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Multivariate models of commodity futures markets: a dynamic copula approach

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Abstract

We apply flexible multivariate dynamic models to capture the dependence structure of various US commodity futures across different sectors between 2004 and 2022; particular attention is paid to the 2008 financial crisis and the COVID-19 pandemic. Our copula-based models allow for time-varying nonlinear and asymmetric dependence by integrating elliptical and skewed copulas with dynamic conditional correlation (DCC) and block dynamic equicorrelation (Block DECO). Flexible copula models that allow for multivariate asymmetry and tail dependence are found to provide the best performance in characterizing co-movements of commodity returns. We also find that the connectedness between commodities has dramatically increased during the financial distress and the COVID-19 pandemic. The impacts of the financial crisis appear to be more persistent than those of the pandemic. We apply our models to some risk management tasks in the commodity markets. Our results suggest that optimal portfolio weights based on dynamic copulas have persistently outperformed the equal-weighted portfolio, demonstrating the practicality and usefulness of our proposed models.

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1 Introduction

The first two decades of the twenty-first century witnessed a severe global financial crisis and an unprecedented pandemic wreaking havoc on people's everyday life and the world economy. There has been a large body of work on the far-reaching and long-lasting impacts of the former and a quickly growing literature on the still-evolving consequences of the latter on the financial markets. However, there exist relatively few studies on their impacts on the commodity futures markets, which have been playing an increasingly important role in the global supply chain and economy.

This study aims to fill in the gap in the literature via a systemic examination of the US commodity futures markets. Specifically, we construct a number of flexible multivariate dynamic models by combining several elliptical and skewed copulas with the dynamic conditional correlation (DCC) of Engle (2002) and block dynamic equicorrelation (Block DECO) proposed by Engle and Kelly (2012) to capture dependence structure of various commodities across different sectors. Particular attention is paid to the time-varying nonlinear dependence and asymmetries of commodity futures with copula models. It shows strong evidence of multivariate asymmetry and tail behavior in commodity returns.

Our dynamic copula models reveal that commodity markets were most connected during the 2008–09 financial crisis and most correlations return to the pre-crisis level as the economy improves. Energy products and industrial metals are the most highly correlated among all commodity sectors. The effects of the COVID-19 pandemic on all sectors of the economy, up to the end of our sample period (January 2022), are suggested to be less severe in terms of magnitude and duration than the financial crisis.

Based on the estimated model, we undertake two tasks of financial risk management of commodity futures. We find that the diversification benefit is diminishing, and the tail dependence is substantially higher in the bearish market during the sample period. Nonetheless, the optimal portfolio weights based on dynamic copulas perform persistently better than the equal-weighted portfolio. Both numerical experiments demonstrate the practicality and usefulness of our proposed models.

The paper proceeds as follows. Section 2 reviews the relevant literature. Section 3 introduces the marginal model and skewed t copulas with two dynamic correlations for the joint distributions of financial assets. Section 4 presents the empirical application for the commodity futures, including the data set description, estimation, and out-of-sample model comparison results. Section 5 discusses two important economic implications of estimated dynamic copulas: diversification benefit and lower tail dependence. Section 6 concludes. Details on multivariate skewed t distributions and implementation of out-of-sample-based model comparison are provided in Appendices A and B.

2 Background

Commodity markets have attracted much attention in both academia and industry since the early 2000s, but some interesting problems remain unanswered and are seldom studied. Commodity exchange-traded funds (ETF), which track specific commodity indices and invest in several different commodities, are increasingly becoming the sole focus of many institutional investors' portfolios since the 2000s. Subsequent large inflows into commodity markets, termed as "financialization of commodity markets," are suggested to have substantially increased the correlations between a large number of commodity futures (Tang and Xiong 2012; Büyükşahin and Robe 2014). Though recent research provides some evidence of structural changes in their correlations, most of them do not fully account for possibly nonlinear dependence among futures returns, partly due to the paucity of flexible multivariate distributions.

Many assets like stocks, bonds, and commodities that historically had low correlations are often used to build well-diversified portfolios in mutual funds, but these assets have shown a tendency to crash together during the recent financial crises. Therefore, understanding time-varying co-movements of a large collection of commodity futures is of great importance to constructing robust dynamic portfolios from the perspective of risk management (Belousova and Dorfleitner 2012; Bessler and Wolff 2015; Daskalaki and Skiadopoulos 2011). However, few previous studies have provided a comprehensive analysis of the benefits of diversification in commodity futures markets over extended periods of time.

There is a fast-growing body of recent literature on the interconnection of commodity markets and the role of financialization in market co-movements. Büyükşahin and Robe (2017) model dynamic correlations between the equity market and commodities in the grain and livestock sector using various specifications of structural vector autoregression (SVAR). They find that global business cycle shocks have a substantial and long-lasting impact on the food market's co-movements with the equity market. In contrast, changes in the intensity of financial speculation have a short-lived and insignificant impact on cross-market return linkages. Tang and Xiong (2012) model the dynamics of correlations of all pairwise combinations of commodities and find increasing correlations since 2004. Adams and Glück (2015) study structural breaks in correlations among eight commodities. Most of these studies focus on specific commodities and are based on low frequency data (monthly or weekly). It is unclear whether their findings apply to relatively high frequency data of futures markets.

The article aims to explore whether the dependence structure of commodity markets is asymmetric and changing over time. To this end, we make use of copula, a flexible and powerful approach to model multivariate distribution, as it allows flexible modeling of the joint distribution of random vectors by estimating marginals and the dependence structure separately. Copula has wide applications in economics and finance. Lee (1983) applies copula to models with selectivity. Prokhorov and Schmidt (2009) propose an improved QMLE in the panel data model with copulas to model the dependence over time while the cross sections are independent, and Amsler et al (2014) use copulas to model time dependence in stochastic frontier models. The last two decades have witnessed numerous applications of the copula in modeling the joint distribution of default probability of credit products in the finance industry, but few of them can successfully model dynamics of joint distribution and high-dimensional data at the same time (Patton 2013).

A few recent studies have proposed some new families of copulas to capture: (1) co-movements of a large number of equity returns and (2) dynamic dependence structure that is robust to various financial and economic circles. For example, Christoffersen et al (2012) propose a new class of dynamic copulas based on the dynamic conditional correlation of Engle (2002) and the multivariate skewed t distribution of Demarta and McNeil (2005) to model co-movements of asset returns in developed and developing markets. Creal and Tsay (2015) introduce time variation into the copula densities via factor models with stochastic loadings. The proposed copula models have flexible dynamics and heavy tails yet remain tractable in high dimensions due to their factor structure. Lucas et al (2016) develop a modeling framework of copulas based on the multivariate skewed t distribution of Demarta and McNeil (2005) and the score-driven dynamic of Creal et al (2013) to estimate joint and conditional tail risk probabilities over time in a financial system with a large number of financial sector firms. Oh and Patton (2018) combine the generalized autoregressive score model (GAS) of Creal et al (2013) and the factor copula model of Oh and Patton (2017) to obtain a tractable and parsimonious time-varying model for high-dimensional conditional distributions.

