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Residual-based cointegration and non-cointegration tests for cointegrating polynomial regressions

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Abstract
Cointegrating polynomial regressions (CPRs), i.e., regressions that include deterministic terms, integrated processes and powers of integrated processes as explanatory variables and stationary errors, have become prominent in several fields of applications, e.g., in the analysis of environmental Kuznets curves. A key issue, as always in cointegration analysis, is testing for the presence or absence of a cointegrating relationship. This paper discusses two complementary tests: one with the null hypothesis of cointegration and one with the null hypothesis of the absence of cointegration. It is shown that (inter alia) for the empirically most relevant case, in which only one of the integrated regressors occurs as regressor also with higher powers, critical values can be simulated and are provided for a variety of specifications. Finally, the usage of the tests is illustrated for the environmental Kuznets curve for carbon and sulfur dioxide emissions. The illustration also investigates the sensitivity of the test decisions with respect to kernel and bandwidth choices, sample size and data vintage.

Keywords Cointegrating polynomial regression · Cointegration · Unit root · Testing · Environmental Kuznets curve · Material Kuznets curve · Exchange rate target-zone

JEL Classification C12 · C13 · Q20

1 Introduction

Recent years have seen growing theoretical and empirical interest in estimating and testing for nonlinear cointegrating relationships. A very voluminous body of applied

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literature in this respect has been and continues to be generated within the environmental economics literature. The so-called environmental Kuznets curve (EKC) hypothesis, which postulates an inverted U-shaped relationship between the level of economic development, typically measured by (the logarithm of) GDP per capita, and (the logarithm of) pollution or emissions per capita is one of the most widely studied empirical relationships in environmental economics. Early survey papers like Stern (2004) and Yandle et al. (2004) already count more than one-hundred refereed publications. The term EKC refers by analogy to the inverted U-shaped relationship between the level of economic development and the degree of income inequality postulated by Kuznets (1955) in his 1954 presidential address to the American Economic Association. Estimation of an EKC entails, in its simplest parametric form, a regression of emissions on GDP and its square to allow for a U or inverted U shape. Given that the logarithm of GDP per capita is often found to be unit root non-stationary, this is thus a regression where as regressors a unit root process and its square are present.[1] In doing so, the empirical EKC literature in large parts fails to acknowledge the fact that if the logarithm of GDP per capita is a unit root process, the square of the logarithm of GDP per capita cannot be a unit root process and continues to use "standard" unit root and cointegration techniques. This means that a large part of the empirical EKC literature (see, e.g., Friedl and Getzner 2003, Perman and Stern 2003 or Galeotti et al. 2009 from a large and still growing list of such contributions) simply treats GDP and its square as two integrated processes and uses standard unit root and cointegration test and estimation methods.[2] This is, notwithstanding its popularity, a problematic practice since tailor-made methods designed for regressions involving integrated processes and nonlinear transformations thereof are available.[3]

Wagner and Hong (2016) develop estimation and inference theory for the special case of *nonlinear* cointegrating relationships including deterministic components as well as unit root processes and their powers as explanatory variables. They refer to such relationships as cointegrating polynomial regressions (CPRs). More precisely, Wagner and Hong (2016) extend the FM-OLS estimator of Phillips and Hansen (1990) from cointegrating linear to cointegrating polynomial regressions.[4] In addition to hypothesis

[1] The seminal empirical study of Grossman and Krueger (1993) uses a third-order polynomial in GDP, whereas the more popular quadratic specification has been initiated by Holtz-Eakin and Selden (1995).

[2] Another literature where this type of relationship occurs is the so-called intensity-of-use or material Kuznets curve (MKC) literature that investigates the potentially inverted U-shaped relationship between GDP and energy or metals use (see, e.g., Labson and Crompton 1993 or Grabarczyk et al. 2018). Almost by definition, the study of inequality, put to the center of attention again by Piketty (2014), when performed with time series data can make use of methods for cointegrating polynomial regressions. The same holds true for the exchange rate target-zone literature, see, e.g., Darvas (2008) or Svensson (1992).

[3] Bradford et al. (2005) and Wagner (2008, 2015) contain critical assessments of this practice. Stypka et al. (2019) derive and discuss the asymptotic properties of using methods, in particular FM-OLS, developed for cointegrating linear regressions in a cointegrating polynomial regression setting. The results of that paper show that "standard" cointegration and non-cointegration tests lead to asymptotically invalid inference, not least because of wrong critical values being used. This problem is overcome with the tests and *proper* critical values provided in this paper.

[4] Also, e.g., Chang et al. (2001), provide extensions of FM-OLS to nonlinear cointegration settings. While these authors consider more general functions than Wagner and Hong (2016), they assume serially uncorrelated errors. Wagner and Hong (2016) allow for serially correlated errors that are allowed to be dynamically correlated with the regressors, thereby considering the same setting as commonly used in linear cointegra-

testing with respect to the estimated parameters, they furthermore also investigate specification tests based on augmented or auxiliary regressions in detail. One important aspect that is not treated exhaustively in that paper is testing for cointegration and for the absence of cointegration, respectively. To be precise, only testing the null hypothesis of cointegration is considered in Wagner and Hong (2016, Propositions 5 and 6), whereas testing the null hypothesis of no cointegration is not considered. Furthermore, Wagner and Hong (2016) do not provide critical values. The present paper considers tests for both null hypotheses in detail and also provides critical values. We (re-)consider, with this test already discussed in Wagner and Hong (2016), in detail an extension of the Shin (1994) cointegration test, itself an extension of the KPSS stationarity test of Kwiatkowski et al. (1992), from cointegrating linear to cointegrating polynomial regressions. Furthermore, we consider an extension of a variance ratio test for the absence of cointegration of Phillips and Ouliaris (1990). The null hypothesis of the latter test is that the relationship is not cointegrating but spurious. It turns out that in the general case of several integrated regressors entering the CPR with powers higher than one, neither of the two tests has a nuisance-parameter-free limiting distribution that could be tabulated. This has already been observed for the KPSS-type test in Wagner and Hong (2016). Consequently, for the Shin- or KPSS-type test Wagner and Hong (2016) and, based on D-OLS estimation rather than FM-OLS estimation, Choi and Saikkonen (2010) consider sub-sampling versions of the KPSS-type test statistic that results in nuisance-parameter-free limiting null distributions, at the expense of a drastically reduced sample size due to sub-sampling. For the relatively small samples available for typical macroeconomic applications, a sub-sampling based test is potentially only of limited practical value. For the extension of the test of Phillips and Ouliaris (1990), this sub-sampling approach does not lead to asymptotically nuisance-parameter-free limiting distribution.

