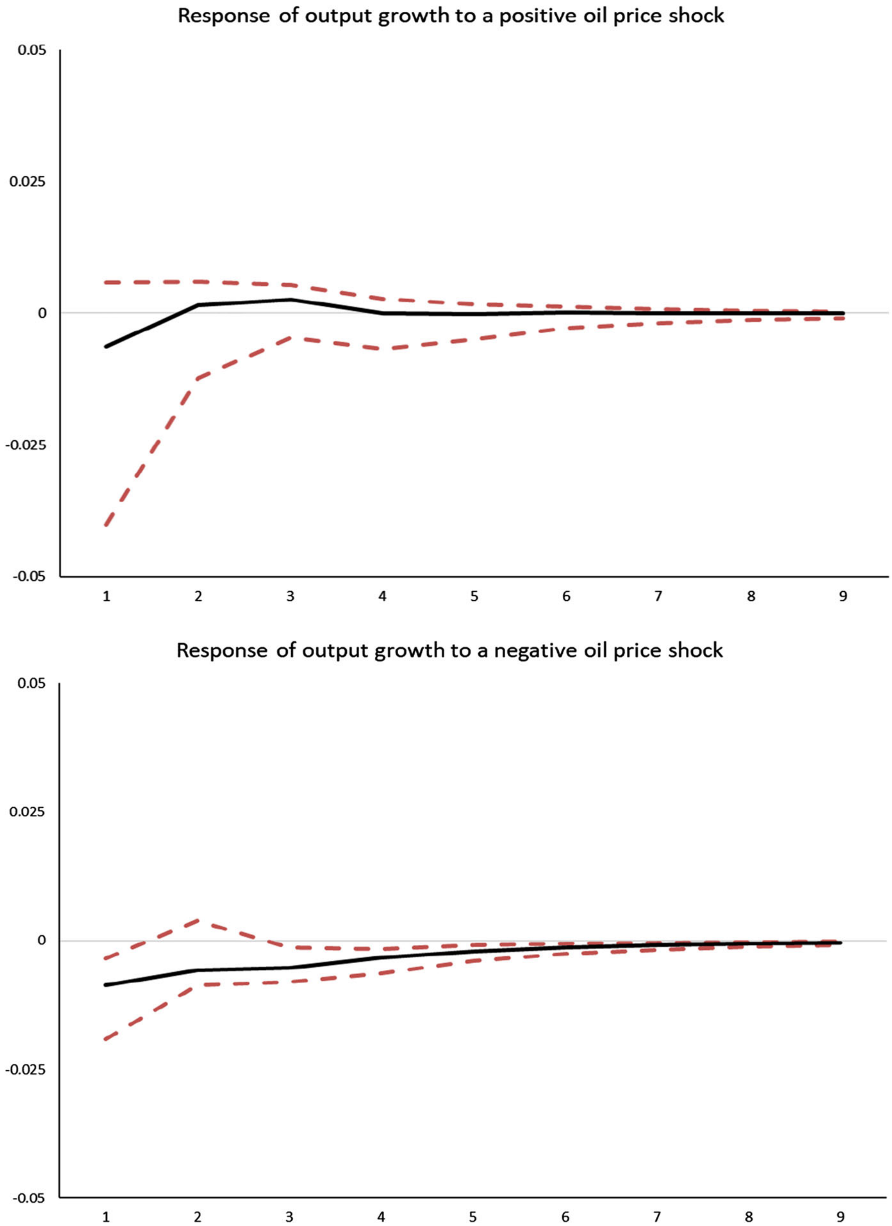


Fig. 6 Impulse response functions for the Markov switching structural GARCH-in-mean copula VAR: Refiners' acquisition cost

- *Step 5* We inject the oil price shock δ into the system at time t for $E(z_i|\bar{\Omega}_{i-1}, s_i, \delta)$, $i \in [t, ..., t+k]$. A new vector of error terms $\hat{\boldsymbol{\epsilon}}_t$ for time t only is constructed by

$$\hat{\boldsymbol{\epsilon}}_{t,s_t} = \boldsymbol{\epsilon}_{t,s_t} + (\delta, 0)'$$

where $\boldsymbol{\epsilon}_{t,s_t}$ is from *Step 2*. We then redo *Step 4* for $E(z_{i+k}|\bar{\Omega}_{i-1}, s_{i+k}, \delta)$, $i \in [t, ..., t+k]$ with the error terms $\hat{\boldsymbol{\epsilon}}_{t,s_t}$ at time period t.
- *Step 6* Take the difference between $E(z_i|\bar{\Omega}_{i-1}, s_i, \delta)$ and $E(z_i|\bar{\Omega}_{i-1}, s_i)$ for $i \in [t, \ldots, t+k]$.
- *Step 7* Average the difference in Step 6 across the sample periods. In other words, we choose each different period to initialize the calculation for its average. See Figures 5 and 6.

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