3 Dynamic multivariate models via copula

In this section, we present the basics of our modeling framework for dynamic multivariate models via copula. Section 3.1 describes how we estimate the marginal distribution of futures returns. Section 3.2 introduces the skewed t copula model that is increasingly used in financial econometrics. Section 3.3 presents the dynamic conditional correlation (DCC) and block dynamic equicorrelation (Block DECO) models that describe the time-varying dependence structure of futures returns. We also briefly discuss how elliptical and skewed copulas and various specifications of dynamic correlations can be integrated using maximum composite likelihood estimation for high-dimensional data.

3.1 Marginal model

Following the common practice in the finance literature, we assume that $y_{i,t}$ is the log return of commodity futures i at period t, dynamic mean $\mu_{i,t}$ is captured by an autoregressive integrated moving average (ARIMA) model, volatility $\sigma_{i,t}$ follows a generalized autoregressive conditional heteroskedasticity (GARCH) process, and $\epsilon_{i,t}$ is the innovation term. The order of ARIMA model is selected according to the Bayesian information criterion (BIC), and the GARCH(1,1) model is estimated by QMLE. The univariate model is given below:

$$y_{i,t} = \mu_{i,t} + \sigma_{i,t}\epsilon_{i,t}, \;\; i = 1, \ldots, N, \;\; t = 1, \ldots, T, \tag{1}$$

$$\sigma^2_{i,t} = \omega_i + \alpha_i (y_{i,t-1} - \mu_{i,t-1})^2 + \beta_i \sigma^2_{i,t-1}, \tag{2}$$

$$\epsilon_{i,t} \sim Skewed\ t(\widetilde{\upsilon}_i, \widetilde{\lambda}_i). \tag{3}$$

We use subscript 'i' in the skewness parameter $\widetilde{\lambda}_i$ and degree of freedom $\widetilde{\upsilon}_i$ to denote the innovation distributions for different commodities and the parameters remain constant over time. Here, the skewed-t distribution is defined as in Hansen (1994):

$$g(\epsilon|\widetilde{\upsilon}, \widetilde{\lambda}) = \begin{cases} bc\left(1 + \frac{1}{\widetilde{\upsilon}-2}(\frac{b\epsilon+a}{1-\widetilde{\lambda}})^2\right)^{-(\widetilde{\upsilon}+1)/2}, & x < -a/b, \\ bc\left(1 + \frac{1}{\widetilde{\upsilon}-2}(\frac{b\epsilon+a}{1+\widetilde{\lambda}})^2\right)^{-(\widetilde{\upsilon}+1)/2}, & x \geq -a/b, \end{cases} \tag{4}$$

where the constants a, b and c are given by

$$a = 4\widetilde{\lambda}c\left(\frac{\widetilde{\upsilon}-2}{\widetilde{\upsilon}-1}\right), \;\; b^2 = 1 + 3\widetilde{\lambda}^2 - a^2, \;\; c = \frac{\Gamma(\frac{\widetilde{\upsilon}+1}{2})}{\sqrt{\pi(\widetilde{\upsilon}-2)\Gamma(\frac{\widetilde{\upsilon}}{2})}}. \tag{5}$$

The skewness parameter $\widetilde{\lambda} \in [-1, 1]$ controls the degree of asymmetry and the degree of freedom $\widetilde{\upsilon} \in (2, \infty]$ governs the thickness of tails. This skewed-t distribution is flexible and nests many distributions. For example, $\widetilde{\lambda} = 0$ yields the standard student t distribution, $\widetilde{\upsilon} \rightarrow \infty$ corresponds to the skewed normal distribution, and with $\widetilde{\upsilon} \rightarrow \infty$ and $\widetilde{\lambda} = 0$ it becomes the standard normal distribution. In our empirical analysis, we estimate the marginal distribution using this ARIMA-GARCH model for each commodity futures returns. We obtain the cumulative distribution function (CDF) by $\eta_{i,t} \equiv \eta_{i,t}(\epsilon_{i,t}|\widetilde{\lambda}_i, \widetilde{\upsilon}_i)$, where $\widetilde{\lambda}_i$ and $\widetilde{\upsilon}_i$ are the estimated parameters for the distribution of innovations $\epsilon_i = [\epsilon_{i,1}, \ldots, \epsilon_{i,T}]'$.

3.2 Dependence modeling via copula

Elliptical copula models, including the normal and student t copulas, are the most popular approach for modeling the dependence structure of high-dimensional data in finance owing to their straightforward interpretation and convenient implementation. However, they suffer from drawbacks that restrict their flexibility to model co-movements in financial markets. For example, the normal copula imposes the restriction of zero tail dependence in equity returns. Although it admits tail dependence, the t copula maintains a symmetric dependence. The Archimedean copula is another popular copula family that includes the Frank, the Clayton, the Gumbel and many other copula specifications. But this family is most successful in modeling bivariate data and is difficult to generalize to high-dimensional data. We refer interested readers to Patton (2013) for a comprehensive coverage of various copula models.

The well-documented stylized facts that financial assets tend to move together, especially during market downturns, call for a more flexible model that accommodates asymmetric tail dependence in multivariate data. In this study, we follow Christoffersen et al (2012, 2018, 2019) and employ the skewed t copula approach proposed by Demarta and McNeil (2005) to model high-dimensional dependence in equity markets.

The skewed t copula at time t is defined as

$$C(\eta_{1,t}, \ldots, \eta_{N,t}; \Gamma_t, \lambda, \upsilon) = t_{\Gamma,\lambda,\upsilon}(t^{-1}_{\lambda_1,\upsilon}(\eta_{1,t}), \ldots, t^{-1}_{\lambda_N,\upsilon}(\eta_{N,t})), \tag{6}$$

where $t_{\Gamma,\lambda,v}$ is the multivariate skewed t distribution with an N-dimensional skewness parameter vector $\lambda = (\lambda_1, ..., \lambda_N)'$, the degree of freedom v, and the correlation matrix Γ_t at time t. Here $t^{-1}_{\lambda_i,v}$ is the quantile function for the univariate skewed t distribution. The skewed t copula has a very flexible specification based on the multivariate skewed t distribution and nests the student t copula (when $\lambda \to \mathbf{0}$) and normal copula (when $\lambda \to \mathbf{0}$ and $v \to \infty$). For simplicity, we assume the same degree of freedom for the multivariate and each univariate skewed t distribution. Demarta and McNeil (2005) define the probability density function (pdf) of the N-dimensional skewed t copula from the asymmetric t distribution at time t as:

$$c(\eta_t|\Gamma_t, \lambda, v) = \frac{2^{\frac{(v-2)(N-1)}{2}} K_{\frac{v+N}{2}}(\sqrt{(v + \epsilon_t^{*\prime}\Gamma_t^{-1}\epsilon_t^*)\lambda'\Gamma_t^{-1}\lambda})e^{\epsilon_t^{*\prime}\Gamma_t^{-1}\lambda}}{\Gamma_t\left(\frac{v}{2}\right)^{1-N}|\Gamma_t|^{\frac{1}{2}}\left(\sqrt{(v + \epsilon_t^{*\prime}\Gamma_t^{-1}\epsilon_t^*)\lambda'\Gamma_t^{-1}\lambda}\right)^{-\frac{v+N}{2}}\left(1 + \frac{1}{v}\epsilon_t^{*\prime}\Gamma_t^{-1}\epsilon_t^*\right)^{\frac{v+N}{2}}}$$

$$\times \prod_{i=1}^{N} \frac{\left(\sqrt{(v + \epsilon_{i,t}^{*2})\lambda_i^2}\right)^{-\frac{v+1}{2}}\left(1 + \frac{\epsilon_{i,t}^{*2}}{v}\right)^{\frac{v+1}{2}}}{K_{\frac{v+1}{2}}\left(\sqrt{(v + \epsilon_{i,t}^{*2})\lambda_i^2}\right)e^{\epsilon_i\lambda_i}},$$

where η_t is an N-dimensional vector of cumulative distribution function obtained from the previous section, $K_{\frac{v+d}{2}}$ is the modified Bessel function of the third kind, and ϵ_t^* is given by $\epsilon_t^* = [t^{-1}_{\lambda_1,v}(\eta_{1,t}), ..., t^{-1}_{\lambda_N,v}(\eta_{N,t})]$.

In the preceding subsection, the parameters $\widetilde{\lambda}_i$ and $\widetilde{v}_i$ in the ARIMA-GARCH model determine the distribution of the innovation for each commodity futures return. While λ_i and v in this subsection are parameters in the univariate skewed t distribution, $t_{\lambda_i,v}(\cdot)$, for each element in the multivariate distribution given in Eq. (6). They are used to construct the skewed t copula in the second stage estimation.

Notice that $\epsilon_{i,t}^*$ is obtained via the quantile function of skewed t distribution, and $\epsilon_{i,t}$ is obtained directly from the ARIMA-GARCH estimation for univariate data. It follows that if the marginal distribution $\eta_{i,t}$ is close to the univariate skewed t distribution $t_{\lambda_i,v}(\cdot)$, then $\epsilon_{i,t}^*$ is close to $\epsilon_{i,t}$ as well. As we shall see in the next section that $\epsilon_{i,t}^*$ is needed since it drives the estimation of dynamic conditional correlation in the copula models. We opt to use the skewed t copula to measure co-movements in commodity markets as it is highly tractable and flexible in modeling dependence structure even for hundreds of equity returns (Christoffersen et al 2018). For the sake of simplicity in our empirical analysis, we shall assume λ as a scalar for the high-dimensional commodity returns.

3.3 Dynamic conditional correlation

Motivated by the seminal paper of Engle (2002) and a recent application of Christoffersen et al. (2014), we propose to integrate the dynamic conditional correlation (DCC) process with skewed t copula to capture the correlation dynamics, asymmetry, and tail behavior in the multivariate distribution of commodity futures returns. Since the original DCC process is driven by a multivariate GARCH process and the copula shocks

$\epsilon_t^* = [t_{\lambda,v}^{-1}(\eta_{1,t}), ..., t_{\lambda,v}^{-1}(\eta_{N,t})]$ that drive the dynamic correlation do not necessarily have zero mean and unit variance from the skewed t copula model, we need to standardize these copula shocks before modeling correlation dynamics. Integrating the DCC process with copulas implies standardization is still needed for t copulas. Notice that standardized copula shocks are only used in the DCC process. In the maximum composite likelihood estimation, we shall use the original copula shocks in the likelihood function.

We assume a GARCH-type process drives the dynamic conditional correlation for elliptical and skewed copulas as

$$\tilde{\Gamma}_t = (1 - \alpha_\Gamma - \beta_\Gamma)\Omega + \alpha_\Gamma \bar{\epsilon}_{t-1}^* \bar{\epsilon}_{t-1}^{*'} + \beta_\Gamma \tilde{\Gamma}_{t-1}, \tag{7}$$

where $\tilde{\Gamma}_t$ is the correlation matrix at time t, α_Γ and β_Γ are scalars, and $\bar{\epsilon}_t^* = (\bar{\epsilon}_{1,t}^*, ..., \bar{\epsilon}_{N,t}^*)'$ is an N-dimensional vector with $\bar{\epsilon}_{i,t}^* = \epsilon_{i,t}^* \sqrt{\tilde{\Gamma}_{ii,t}}$. We then adopt the following normalization to ensure correlations are always in the $[-1, 1]$ interval,

$$\Gamma_{ij,t} = \tilde{\Gamma}_{ij,t} / \sqrt{\tilde{\Gamma}_{ii,t} \tilde{\Gamma}_{jj,t}}. \tag{8}$$

As shown by Aielli (2013), the transform of $\epsilon_{i,t}^*$ to $\bar{\epsilon}_{i,t}^*$ allows us to estimate Ω consistently as follows

$$\hat{\Omega} = \frac{1}{T} \sum_{t=1}^{T} \bar{\epsilon}_t^* \bar{\epsilon}_t^{*'}. \tag{9}$$

Since Ω is a copula correlation matrix, all diagonal elements of Ω equal one and we only need $\tilde{\Gamma}_{ii,t}$ for all i to calculate $\bar{\epsilon}_{i,t}^*$. Aielli (2013) shows that we can first obtain the diagonal elements of Eq. (7) for all i and t by

$$\tilde{\Gamma}_{ii,t} = (1 - \alpha_\Gamma - \beta_\Gamma) + \alpha_\Gamma \bar{\epsilon}_{i,t-1}^{*2} + \beta_\Gamma \tilde{\Gamma}_{ii,t-1}, \tag{10}$$

which is then used to compute $\bar{\epsilon}_{i,t}^*$ to calculate $\hat{\Omega}$ from Eq. (9). This is a direct modeling of correlation dynamics and has the potential to capture precisely the time-varying nature of correlations.