However, using the terminology of Vogelsang and Wagner (2016), in case of a *full design* CPR relationship it can be shown that the limiting distributions of both tests can be tabulated. Full design refers to a situation in which the limiting distribution of the FM-OLS estimator—and a fortiori the limiting partial sum processes of the residuals—can be expressed as a functional of standard Brownian motions rather than of non-standard Brownian motions. The empirically most relevant case, with only one of the integrated processes entering the CPR with powers higher than one is straightforwardly seen to be of full design. "Appendix B" of the paper provides tables with critical values for this case, for up to four integrated regressors, up to power four of the one integrated regressor entering with higher powers for the usual specifications of the deterministic component (no deterministic component, intercept only, intercept and linear trend). Thus, the results of the paper allow to perform non-/cointegration testing in typical applications like EKCs, MKCs or exchange rate target-zone relationships.[5]

tion analysis.

A basic observation that is not always sufficiently acknowledged is the fact that nonlinear transformations change the properties of integrated processes *fundamentally*, depending upon the type of transformation. It is a basic mathematical property that only linear transformations "commute" with summation. Wagner (2012) or Stypka and Wagner (2019) exemplify this issue for polynomial transformations.

[5] For further discussion concerning full design and FM-OLS type estimation for cointegrating multivariate polynomial regressions (CMPRs), that potentially include arbitrary cross-products of powers of integrated

The tests discussed are illustrated with annual carbon dioxide (CO_2) and sulfur dioxide (SO_2) emissions data for 18 early industrialized countries over the period 1870–2016, with New Zealand data starting only in 1878. For CO_2 emissions an EKC relationship is found for six of the 18 countries, Austria, Belgium, Finland, Germany, Switzerland and the UK. For all countries but Germany a quadratic specification suffices, whereas for Germany a cubic specification is required. For SO_2 emissions an EKC relationship is found for none of the countries. In this respect note that the logarithm of GDP per capita and its powers are the only regressors included in the *simple "reduced-form"* equation and the absence of an EKC for SO_2 emissions could partly be driven by important explanatory variables and legislative changes not entirely captured by GDP. A more structural investigation of the EKC relationship is beyond the scope of this paper that is merely devoted to discussing the (non-)cointegration tests rather than to a fully fledged empirical analysis. A second observation that emerges from the illustration is that the usage of appropriate (non-)cointegration tests, designed for testing in a CPR context, reduces the evidence for a cointegrating EKC relationship compared to using (non-)cointegration tests designed for linear cointegrating relationships. By means of robustness checks we also illustrate that the results obtained with the two nonparametric tests are quite insensitive to kernel and bandwidth choices, in particular with respect to the evidence for the prevalence of an EKC. Changing the sample size and, in particular, using different data vintages lead to more variation in the test results.

The paper is organized as follows: The following section discusses and presents the tests, Sect. 3 briefly illustrates the tests with CO_2 and SO_2 emissions data, and Sect. 4 briefly summarizes and concludes. Two appendices follow the main text: "Appendix A" contains the proofs, and "Appendix B" contains the tables with the critical values. "Supplementary Appendix C" contains the detailed test results for the robustness checks.

2 The tests

Following Wagner and Hong (2016), we consider a cointegrating polynomial regression (CPR), i.e., an equation including a constant and polynomial time trends up to power q, integer powers of integrated regressors x_{jt}, $j = 1, \ldots, m$ up to degrees p_j and a stationary error term u_t:

$$y_t = D_t'\theta_D + \sum_{j=1}^{m} X_{jt}'\theta_{X_j} + u_t \quad , \quad \text{for } t = 1, \ldots, T, \tag{1}$$

with $D_t := [1, t, t^2, \ldots, t^q]'$, $X_{jt} := [x_{jt}, x_{jt}^2, \ldots, x_{jt}^{p_j}]'$ and the parameter vectors $\theta_D \in \mathbb{R}^{q+1}$ and $\theta_{X_j} \in \mathbb{R}^{p_j}$.

The following assumption on the regressors and the errors is put in place:

processes as explanatory variables, see Stypka and Wagner (2020). Vogelsang and Wagner (2016) effectively consider IM-OLS estimation for CMPRs.

Assumption 1 The process $\{\eta_t\}_{t\in\mathbb{Z}} := \left\{[u_t, v_t']'\right\}_{t\in\mathbb{Z}}$ with $\{v_t\}_{t\in\mathbb{Z}} := \{\Delta x_t\}_{t\in\mathbb{Z}}$, with $x_t := [x_{1t}, \ldots, x_{mt}]'$, is generated as:

$$\eta_t := C(L)\eta_t^0 = \textstyle\sum_{j=0}^{\infty} C_j \eta_{t-j}^0,$$

with $\sum_{j=0}^{\infty} j||C_j|| < \infty$ and $\det(C(1)) \neq 0$. Furthermore, we assume that the process $\left\{\eta_t^0\right\}_{t\in\mathbb{Z}}$ is a stationary and ergodic martingale difference sequence with natural filtration $\mathcal{F}_t = \sigma\left(\left\{\eta_s^0\right\}_{-\infty}^t\right)$, positive definite covariance matrix $\Sigma_{\eta^0\eta^0}$ and $\sup_{t\geq 1} \mathbb{E}(\|\eta_t^0\|^r|\mathcal{F}_{t-1}) < \infty$ *a.s.* for some $r > 4$.