In a recent paper, Engle and Kelly (2012) extend dynamic conditional correlation to dynamic equicorrelation (DECO) and block dynamic equicorrelation (Block DECO) to further reduce computational burden and utilize potential group information in the data. As its name suggests, DECO implies an equal correlation between any pairwise combination of data at period t, and Block DECO implies the data belong to different groups; hence, the correlation matrix has a group structure and time-varying elements at each period. They show that consistent estimates of DECO and Block DECO can be readily obtained from the dynamic conditional correlation Γ_t from Eq. (8):

$$\Gamma_{\text{DECO},t} = \left(\frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j>i} \Gamma_{ij,t} \right) \boldsymbol{I}_{\boldsymbol{N \times N}}, \tag{11}$$

and the off-diagonal ijth element of Block DECO at period t is:

$$\Gamma_{\text{Block DECO},ij,t} = \begin{cases} \frac{1}{N_k(N_k-1)} \sum_{r,s\in k, r\neq s} \Gamma_{rs,t}, & i, j \in \text{group } k, \\ \frac{1}{N_k \times N_l} \sum_{r\in k, s\in l} \Gamma_{rs,t}, & i \in \text{group } k, \quad j \in \text{group } l, \end{cases} \tag{12}$$

where N_k and N_l represent the number of members in group k and group l, respectively. We are particularly interested in the Block DECO as an alternative specification for dynamic correlation in elliptical and skewed copulas, as commodity futures have been traditionally clustered by their industrial groupings and show evident group behaviors across financial and economic cycles. To the best of our knowledge, we are not aware of studies that take group behavior of commodity futures into account when modeling their dynamic dependence structure.

Pakel et al. (2021) and Engle and Kelly (2012) propose to estimate the DCC, DECO, and Block DECO models for high-dimensional data with composite likelihood to reduce computational burden under the full likelihood. Christoffersen et al. (2012; 2018) adopt this approach and demonstrate that it is highly reliable for modeling dependence structures with up to hundreds of variables using dynamic copulas. Specifically, the composite likelihood in our context is defined as:

$$CL(\lambda, v, \alpha_\Gamma, \beta_\Gamma) = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j>i} \ln f(\lambda, v, \alpha_\Gamma, \beta_\Gamma; \epsilon^*_{i,t}, \epsilon^*_{j,t}), \tag{13}$$

where $f(\alpha_\Gamma, \beta_\Gamma; \epsilon^*_{i,t}, \epsilon^*_{j,t})$ denotes a bivariate elliptical and skewed copula density of pair i and j with correlation defined according to the DCC, DECO or Block DECO model. We maximize the composite likelihood by summing over all possible pairs in each period t, and find this consistent estimator numerically fast and efficient.[1] In the next section, we shall combine normal, student t, and skewed t copulas with the DCC and Block DECO models to capture the dynamic dependence structure in the commodity futures returns.

4 Empirical investigation of commodity futures

We apply the models described in the previous section to US commodity futures markets. This section presents the main results of dynamic copulas estimation and

[1] A two-stage approach is employed to implement the proposed models. The first stage estimates the ARIMA-GARCH model for each margin, and imputes the CDF of the copula shocks based on the estimated marginal skewed t distributions. In the second stage, the parameters of the skewed t copula and those of the dynamic correlation matrix (under various specifications) are estimated through maximizing the composite likelihood.

out-of-sample-based model comparison. Section 4.1 introduces the data; Sect. 4.2 discusses the estimation results of dynamic copulas with various elliptical and skewed copula specifications with dynamic correlations. In Sect. 4.3, we use a prediction comparison test to rank the performance of dynamic copulas based on out-of-sample copula density.

4.1 Data

The S&P GSCI is the first major investable commodity index and serves as a benchmark for investment in the commodity markets. The index includes the most liquid commodity futures from all commodity sectors. Given this property, we consider 23 commodity futures in the S&P GSCI from all sectors. Specifically, we have 3 commodities in the energy sector (WTI crude oil, Brent crude oil, and natural gas), 6 commodities in the grain sector (corn, soybean, soybean oils, oats, wheat, and rough rice), 6 commodities in the soft sector (coffee, cotton, sugar, cocoa, lumber, and orange juice), 3 commodities in the livestock sector (feeder cattle, lean hogs, and live cattle) and 5 commodities in the metal sector (gold, platinum, palladium, silver, and copper). We use these five sectors to determine the grouping in Block DECO copulas.

For each commodity, we collect the daily last price of the generic 1st future, a continuous contract constructed by the front-month futures contract of that commodity futures, from Bloomberg. We calculate the daily log returns from January 5, 2004, to January 31, 2022, for all commodity futures. We focus on this period as the commodity markets have experienced the so-called financialization after 2004, and it is not affected by the recent Russia-Ukraine war. Understanding how the market dependence structure varies over time is critical to diversifying the systemic risk of an extensive portfolio with commodity futures. Therefore, within this time range, the effects of the financial crisis and the COVID-19 pandemic on the market dependence structure are of great interest and may provide useful insight regarding how to diversify systemic risk of a large portfolio of commodity futures.

4.2 Estimation results

As is explained in Sect. 3, the estimation of dynamic copulas is a two-stage process. In the first stage, we use the quasi maximum likelihood estimation (QMLE) to obtain the GARCH volatility of commodity futures returns, and in the second stage, we use estimated marginal distributions and the proposed maximum composite likelihood estimation to obtain α_Γ, β_Γ, λ and υ that drive the dynamics of high-dimensional time-varying copulas. For the sake of parsimony, we omit the univariate GARCH results for all 23 commodities below and focus on the DCC and Block DECO copulas. Table 1 reports the estimation results for the normal, student t, and skewed t copulas, wherein the dependence persistence ($\alpha_\Gamma + \beta_\Gamma$) represents the degree of mean-reversion in copula correlations, and the last column reports the copula likelihood.[2] The estimated

[2] The full likelihood function of copula model is computed with both copula likelihood and marginal likelihood for univariate returns. Since the marginal models are identical across all six models we only report the copula likelihood here.

parameters of all dynamic copulas are statistically significant at the 1% significance level. The skewed t copula has a negative asymmetry parameter λ in both correlation specifications, manifesting the multivariate asymmetry typically seen in stock returns. Since v of the student and skewed t copulas with the same dynamic correlation is rather close, and the only difference between them is the presence of λ in skewed t copula, our models strongly reject multivariate symmetry in commodity futures returns.

Another noteworthy result is the dependence persistence being close to one in all models, which suggests a slow mean-reversion in copula correlations according to Eq. (7). To better understand the empirical results, we compare our estimates based on the commodity futures returns with those of Christoffersen et al (2012) based on the equity returns. In Christoffersen et al (2012), the estimated degree of freedom parameters v and asymmetry parameter λ from skewed t copula with DCC specification for 16 developed markets, 13 emerging markets, and all markets are [17.64, 22.37, 21.83] and [−0.48, −0.49, −0.41], respectively. Our estimates of both parameters are around 24.66 and −0.16, suggesting that distributions of equity returns have fatter tails and more asymmetry than those of commodities.

Next, we undertake a simple yet informative in-sample model selection procedure by comparing the composite likelihood of dynamic copulas. The likelihood results in Table 1 suggest two noteworthy points: (1) with identical correlation specification (DCC or Block DECO) the skewed t copula performs the best among all three copulas and it is marginally better than student t copula under Block DECO, (2) the DCC is preferred to the Block DECO for all copula specifications. The first result is not unexpected as the skewed t copula is more flexible and able to capture both multivariate asymmetry and tail behavior of commodity returns. The second result is interesting as it suggests that the DCC copula generally has better performance than the Block DECO copula, and we attribute this finding to the fact that the DCC is an unrestricted specification for correlation while the number of groups somewhat restricts the Block DECO. Although modeling the dependence structure of 23 commodity futures is a high-dimensional application, this dimension perhaps is still not high enough to fully reap the benefits of the Block DECO structure that exploits grouping information (for comparison, Engle and Kelly (2012) demonstrate the advantage of the Block DECO model on all constituents of the S&P 500 Index.)

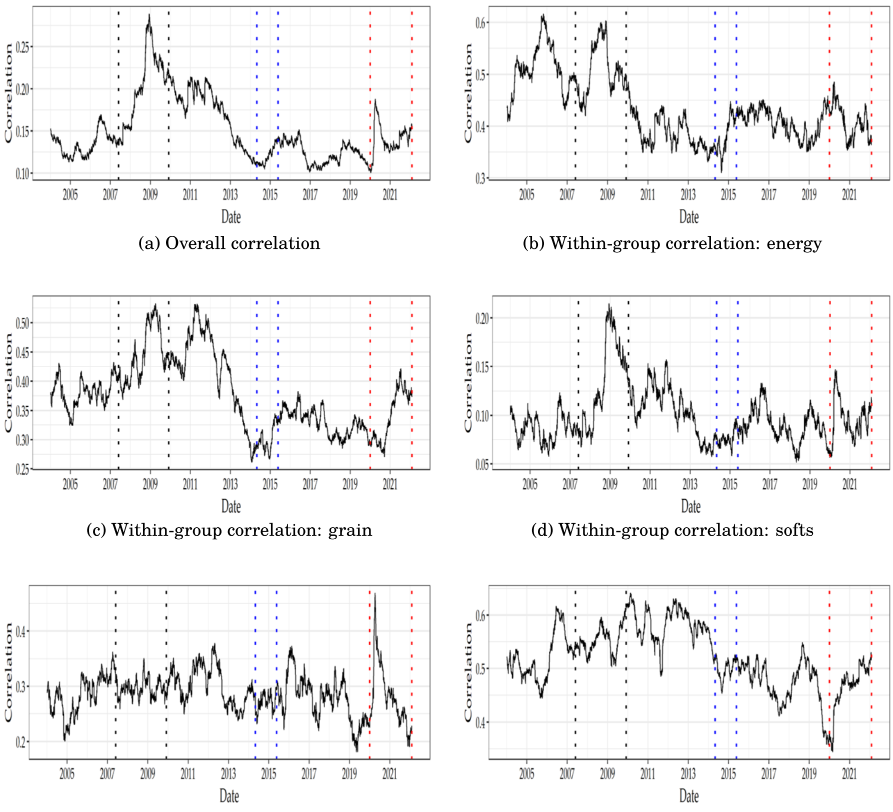
To investigate how dependence structure changes over time, we show the estimated dynamic correlation from DCC skewed t copula in Figs. 1 and 2. Since there are $N(N-1)/2 = 253$ correlations at each period, it is extremely difficult to detect patterns among all correlations. Therefore we cluster commodities by their groups and present them at the group level, reducing the number of within-group and cross-group correlations to 15. We take the average of all dynamic correlations at the same period t to have an overall dynamic dependence measure for the commodities under consideration. For all figures in this paper, we use vertical dashed lines to mark the financial crisis (from June 1, 2007 to December 1, 2009), a sharp decline in crude oil prices (from May 1, 2014 to May 22, 2015), and the COVID-19 pandemic (from January 1, 2020 to January 31, 2022).

Inspection of these results suggests that the overall and the majority of the group-based and cross-group correlations increased during the global financial crisis (2007–2009). Most of them returned to the pre-crisis level as the economy recovered,

Table 1 Parameter estimates for dynamic copula models

	α_Γ	β_Γ	Persistence	v	λ	Likelihood
DCC Copulas						
Normal	0.00707**	0.98730**	0.99437	–	–	32336.86
	(0.00021)	(0.00047)				
Student *t*	0.00697**	0.98787**	0.99484	24.704**	–	33354.64
	(0.00021)	(0.00045)		(0.52397)		
Skewed *t*	0.00693**	0.98791**	0.99484	24.661**	−0.16441**	33456.34
	(0.00021)	(0.00051)		(0.50772)	(0.00599)	
Block-DECO Copulas						
Normal	0.01363**	0.97926**	0.99289	–	–	24500.52
	(0.00056)	(0.00101)				
Student *t*	0.01350**	0.98019**	0.99369	21.160**	–	25799.85
	(0.00056)	(0.00096)		(0.29568)		
Skewed *t*	0.01325**	0.98053**	0.99378	21.102**	−0.13454**	25893.60
	(0.00063)	(0.00110)		(0.00175)	(0.00024)	

This table reports parameter estimates for DCC-based copula models and Block-DECO based copula models with full sample. Standard errors are reported below the estimated parameters. ** denotes the 1% significance level