Assumption 1 is analogous to the corresponding assumption in Wagner and Hong (2016). Quite similar assumptions have been used in several places in the literature, e.g., Chang et al. (2001), Park and Phillips (1999, 2001) and Hong and Phillips (2010).[6] The key result that this type of assumption is required for is an invariance principle for terms of the form $T^{-\frac{k+1}{2}} \sum_{t=1}^T x_{jt}^k u_t$. Alternative sets of assumptions that could be used instead are formulated in Ibragimov and Phillips (2008) in a martingale framework or in de Jong (2002, Assumptions 1 and 2) who combines near-epoch dependence and appropriate moment assumptions. For the present paper any such set of assumptions leading to the required invariance principle can be put in place. The assumption $\det(C(1)) \neq 0$ together with positive definiteness of $\Sigma_{\eta^0\eta^0}$ implies that $\{x_t\}$ is an integrated but not cointegrated process. Assumption 1 implies an invariance principle for $\{\eta_t\}_{t\in\mathbb{Z}}$:

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{\lfloor Tr \rfloor} \eta_t \Rightarrow B(r) = \begin{bmatrix} B_u(r) \\ B_v(r) \end{bmatrix}, \tag{2}$$

with $B_v(r) := [B_{v_1}(r), \ldots, B_{v_m}(r)]'$. It holds that $B(r) = \Omega^{1/2}W(r)$ with the long-run covariance matrix $\Omega := \sum_{h=-\infty}^{\infty} \mathbb{E}\left(\eta_0^0\eta_h^{0\prime}\right)$ and where $W(r)$ is an $m+1$-dimensional vector of standard Brownian motions. For later usage define the one-sided long-run covariance $\Delta := \sum_{h=0}^{\infty} \mathbb{E}\left(\eta_0^0\eta_h^{0\prime}\right)$ and partition both matrices according to the partitioning of η_t:

$$\Omega = \begin{bmatrix} \Omega_{uu} & \Omega_{uv} \\ \Omega_{vu} & \Omega_{vv} \end{bmatrix}, \quad \Delta = \begin{bmatrix} \Delta_{uu} & \Delta_{uv} \\ \Delta_{vu} & \Delta_{vv} \end{bmatrix}.$$

Similarly to the linear case (compare Phillips and Hansen 1990), the OLS estimator of the parameter vector in (1) is consistent but its limiting distribution is contaminated by second-order bias terms, the presence of which renders standard asymptotic inference based on the OLS estimator invalid. Consequently, Wagner and Hong (2016, Proposition 1) extend the FM-OLS estimation principle from the cointegrating linear to the cointegrating polynomial case. The FM-OLS estimator is based

[6] The key difference between the assumption used here—and in Wagner and Hong (2016)—and those in the other papers mentioned is that the other papers assume that the errors $\{u_t\}$ are serially uncorrelated, as mentioned already in Footnote 4.

on two modifications of the OLS estimator. First, the dependent variable y_t is replaced by $y_t^+ := y_t - \Delta x_t' \hat{\Omega}_{vv}^{-1}\hat{\Omega}_{vu}$, $y^+ := [y_1^+, \ldots, y_T^+]'$ and, second, a correction term is subtracted. The modified dependent variable y_t^+ is exactly as in Phillips and Hansen (1990), but the correction terms are different in the CPR case and given by:

$$M^* := \begin{bmatrix} M_1^* \\ \vdots \\ M_m^* \end{bmatrix}, \quad M_j^* := \hat{\Delta}_{v_j u}^+ \begin{bmatrix} T \\ 2\sum x_{jt} \\ \vdots \\ p_j \sum x_{jt}^{p_j - 1} \end{bmatrix}. \tag{3}$$

Throughout, we rely upon consistent estimators of the required long-run variances $\hat{\Omega}_{vv}$, $\hat{\Omega}_{vu}$, $\hat{\Delta}_{v_j u}$ and $\hat{\Delta}_{v_j u}^+ := \hat{\Delta}_{v_j u} - \hat{\Omega}_{uv}\hat{\Omega}_{vv}^{-1}\hat{\Delta}_{v v_j}$. In this respect, OLS consistency is important since it allows for consistent long-run variance estimation based on the OLS residuals (it is straightforward to verify that the necessary assumptions of Jansson 2002, Corollary 3 are fulfilled).

To (re-)state the asymptotic distribution of the FM-OLS estimator developed in Wagner and Hong 2016, Proposition 1), we need to define a few more quantities: $D(r) := [1, \ldots, r^q]'$, $\mathbf{B}_{v_j}(r) := [B_{v_j}(r), \ldots, B_{v_j}^{p_j}(r)]'$, $\mathbf{B}_v(r) := [\mathbf{B}_{v_1}(r)', \ldots, \mathbf{B}_{v_m}(r)']'$, $D := [D_1, \ldots, D_T]'$, $X_t := [X_{1t}', \ldots, X_{mt}']'$, $X := [X_1, \ldots, X_T]'$, $Z := [D, \ X]$, $G_D := \text{diag}(T^{-1/2}, \ldots, T^{-(q+1/2)})$, $G_{X_j} := \text{diag}(T^{-1}, \ldots, T^{-\frac{p_j+1}{2}})$, $G_X := \text{diag}(G_{X_1}, \ldots, G_{X_m})$, $G := \text{diag}(G_D, G_X)$, and $\theta := [\theta_D', \theta_X']'$ with $\theta_X := [\theta_{X_1}', \ldots, \theta_{X_m}']'$.

Proposition 1 *Let $\{y_t\}$ be generated by (1) with the regressors $\{x_t\}$ and errors $\{u_t\}$ satisfying Assumption 1. Define the FM-OLS estimator of θ as:*

$$\hat{\theta}^+ := (Z'Z)^{-1}\left(Z'y^+ - A^*\right),$$

with:

$$A^* := \begin{bmatrix} 0_{(q+1)\times 1} \\ M^* \end{bmatrix}$$

and M^* as given in (3) with consistent estimators of the required long-run (co)variances. Then, $\hat{\theta}^+$ is consistent and has a zero mean Gaussian mixture asymptotic distribution given by:

$$G^{-1}\left(\hat{\theta}^+ - \theta\right) \Rightarrow \left(\int_0^1 J(r)J(r)'dr\right)^{-1} \int_0^1 J(r)dB_{u\cdot v}(r), \tag{4}$$

with $J(r) := \left[D(r)', \ \ \mathbf{B}_v(r)'\right]'$ and $B_{u\cdot v}(r) := B_u(r) - B_v(r)'\Omega_{vv}^{-1}\Omega_{vu}$.