(a) Overall correlation

(b) Within-group correlation: energy

(c) Within-group correlation: grain

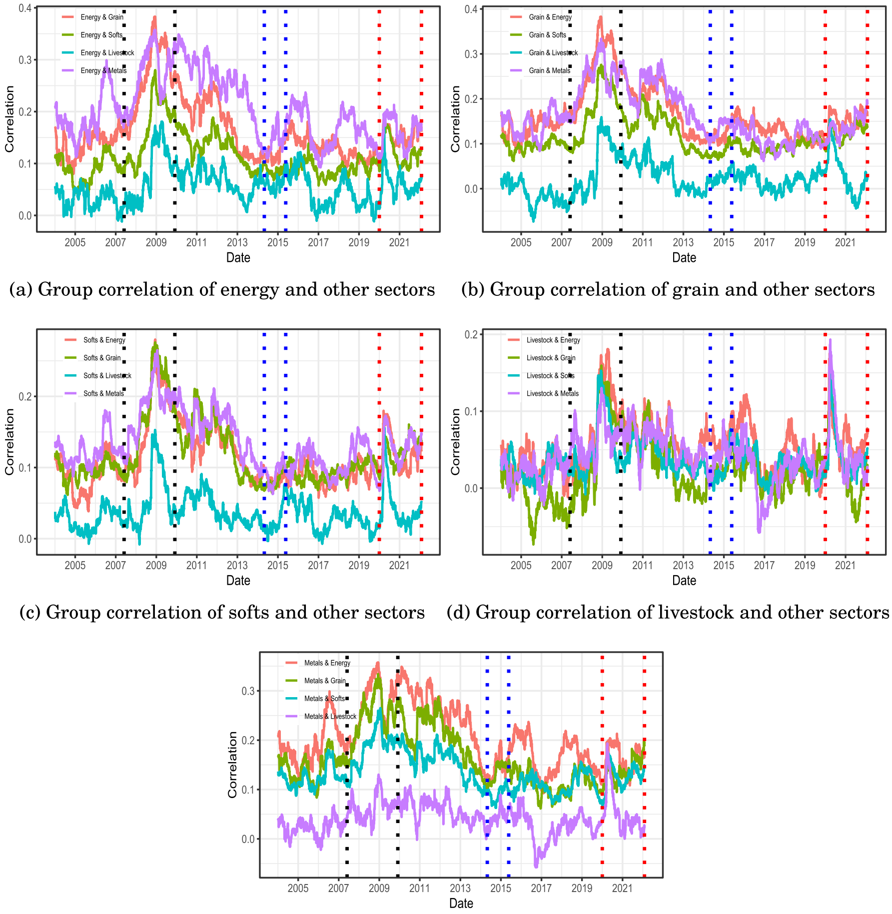
(d) Within-group correlation: softs

(e) Within-group correlation: livestock

(f) Within-group correlation: metal

Fig. 1 Dynamic correlations of skewed t copula: overall and within-group

except for the livestock and metal groups. Since the markets have different expectations for metals during the financial crisis, the correlation change is not obvious for the metal group. As for the livestock sector, the within-group correlation did not vary much during the financial crisis but shot up to an extremely high level at beginning of the outbreak of the COVID-19. During the pandemic, the livestock markets experienced massive disruptions as most countries issued lockdown orders and restaurants shut down, reducing the demand for the livestock sector considerably. As a result, the commodity futures prices for feeder cattle, lean hogs, and live cattle declined simultaneously. Almost all correlations increased at the outset of the pandemic. This is to be expected as this pandemic has affected all sectors of the economy dramatically. It is also observed that when crude oil prices declined sharply, the correlation between the energy and metal sectors increased. Hence, these two sectors are closely related.



(a) Group correlation of energy and other sectors

(b) Group correlation of grain and other sectors

(c) Group correlation of softs and other sectors

(d) Group correlation of livestock and other sectors

(e) Group correlation of metals and other sectors

Fig. 2 Cross-group correlations of dynamic skewed t copula

4.3 Out-of-sampling forecasting

In this section, we conduct a model comparison based on the predictive performance of dynamic copulas during stress periods and various economic cycles in recent years through an out-of-sample (OOS) forecast. In particular, we use the data from June 1, 2007 to December 1, 2009 (financial crisis), from May 1, 2014 to May 22, 2015 (sharp decline in crude oil prices), as well as those from January 1, 2020 to January 31, 2022 (COVID-19 pandemic) with a total of 1418 days as the OOS test set. The reason for choosing these days is that they represent volatile periods in the commodity markets. It is of interest to explore if the proposed dynamic copulas may capture market co-movements in time and prove useful with good prediction performances during these periods. We adopt a fixed rolling window of 800 days to fit the dynamic copulas and predict the one-day ahead copula density (see Appendix B for implementation

Table 2 Out-of-sample model comparisons for dynamic copulas

Correlation Copula	DCC Normal	Student t	Skewed t	Block DECO Normal	Student t	Skewed t
DCC						
Normal	–					
Student t	4.16	–				
Skewed t	4.09	2.44	–			
Block DECO						
Normal	−17.33	−17.82	−16.95	–		
Student t	−10.59	−17.81	−18.92	4.19	–	
Skewed t	−9.09	−15.30	−17.78	3.98	**1.82**	–

This table presents t-statistics from out-of-sample pair-wise comparisons of the log-likelihood values for six dynamic copula models. A positive value suggests the model to the left is better than the model above, and a negative value suggests the opposite. t values which fall between [−1.96, 1.96] are in bold

details of the OOS-based model comparison). Following Diebold and Mariano (1995), Giacomini and White (2006) and Patton (2013), we compare two competing dynamic copulas conditional on their estimated parameters:

$$\begin{aligned} H_0 &: \; E[\log c_1(\hat{\eta}_t; \hat{\theta}_{1t}) - \log c_2(\hat{\eta}_t; \hat{\theta}_{2t})] = 0, \\ \text{vs } H_1 &: \; E[\log c_1(\hat{\eta}_t; \hat{\theta}_{1t}) - \log c_2(\hat{\eta}_t; \hat{\theta}_{2t})] > 0, \\ H_2 &: \; E[\log c_1(\hat{\eta}_t; \hat{\theta}_{1t}) - \log c_2(\hat{\eta}_t; \hat{\theta}_{2t})] < 0, \end{aligned} \tag{14}$$

where $\hat{\eta}_t$ is the predictive CDF at period t, and $\hat{\theta}_{it}$ for $i = 1, 2$ is the estimated copula parameter from the fixed-length rolling window. The asymptotic framework of the predictive log-likelihood test is based on Giacomini and White (2006), which does not require any adjustments for the estimated parameters of the competing copulas and the limiting distribution of the test statistic is $N(0, 1)$. Giacomini and White (2006) show that as the differences in log-likelihoods is potentially heteroskedastic and serially correlated, we need to first estimate the heteroskedasticity and autocorrelation consistent (HAC) covariance for the predictive test. Diebold (2015) suggests this test statistic can be simply computed by regressing the differences in log-likelihoods on an intercept using HAC robust standard error. If t values of the intercept estimator are greater than 1.96 or less than −1.96, we may argue there is a significant difference in predictive ability between competing models under consideration. We report the model comparison results for all dynamic copulas in Table 2.

In Table 2, 14 out of 15 model comparisons imply the significant better performance of one dynamic copula against another in terms of predictive copula density. The results are consistent with what we have found in Sect. 4.2 based on the composite likelihood, where all DCC copulas have much higher composite likelihood than the Block DECO copulas, and the dynamic skewed t copula dominates the other two among all three copulas under the same correlation specification. The only case without a significant difference is the comparison between the Block DECO skewed t copula and the Block

DECO student t copula. With a test statistic of 1.82, it is suggested that the former performs only marginally better than the latter.

5 Economic implications

We apply our models to two important tasks of risk management in the commodity markets. In Sect. 5.1, we explore the benefits of portfolio diversification based on our estimate of DCC skewed t copula and a dynamic measure derived from expected shortfall by Christoffersen et al (2012). In Sect. 5.2, we measure the lower tail dependence of dynamic copula to investigate how the systemic risk of commodity markets varies over time during the volatile markets.