The limiting distribution given in (4) provides the basis for asymptotic standard, i.e., standard normal or chi-squared, inference on the coefficients *in case* the error term $\{u_t\}$ is stationary, i.e., in case of prevalence of a CPR relationship. Thus, testing for the presence of a CPR relationship is of prime importance and as always in cointegration testing two null hypotheses are conceivable: the null hypothesis of cointegration, i.e., the null hypothesis of stationarity of $\{u_t\}$, or the null hypothesis of no cointegration, i.e., testing whether (1) is in fact a spurious regression. The first null hypothesis can be tested by an extension of the Shin (1994) test, which itself is an extension of the stationarity test of Kwiatkowski et al. (1992), from cointegrating linear to cointegrating polynomial regressions. Since, of course, the errors u_t are not observed, the test statistic has to be based on observable residuals, in particular the FM-OLS residuals, $\hat{u}_t^+$ say, can be used. The KPSS-type test statistic for the null hypothesis of cointegration is defined as:

$$CT := \frac{1}{T\hat{\Omega}_{u\cdot v}} \sum_{t=1}^{T} \left(\frac{1}{\sqrt{T}} \sum_{j=1}^{t} \hat{u}_j^+ \right)^2, \tag{5}$$

with $\hat{\Omega}_{u\cdot v}$ a consistent estimator of the long-run variance $\Omega_{u\cdot v} := \Omega_{uu} - \Omega_{uv}\Omega_{vv}^{-1}\Omega_{vu}$ of $\{\hat{u}_t^+\}$.

For testing the null hypothesis of no cointegration, we consider an extension of a variance ratio test statistic of Phillips and Ouliaris (1990), their $\hat{P}_u$ statistic, and we continue to use this name for the extension. Denoting the OLS residuals of (1) as $\hat{u}_t$, the denominator of the test statistic is the properly scaled estimated "variance" of $\{\hat{u}_t\}$, which is an integrated process under the null hypothesis. Under the null hypothesis of a spurious regression, with $\{y_t\}$ being an I(1) process, the stacked vector $\{m_t\} := \{[y_t, x_t']'\}$ is a non-cointegrated I(1) vector and for later use we denote its first difference as $\{\Delta m_t\} := \{[w_t, v_t']'\}$. The second element of $\hat{P}_u$ is a long-run variance estimate based on the OLS residuals of a vector autoregression (VAR) of order one for $\{m_t\}$, taking into account the deterministic components D_t considered:

$$m_t = \Pi_0 D_t + \Pi_1 m_{t-1} + \xi_t. \tag{6}$$

The required conditional long-run variance is computed using VAR(1) OLS residuals from (6) rather than from the vector of first differences Δm_t. Both versions lead to the same asymptotic behavior of the test statistic under the null hypothesis, but the asymptotic behavior differs under the alternative, compare Phillips and Ouliaris (1990, Theorems 5.2 and 5.3). Denoting the long-run variance estimated from $\{\hat{\xi}_t\}$ as $\tilde{\Omega} = \begin{bmatrix} \tilde{\Omega}_{ww} & \tilde{\Omega}_{wv} \\ \tilde{\Omega}_{vw} & \tilde{\Omega}_{vv} \end{bmatrix}$, the estimated conditional long-run variance corresponding to the first component (the conditional long-run variance of $\{w_t\}$ given $\{v_t\}$) is given by $\tilde{\Omega}_{w\cdot v} := \tilde{\Omega}_{ww} - \tilde{\Omega}_{wv}\tilde{\Omega}_{vv}^{-1}\tilde{\Omega}_{vw}$.

The test statistic for the null hypothesis of no cointegration is defined as:

$$\hat{P}_u := \frac{\tilde{\Omega}_{w\cdot v}}{\frac{1}{T^2}\sum_{t=1}^{T} \hat{u}_t^2}. \tag{7}$$

The following proposition characterizes the asymptotic behavior of the two test statistics, with the CT test already discussed in Wagner and Hong (2016, Proposition 5).

Proposition 2 *(i) Let $\{y_t\}$ be generated by (1) with the regressors $\{x_t\}$ and errors $\{u_t\}$ satisfying Assumption 1 and let $\hat{\Omega}_{u\cdot v}$ be a consistent estimator of $\Omega_{u\cdot v}$, then the asymptotic distribution of the test statistic (5) defined above is:*

$$CT \Rightarrow \int_0^1 (W_{u\cdot v}^J(r))^2 dr, \tag{8}$$

with:

$$W_{u\cdot v}^J(r) := W_{u\cdot v}(r) - \int_0^r J(s)' ds \left(\int_0^1 J(s)J(s)' ds \right)^{-1} \int_0^1 J(s) dW_{u\cdot v}(s). \tag{9}$$

(ii) Let (1) be a spurious regression, i.e., let $\{y_t\}$ be an integrated process not related to $\{X_t\}$ in a cointegrating polynomial relationship, then the asymptotic distribution of the test statistic (7) defined above is:

$$\hat{P}_u \Rightarrow \frac{\Omega_{w\cdot v}}{\tau' \int_0^1 J^*(r) J^*(r)' dr \tau}, \tag{10}$$

with $\tau := \begin{pmatrix} 1 \\ -(\int_0^1 J(r)J(r)'dr)^{-1} \int_0^1 J(r) B_w(r) dr \end{pmatrix}$, $J^*(r) := [B_w(r), J(r)']'$ *and $\tilde{\Omega}_{w\cdot v}$ as defined above.*

The asymptotic distributions of the CT and $\hat{P}_u$ test statistics cannot—in general—be tabulated due to their nuisance parameter dependency related to the correlation structure between the variables. Wagner and Hong (2016, Proposition 6), following Choi and Saikkonen (2010), consider a sub-sample version of the CT test statistic that has a nuisance-parameter-free limiting distribution. They show, in particular, that the limiting distribution of the sub-sample CT test statistics is $\int_0^1 W_{u\cdot v}(r)^2 dr$, if the sub-samples of size b are chosen such that $b \to \infty$ and $b/T \to 0$, i.e., sub-sampling achieves that the second term in (9) vanishes asymptotically. Wagner and Hong (2016), in addition, discuss how the set of sub-sample test statistics can be used in conjunction with modified Bonferroni bounds to improve the performance of the sub-sample-based test. An inspection of the proof of Proposition 1 reveals that sub-sampling does not lead to similar simplifications for the $\hat{P}_u$ test, with this "unfortunate difference" caused by the too slow convergence rate of $\hat{\tau}$ in the spurious regression case. Thus, for the general case only the CT test, when used in a sub-sampling fashion, is available.[7]