5.1 Dynamic diversification benefit

The dynamic measure of diversification benefit is closely related to the expected shortfall, which is defined as:

$$ES_t^q(y_{i,t}) = -E[y_{i,t}|y_{i,t} \leq \eta_{i,t}^{-1}(q)], \quad (15)$$

where $\eta_{i,t}^{-1}(q)$ is the quantile function of returns i at period t, and q is a probability that we set as 5% in the following analysis. As Artzner et al (1999) point out that the expected shortfall has the sub-additivity property such that

$$ES_t^q(w_t) \leq \sum_{i=1}^{N} w_{i,t} ES_t^q(y_{i,t}), \text{ for all } w_t, \quad (16)$$

where $ES_t^q(w_t)$ is the expected shortfall of a portfolio with an N-dimensional weight vector w_t.

For a given w_t, the upper and lower bounds for the expected shortfall are defined as $\overline{ES}_t^q(w_t) \equiv \sum_{i=1}^{N} w_{i,t} ES_t^q(y_{i,t})$, and $\underline{ES}_t^q(w_t) \equiv -\eta_t^{-1}(w_t, q)$, respectively. The upper bound on the portfolio's expected shortfall reflects the weighted average of returns' individual expected shortfalls, while the lower bound represents an extreme case such that the portfolio return is never below the q^{th} quantile of a portfolio distribution with weight w_t, which is denoted by $\eta_t^{-1}(w_t, q)$.

Christoffersen et al (2012) propose a dynamic measure of diversification benefit as:

$$CDB_t(w_t, q) \equiv \frac{\overline{ES}_t^q(w_t) - ES_t^q(w_t)}{\overline{ES}_t^q(w_t) - \underline{ES}_t^q(w_t)}, \quad (17)$$

which takes values on the [0, 1] interval and is not conditional on the level of returns $y_{i,t}$. Notice also that this measure is an increasing function in the degree of diversification benefit. In our analysis we maximize $CDB_t(w_t, q)$ by choosing optimal w_t

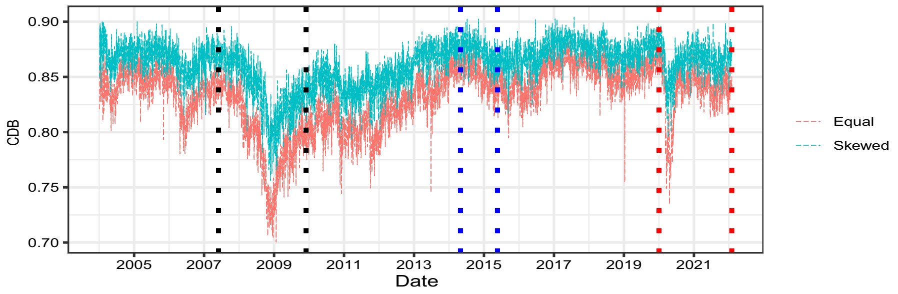


Fig. 3 Conditional diversification benefits with skewed t copula

with constraints $\sum_{i=1}^{N} w_{i,t} = 1$ and $w_{i,t} \geq 0$ for all i to mimic dynamic optimization process for portfolio re-balancing. Since the expected shortfall $ES_t^q(w_t)$ is not known in closed form based on dynamic copulas, we follow Christoffersen et al (2012) to construct $CDB_t(w_t, q)$ as follows:

Step 1 Simulate 2000 returns for each commodity futures at period t using the estimated univariate ARIMA-GARCH model and DCC skewed t copula with conditional returns, volatility and correlation that are available at period $t-1$.

Step 2 Maximize $CDB_t(w_t, q)$ in (17) over w_t with constraints $\sum_{i=1}^{N} w_{i,t} = 1$ and $w_{i,t} \geq 0$ for all i using the simulated returns from step 1.

Step 3 Save the optimal weights w_t and corresponding $CDB_t(w_t, q)$ and repeat the simulation and optimization process in step 1 and 2 for period $t+1$.

Figure 3 reports the CDB results from our estimates in Sect. 4.2. To compare the diversification benefits between the DCC skewed t copula based portfolio and equal-weighted portfolio, we denote the former by "Skewed" and the latter by "Equal" in subsequent discussions. Evidently the CDB from both models are quite close before 2006. After 2006 $\text{CDB}_t^{\text{Equal}}$ declined sharply; $\text{CDB}_t^{\text{Skewed}}$ also declined but to a less extent. These decreases seem to be temporary, and both benefit measures have gradually risen to their previous levels before 2008. During the 2008–2009 financial crisis, diversification benefits have dropped markedly and reached the bottom at the beginning of 2009. Note also that the discrepancy between both CDBs are also greatest at this time. From 2009, CDB once again rose mildly and reached the pre-crisis level in 2014. Then, both CDBs moved downward since mid-2014, when global energy prices declined. Starting from 2020, both benefit measures dropped again, which enlarged the discrepancy again. The results indicate that the skewed t copula is more apt to capture multivariate asymmetry, nonlinear dependence, and higher-order moments during the distress periods when equity returns tend to crash together.

5.2 Dynamic tail dependence

One advantage of skewed t copula over other elliptical copulas is that it allows for nonzero dependence and asymmetric dependence in the upper and lower tails, while

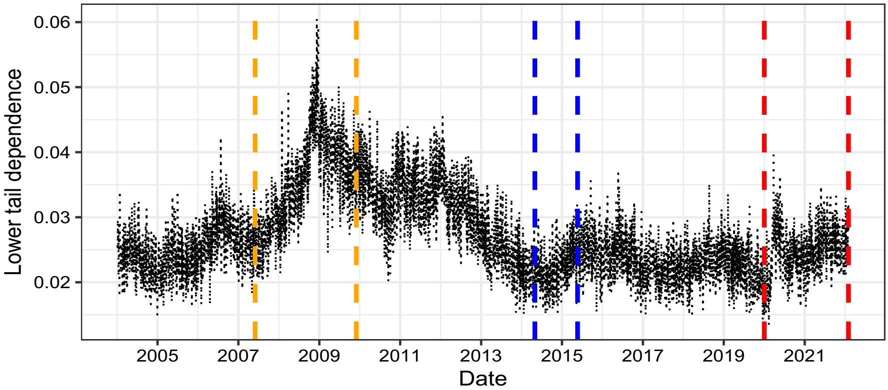


Fig. 4 Dynamic average bivariate tail dependence for skewed t copula

normal copula assumes zero tail dependence and student t copula implies symmetry on both tails. Since the equity returns are usually more connected and have higher co-movements during downturns, we are especially interested in the lower tail dependence to see whether this still holds for commodities. Lower tail dependence is defined by the probability limit:

$$\tau_{i,j,t}^{L} = \lim_{\xi \to 0} \Pr[\eta_{i,t} \leq \xi | \eta_{j,t} \leq \xi] = \lim_{\xi \to 0} \frac{C_t(\xi, \xi)}{\xi}, \tag{18}$$

where ξ is the tail probability. We note that the measure of lower tail dependence above is only bivariate and difficult to generalize to higher dimensions. To display the dynamics of tail dependency among commodity futures, we take the average of bivariate tail dependence across all pairs of commodities. As the skewed t copula does not have an analytical solution for the lower tail dependence, we use numerical integration with $\xi = 0.001$ to approximate the lower tail dependence.