The problem with the nuisance parameter dependency of the limiting distributions of the test statistics originates in $J(r)$ as defined in Proposition 1. In case it is possible

[7] An alternative route may be to develop bootstrap inference.

to write:

$$J(r) = \begin{bmatrix} D(r) \\ \mathbf{B}_v(r) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & \Pi \end{bmatrix} \begin{bmatrix} D(r) \\ \mathbf{W}_v(r) \end{bmatrix}, \tag{11}$$

with a regular matrix Π and $\mathbf{W}_v(r)$ a functional of standard Brownian motions one can rewrite both test statistics as functions of standard Brownian motions and a scalar (long-run) variance that can be scaled out. If a transformation as just described exists, the CPR is said to exhibit full design, a terminology coined by Vogelsang and Wagner (2016). CPRs with only one of the integrated processes present as regressor also with higher powers are straightforwardly seen to be of full design, other cases include, e.g., Translog-type relationships (see the discussion in Stypka and Wagner 2020). For brevity, we provide the corresponding result below for the *one integrated regressor with higher power only* case, which—as mentioned before—allows to test in the empirically most relevant case covering EKCs, MKCs, intensity-of-use and exchange rate target-zone relationships. As before in Proposition 2, the result for the CT test has already been discussed in Wagner and Hong (2016).

Proposition 3 *Consider the special case of* (1) *when* $p_1 = \cdots = p_{m-1} = 1$ *and let otherwise the assumptions of Proposition 2 be fulfilled.*

(i) Under the null hypothesis of cointegration in (1) *the limiting distribution of the* CT *statistic is given by:*

$$CT \Rightarrow \int_0^1 (W_{u\cdot v}^{J^W}(r))^2 dr, \tag{12}$$

with:

$$W_{u\cdot v}^{J^W}(r) := W_{u\cdot v}(r) - \int_0^r J^W(s)'ds \left(\int_0^1 J^W(s)J^W(s)'ds\right)^{-1} \int_0^1 J^W(s)dW_{u\cdot v}(s) \tag{13}$$

and $J^W(r) := [D(r)', \mathbf{W}(r)', W_m(r)^2, \ldots, W_m(r)^{p_m}]'$, *where* $\mathbf{W}(r) := [W_1(r)', \ldots, W_m(r)']'$ *is a vector of standard Brownian motions independent of* $W_{u\cdot v}(r)$.

(ii) Under the null hypothesis of (1) *being a spurious regression, the limiting distribution of the* $\hat{P}_u$ *statistic is given by:*

$$\hat{P}_u \Rightarrow \frac{1}{\int_0^1 W_{w\cdot v}^2(r)dr - \int_0^1 W_{w\cdot v}(r)J^W(r)'dr \left(\int_0^1 J^W(r)J^W(r)'dr\right)^{-1} \int_0^1 J^W(r)W_{w\cdot v}(r)dr}, \tag{14}$$

with $W_{w\cdot v}(r)$ *independent of* $\mathbf{W}(r)$.

The limiting distributions (12) and (14) can be tabulated because they are functionals of standard Brownian motions. The corresponding critical values depend upon the deterministic component, the number of integrated regressors and the included powers

Table 1 List of countries included in the empirical analysis

Australia	Austria	Belgium	Canada	Denmark	Finland
France	Germany	Italy	Japan	New Zealand	Norway
Portugal	Spain	Sweden	Switzerland	UK	USA

The sample range is 1870–2016 with the exception of New Zealand with sample range 1878–2016

of the single integrated regressor that enters the CPR with higher powers. Critical values for the CT statistic are given in Table 6 and for the $\hat{P}_u$ statistic in Table 7 in "Appendix B." These tables extend the corresponding tables provided in Shin (1994) and Phillips and Ouliaris (1990) for linear cointegrating relationships to the CPR case. The tables contain critical values for up to four integrated regressors, up to power four of the integrated regressor entering with powers for the usual deterministic components, i.e., no deterministic component, an intercept only and intercept and linear trend.[8]

3 An illustration with the environmental Kuznets curve

We illustrate the tests discussed with annual data for real GDP per capita, carbon dioxide (CO_2) in metric tons per capita and sulfur dioxide (SO_2) emissions in kilograms per capita for 18 early industrialized countries, listed in Table 1, over the period 1870–2016; with the exception of New Zealand where the sample range is 1878–2016. Real GDP is measured in 2011 US dollars and calculated from the GDP and population data in the 2018 version of the Maddison Project Database (see Bolt et al. 2018). The CO_2 per capita emissions are calculated from total CO_2 emissions from fossil fuel usage, downloaded from the webpage of the Carbon Dioxide Information Analysis Center (CDIAC) at the Appalachian State University (see Boden et al. 2018). The SO_2 per capita emissions are calculated from combining total SO_2 emissions taken from the NASA Socioeconomic Data and Applications Center (see Smith et al. 2011), which provides data for 1870–2005, with OECD (2020) data for the period 2006–2016.[9]

We use the data described in logarithmic specifications and test for cointegration and non-cointegration in both the quadratic and the cubic EKC relationship including an intercept and a linear trend:

$$e_t = c + \delta t + \beta_1 y_t + \beta_2 y_t^2 + u_t, \tag{15}$$

$$e_t = c + \delta t + \beta_1 y_t + \beta_2 y_t^2 + \beta_3 y_t^3 + u_t, \tag{16}$$

with e_t denoting the logarithm of emissions (CO_2 or SO_2) per capita and y_t the logarithm of GDP per capita.

[8] MATLAB code to generate (additional) critical values as well as to perform the tests is available upon request.

[9] To be precise, the OECD provides data for SO_x emissions. The National Research Council (1975) estimates the share of SO_2 in SO_x emissions at about 98% and this is the factor we also use to transform OECD SO_x to SO_2 data.

Performing the usual battery of unit root tests on the logarithm of GDP per capita does not lead to a rejection of the null hypothesis for any of the 18 countries. Thus, a "necessary condition" for embarking on CPR analysis, that of an integrated regressor, appears to be fulfilled. The CT test is performed using the FM-OLS residuals (i.e., is based on using the estimator of Wagner and Hong 2016) with the Bartlett kernel and the Newey and West (1994) bandwidth rule.[10]

Tables 2 and 3 contain the test results for CO_2 and SO_2 emissions, respectively. We use the following (quite obvious) "decision rule": If CT does not lead to a rejection, but $\hat{P}_u$ does, this is evidence for the presence of a cointegrating polynomial EKC relationship. If CT does lead to a rejection, but $\hat{P}_u$ does not, this is evidence against the presence of an EKC. If either both or none of the tests lead to a rejection, this is regarded as conflicting evidence.[11] Given this decision rule, let us now turn to the results, first for CO_2 emissions given in Table 2. For six of the 18 countries, Austria, Belgium, Finland, Germany, Switzerland, and the UK we find evidence for an EKC relationship. For Germany, only for the cubic specification, for the other five countries for both the quadratic and the nesting cubic specification (which is, of course, a necessary theoretical implication of the prevalence of a quadratic CPR). The test decisions are also consistent between the cubic and quadratic specifications in the sense that evidence against the presence of a cubic EKC for a certain country comes along with evidence also against a quadratic EKC for that country. The one exception here is Sweden for which there is evidence against an EKC for the cubic specification and "only" conflicting evidence for the quadratic specification. Also note that conflicting evidence for the cubic specification occurs typically in conjunction with evidence against a quadratic EKC. The results for SO_2 emissions given in Table 3 are quite clear: There is no evidence for a quadratic or cubic EKC for any of the 18 countries considered. For 16 of the 18 countries, however, the evidence is conflicting for the cubic specification.

The limited evidence for the prevalence of an EKC is akin to an observation made earlier in Wagner (2015), i.e., that the usage of adequate (non-)cointegration tests reduces the evidence for an EKC compared to using (non-)cointegration tests (critical values) developed for linear cointegrating relationships. This is exemplified here by including also the *asymptotically invalid* test results obtained by applying the residual-based augmented Dickey–Fuller non-cointegration t-type test of Phillips and Ouliaris (1990) developed for cointegrating linear relationships, labeled PO_t. Using this test, for CO_2 emissions the null hypothesis of non-cointegration is rejected for twelve of the 18 countries at the 5% significance level compared to only five rejections obtained with the asymptotically valid $\hat{P}_u$ test. For SO_2 emissions, the differences between the $\hat{P}_u$ and PO_t results are less pronounced, but point in the same direction. These findings illustrate the importance of using (non-)cointegration tests—or more precisely

10 The following subsection investigates inter alia the impact of kernel and bandwidth choices on the test results.

11 In those cases where one of the tests rejects at the 5% significance level, but the other only at the 10% significance level, we put higher weight on the "stronger" 5% rejection. Considering also such cases as conflicting evidence does not lead to substantial differences in the overall picture. In Tables 4 and 5 that summarize the results from the robustness checks, evidence for an EKC is labeled with "y," evidence against an EKC with "n" and conflicting evidence with "o."

Table 2 Cointegration (CT) and non-cointegration ($\hat{P}_u$) test results for CO_2 emissions for the sample range 1870–2016 (for New Zealand 1878–2016)

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Australia	**0.107**	11.294	−2.623	**0.106**	11.306	−2.530
Austria	0.055	**57.186**	−3.816	0.042	**57.988**	−3.822
Belgium	0.061	*51.074*	**−5.710**	0.057	*55.199*	**−5.667**
Canada	**0.144**	11.882	−3.355	0.056	25.542	**−4.878**
Denmark	0.047	36.027	**−4.673**	0.049	36.029	**−4.670**
Finland	0.048	**75.487**	**−5.719**	0.033	**83.542**	**−6.136**
France	0.066	27.859	**−4.929**	0.062	28.041	**−4.863**
Germany	**0.108**	**69.262**	**−8.001**	*0.090*	**69.349**	**−8.089**
Italy	**0.138**	34.120	**−4.174**	*0.088*	*50.661*	**−5.520**
Japan	**0.149**	8.718	**−5.889**	0.065	12.995	**−6.009**
New Zealand	**0.112**	13.698	**−5.422**	*0.098*	14.203	**−5.809**
Norway	**0.116**	20.583	−3.334	*0.092*	26.063	−3.617
Portugal	**0.108**	20.919	**−9.181**	**0.111**	21.327	**−9.412**
Spain	*0.091*	42.031	−3.315	*0.089*	42.112	−3.384
Sweden	0.084	29.150	**−4.312**	*0.084*	29.988	*−4.323*
Switzerland	*0.097*	**86.476**	**−6.291**	0.056	**107.442**	**−6.763**
UK	0.070	**91.459**	**−6.894**	0.067	**91.704**	**−6.848**
USA	**0.148**	12.620	−2.337	0.077	23.531	−3.558

The columns PO_t display the test results of the residual-based augmented Dickey–Fuller non-cointegration t-type test of Phillips and Ouliaris (1990) developed for cointegrating linear relationships. *Italic* entries indicate rejection of the null hypothesis at the 10% level and bold entries rejection at the 5% level. The left panel displays the results for the quadratic specification and the right panel for the cubic specification. For both polynomial degrees intercept and linear trend are included. The results are based on the Bartlett kernel with bandwidth according to Newey and West (1994)

appropriate critical values—constructed for the CPR setting. For the CT test, this issue has already been illustrated in Wagner (2015) and is discussed more thoroughly from a theoretical perspective in Stypka et al. (2019).