Figure 4 shows the average dynamic lower tail dependence of all pairs of commodity futures returns, which increased dramatically during the 2008–2009 financial crisis and peaked at the beginning of 2009. However, it declined rapidly from the peak but mildly rose again during the 2011–2012 European debt crisis, followed by another gradual decline. When the global energy prices dropped during 2014–15, the tail dependence shifted upward slightly and then remained at the pre-crisis level until the outset of COVID-19. The reverse of this pattern is consistent with what we have seen in Sect. 5.1, where CDB_t went through a similar process during the crisis periods. It appears that the conditional diversification benefit and dynamic tail dependence can be complementary measures of risk management.

In Christoffersen et al. (2012), the dynamic measure of average lower tail dependence between 29 emerging and developed markets during 1989–2009 has trended upward, rising from near 0 to around 0.18. This is considerably higher than our measure for commodity markets, which reached peaks at 0.06 during the financial crisis. This comparison suggests that though commodity markets have shown strong evidence of increasing dependence, their dependence level remains significantly lower than that of the equity markets.

6 Concluding remarks

We develop copula-based flexible multivariate dynamic models that allow for time-varying nonlinear and asymmetric dependence. They are employed to model the dynamic dependence structure of a large collection of daily commodity returns from various sectors during the past two decades. Our flexible, dynamic copula models reveal strong evidence of multivariate asymmetry, fat tails, and time-varying co-movements in the commodity markets. We show that copula correlations between various commodities increased substantially during the 2008–2009 financial crisis, and most correlations returned to the pre-crisis level as the economy's growth improved. They rose instantaneously at the outset of the COVID-19 pandemic, which impacted all sectors of the global economy. Estimation results from the dynamic copulas provide some useful insights. First, although conditional diversification benefits for a portfolio of commodity futures have declined dramatically during the crisis period, DCC skewed t copula-based optimal portfolio has dominated the equal-weighted portfolio by a large margin in the bearish market. Second, the tail dependence of commodity futures, a useful measure for systemic risk, had trended upward when diversification benefit reached the bottom. These results suggest that the dynamic copula model is able to inform risk management, facilitating the construction of a well-diversified portfolio with a large number of commodity futures.

It may prove interesting to investigate and extend the models we use for other topics in the future. First, understanding how co-movements of commodity markets are affected by macroeconomy appears to be difficult, as macroeconomic variables are usually released at a relatively low frequency (monthly or quarterly) compared to asset returns. To handle this problem, Ghysels et al (2004) propose a mixed-data sampling regression model (MIDAS) that has attracted growing interest in recent years. Combining dynamic copulas and MIDAS seems to be a promising approach to model interactions between macroeconomy and commodity markets without sacrificing much information in the data. Second, Diebold and Yilmaz (2014) develop a variety of connectedness measures of equity returns based on network topology and VAR models. One might want to know how and if the empirical results of dynamic copulas and connectedness measures could be integrated to produce useful insights for risk management. To the best of our knowledge, there is no unifying framework to answer this important question. We may consider extending the present study in these directions in our future research.

Data Availability The data that support the findings of this study are available from the corresponding author upon request.

Declarations

Conflict of interest The authors have no conflict of interest.

Ethical approval This article complies with all the ethical standards.

Appendix

We provide in this Appendix some technical details pertinent to the formulation and implementation of the dynamic copula models outlined in the main text.

A The skewed *t* distribution

As discussed in Sect. 3.3, standardization of copula shocks is required in the estimation of dynamic conditional correlation. In this section, we briefly introduce the basics of skewed *t* copula which are used to drive dynamic correlations in our empirical analysis. Demarta and McNeil (2005) show that the skewed *t* distribution has the following stochastic representation:

$$X = \sqrt{W} Z + \lambda W \tag{A.1}$$

where w is an inverse gamma variable such that $W \sim IG(v/2, v/2)$, Z is a normal variable such that $Z \sim N(0_N, \Gamma)$, and λ is the asymmetry parameter. Equation (A.1) suggests skewed *t* distribution has a normal mixture structure which implies the expected value of X is:

$$E(X) = E(E[X|W]) = E(W)\lambda = \frac{v}{v-2}\lambda \tag{A.2}$$

and the covariance matrix is:

$$\begin{aligned} Cov(X) &= E(Var(X|W)) + Var(E(X|W)) \\ &= \frac{v}{v-2}\Gamma + \frac{2v^2\lambda^2}{(v-2)^2(v-4)} \end{aligned} \tag{A.3}$$

The expected value and covariance matrix show how the skewed *t* distribution is linked with the copula correlation Γ. Standardization of student *t* copula shocks can be implemented using these moments with $\lambda = 0$.

B Out-of-sample model comparison

Out-of-sample period spans from June 1, 2007 to December 1, 2009 (financial crisis), from May 1, 2014 to May 22, 2015 (sharp decline in crude oil prices) as well as those from January 1, 2020 to January 31, 2022 (COVID-19 pandemic) which includes 1418 days as the test set. We use a fixed rolling window of 800 days to fit the dynamic copulas and forecast one-day ahead copula density. To reduce the high computational burden in the composite likelihood estimation of dynamic copulas, we split the test set every six months into twelve separate periods and do the one step ahead out-of-sample prediction for each copula model twelve times. The starting dates for each separate test set are June 1, 2007, December 4, 2007, June 3, 2008, December 2, 2008, June 2, 2009, May 1, 2014, November 4, 2014, January 2, 2020, July 1, 2020, January 4, 2021, July 1, 2021, and January 3, 2022. We re-estimate the dynamic copula model using 800 days before each date to predict a copula density for the next six months. To reliably forecast one-day ahead volatility and recover copula shocks, we re-estimate the univariate ARIMA-GARCH model for each day in the test set. Denoted by t one of the twelve dates mentioned above, the detailed procedure of OOS model comparison is summarized below:

Step 1 Estimate the univariate ARIMA-GARCH model for each commodity returns and dynamic copulas using data in period $[t-801, t\text{-}1]$ as the training set and save all estimates;

Step 2 Predict one-day ahead volatility $\sigma_{i,t}$ and recover the error $\epsilon_{i,t}$ using return $y_{i,t}$ for each i. Compute the cumulative distribution $\eta_{i,t}$ and save the copula density from the estimated dynamic copula;

Step 3 At the end of date t re-estimate the univariate ARIMA–GARCH model using data in period $[t-800, t]$, and repeat Step 2;

Step 4 Repeat Steps 2 and 3 until we approach the next copula re-estimation date on which we begin from Step 1.

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