3.1 Robustness checks

Both tests, being nonparametric, depend upon kernel and bandwidth choices. Consequently, the results might be sensitive to these choices. To gauge the extent of this potential sensitivity, we consider four combinations of bandwidths and kernels. The Bartlett kernel and the Newey and West (1994) bandwidth (the baseline, used above), labeled I, the Bartlett kernel and the Andrews (1991) bandwidth, labeled II, and the Quadratic Spectral kernel with these two bandwidth rules, labeled III and IV in Tables 4 for CO_2 emissions and 5 for SO_2 emissions. It turns out that the results are in fact highly robust with respect to these choices, most importantly the evidence for the prevalence of an EKC for CO_2 emissions. For the five countries with evidence for a

Table 3 Cointegration (CT) and non-cointegration $\hat{P}_u$ test results for SO_2 emissions for the sample range 1870–2016 (for New Zealand 1878–2016)

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Australia	0.051	38.885	−3.402	0.048	40.506	−3.521
Austria	0.080	28.695	−3.776	0.055	39.188	−3.695
Belgium	*0.100*	17.752	−3.738	0.043	27.436	−3.556
Canada	0.055	22.976	*−3.865*	0.053	23.118	−3.888
Denmark	*0.101*	10.010	−3.160	0.057	20.890	−4.006
Finland	0.078	25.292	*−4.082*	0.077	25.701	−3.905
France	**0.112**	5.524	−2.353	0.067	11.725	−2.455
Germany	*0.098*	6.057	−2.584	0.062	11.524	−2.782
Italy	*0.089*	3.835	−2.296	*0.090*	4.090	−1.729
Japan	**0.113**	15.237	−3.219	0.036	19.190	*−4.447*
New Zealand	*0.090*	11.831	**−4.849**	0.054	16.130	**−6.941**
Norway	0.069	22.719	−3.216	0.067	22.931	−3.283
Portugal	*0.100*	11.880	−3.175	**0.102**	11.933	−3.271
Spain	0.074	12.060	−1.799	0.080	13.046	−1.761
Sweden	0.066	11.155	−3.709	0.067	12.278	−2.908
Switzerland	**0.203**	6.766	−1.703	0.060	37.684	**−5.067**
UK	0.075	0.841	−1.510	0.066	1.731	−2.647
USA	0.085	10.517	−1.230	0.074	10.574	−1.200

For further explanations see caption of Table 2

quadratic (and cubic) EKC, this evidence is present for all four combinations of kernel and bandwidth. For Germany, with evidence for the cubic specification only and mixed evidence for the quadratic specification in the baseline combination I, the evidence is scattered a bit between "y" and "o" across kernel and bandwidth choices. Looking at columns I to IV in Table 4 indicates some limited variation between "n" and "o" across combinations for some of the countries. For SO_2 emissions, there is—similarly to CO_2 emissions—some variability for some countries between "n" and "o" decisions. However, for Switzerland using the Andrews (1991) bandwidth rule leads to evidence for an EKC in the cubic specification, whereas the evidence is conflicting when using the Newey and West (1994) bandwidth rule. The differences in conclusions are driven by $\hat{P}_u$ rejecting with the Andrews (1991) bandwidth. Altogether, however, the test results are quite insensitive—most importantly with respect to evidence for the prevalence of an EKC—to kernel and bandwidth choices.

Columns N_0 and O_0 of Tables 4 and 5 shed light on two other *empirical* dimensions of robustness of the test results: N_0 with respect to the sample size, using the data used so far but only until 2000 and O_0 using the same data as used in Wagner (2015). Sample size and data vintage robustness do not directly relate to robustness of the tests, but it may still be informative to see their impact on the test results for our EKC illustration. The results in columns N_0 and O_0 are based on using the Bartlett kernel and the Newey

Table 4 Test decisions, as explained above, for CO_2 emissions, with "y" indicating evidence for the presence of an EKC, "n" evidence against and "o" indicating conflicting evidence

	Quadratic						Cubic					
	I	II	III	IV	N_0	O_0	I	II	III	IV	N_0	O_0
Australia	n	o	n	o	n	n	n	o	n	n	n	n
Austria	y	y	y	y	y	y	y	y	y	y	y	y
Belgium	y	y	o	y	y	y	y	y	o	y	y	y
Canada	n	n	n	o	n	n	o	o	o	o	o	n
Denmark	o	o	o	o	o	o	o	o	o	o	o	o
Finland	y	y	y	y	y	y	y	y	y	y	y	y
France	o	o	o	o	o	n	o	o	o	o	o	n
Germany	o	o	y	o	o	o	y	o	y	y	y	o
Italy	n	n	n	o	n	n	o	o	n	o	o	n
Japan	n	n	n	o	n	n	o	o	o	o	o	n
New Zealand	n	o	n	o	n	n	n	o	n	o	n	n
Norway	n	o	n	o	n	n	n	o	n	o	n	n
Portugal	n	n	n	n	n	n	n	n	n	n	n	n
Spain	n	o	n	o	n	n	n	o	n	o	n	n
Sweden	o	o	o	o	o	n	n	o	n	o	n	n
Switzerland	y	y	y	y	y	n	y	y	y	y	y	n
UK	y	y	y	y	y	o	y	y	y	y	y	o
USA	n	o	n	o	n	n	o	o	o	o	o	n

I, II, III and IV indicate test results based on different combinations of kernels and bandwidths as detailed in the main text. N_0 indicates results based on the data used in this paper until 2000 only and O_0 indicates test results using an earlier data vintage until 2000. All test results using data until 2000 are based on the Bartlett kernel and the Newey and West (1994) bandwidth rule

and West (1994) bandwidth. Comparing columns N_0 with columns I leads to only one difference, for SO_2 emissions of Canada in the quadratic specification. Using the full data range the evidence points against the prevalence of an EKC, whereas the reduced sample leads to conflicting evidence. Thus, it appears that the additional 16 years of data help to sharpen inference. We close the robustness check by comparing the results for the sample period until 2000 with the results obtained using the data of Wagner (2015) that also range until 2000, i.e., we compare the results in columns N_0 and O_0. For CO_2 emissions, using the new data vintage leads to evidence for the prevalence of an EKC for three more countries than for the old data vintage, for Switzerland and the UK for both specifications and for Germany for the cubic specification. Additionally, the evidence for four countries is changed from negative to conflicting, with these changes occurring primarily in the cubic specification. For SO_2 emissions the evidence changes from negative to conflicting for a number of countries in both the quadratic and the cubic specification. Additionally, the evidence changes for the UK from positive to conflicting for the cubic specification.[12] Whether these changes relate to some form

[12] The UK was the sole country for which Wagner (2015) finds an EKC for SO_2 emissions.

Table 5 Test decisions for SO_2 emissions,emissions, with "y" indicating evidence for the presence of an EKC, "n" evidence against and "o" indicating conflicting evidence

	Quadratic						Cubic					
	I	II	III	IV	N_0	O_0	I	II	III	IV	N_0	O_0
Australia	o	o	o	o	o	n	o	o	o	o	o	n
Austria	o	o	o	o	o	n	o	o	o	o	o	n
Belgium	n	o	n	o	n	n	o	o	o	o	o	n
Canada	n	o	o	o	o	o	o	o	o	o	o	o
Denmark	n	o	n	o	n	n	o	o	o	o	o	n
Finland	o	o	o	o	o	n	o	o	o	o	o	n
France	n	n	n	n	n	n	o	n	o	n	o	n
Germany	n	o	n	n	n	n	o	o	o	n	o	n
Italy	n	n	o	n	n	n	n	n	n	n	n	n
Japan	n	o	n	o	n	n	o	o	o	o	o	n
New Zealand	n	o	n	n	n	n	o	o	o	o	o	n
Norway	o	o	o	o	o	n	o	o	o	o	o	n
Portugal	n	o	n	o	n	n	n	o	n	n	n	n
Spain	o	n	o	n	o	n	o	n	o	n	o	n
Sweden	o	o	o	o	o	n	o	o	o	o	o	n
Switzerland	n	n	n	n	n	n	o	y	o	y	o	n
UK	o	n	o	n	o	n	o	o	o	o	o	y
USA	o	n	o	n	o	n	o	n	o	n	o	n

For further explanations see caption of Table 4

of sensitivity of the tests to data with unchanged CPR characteristics or because the data revision truly changes the nature of the data is, of course, not clear. What we can, however, take home from our robustness checks is that the tests are not very sensitive to kernel and bandwidth choices.

4 Summary and conclusions

This paper discusses two nonparametric tests: one for the null hypothesis of a cointegrating polynomial regression and one for the null hypothesis of the absence of cointegration. Both tests are extensions from tests developed for cointegrating linear regressions to the cointegrating polynomial regression setting. Specifically, they are extensions of the Shin (1994) test and of a variance ratio test of Phillips and Ouliaris (1990).[13] The word extension hereby refers to deriving the null limiting distributions of the test statistics in the CPR setting. It turns out that, in the general case, neither of the two tests has a nuisance-parameter-free limiting distribution. However, in case

[13] Note again that the CT test has already been discussed in Wagner and Hong (2016) and used in Wagner (2015). Both, the CT and the $\hat{P}_u$ test have already been used in an analysis of the material Kuznets Curve hypothesis in Grabarczyk et al. (2018).

of *full design* of the regression model, both tests' limiting distributions are nuisance parameter free and can hence be tabulated. The empirically most relevant case with only one of the integrated regressors appearing also with higher powers is easily seen to be of full design. For this case, "Appendix B" provides tables with critical values for up to four integrated regressors, up to power four of the single integrated regressor entering with higher powers and the usual deterministic specifications.

The tests are briefly illustrated—which is by no means a fully fledged EKC analysis—for the environmental Kuznets curve for both CO_2 and SO_2 emissions for 18 early industrialized countries over, with the exception of New Zealand, the period 1870–2016. For CO_2 emissions an EKC relationship appears to be present for six of the 18 countries, i.e., Austria, Belgium, Finland, Germany, Switzerland and the UK. For SO_2 emissions there is no evidence for the prevalence of a quadratic or cubic cointegrating EKC relationship. Little or reduced evidence for an EKC is a typical finding when using adequate tests compared to using tests designed for linear cointegrating relationships, as is discussed in this paper and already earlier in Wagner (2015). The illustration also indicates that the test results are not (really) sensitive with respect to kernel and bandwidth choices, most importantly the results for the prevalence of an EKC. Our illustration also shows that the findings are more sensitive with respect to sample size and in particular data vintage used.

The tests discussed are important inputs for developing (non-)cointegration tests in panel (see, e.g., de Jong and Wagner 2022; Wagner and Reichold 2022) or seemingly unrelated CPR settings (see, e.g., Wagner et al. 2020).

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Declaration

Conflict of interest The author declares that he has no conflict of interest related to this research. This article does not contain any studies with human participants or animals performed by the author.

Appendix A: Proofs

Proof of Proposition 2 The proof of item (i) is contained in Wagner and Hong (2016, Proposition 5) and it thus only remains to show the second item. By definition, it holds that:

$$\hat{u}_t = \left(y_t,\ Z_t' \right) \begin{pmatrix} 1 \\ -\hat{\theta} \end{pmatrix} =: \tilde{Z}_t' \hat{\tau}, \tag{17}$$

Table 6 Critical values for the CT test for the case of only one regressor entering the CPR with powers

	0.010	0.025	0.050	0.100	0.500	0.900	0.950	0.975	0.990
	$m = 1, p = 1$								
	0.0283	0.0354	0.0438	0.0576	0.2016	0.8575	1.2087	1.6005	2.1413
c	0.0206	0.0247	0.0292	0.0357	0.0832	0.2325	0.3164	0.4084	0.5495
c,t	0.0155	0.0180	0.0206	0.0243	0.0467	0.0977	0.1212	0.1450	0.1786
	$m = 1, p = 2$								
	0.0247	0.0311	0.0379	0.0492	0.1591	0.6639	0.9465	1.2649	1.7119
c	0.0188	0.0224	0.0263	0.0321	0.0742	0.2133	0.2927	0.3786	0.5044
c,t	0.0144	0.0165	0.0189	0.0221	0.0416	0.0858	0.1063	0.1280	0.1568
	$m = 1, p = 3$								
	0.0229	0.0282	0.0346	0.0442	0.1382	0.5605	0.8039	1.0792	1.4731
c	0.0177	0.0210	0.0246	0.0299	0.0699	0.2044	0.2807	0.3651	0.4904
c,t	0.0137	0.0157	0.0179	0.0209	0.0391	0.0812	0.1010	0.1221	0.1501
	$m = 1, p = 4$								
	0.0216	0.0266	0.0322	0.0413	0.1262	0.4962	0.7079	0.9386	1.2753
c	0.0169	0.0200	0.0234	0.0286	0.0673	0.1987	0.2743	0.3571	0.4767
c,t	0.0131	0.0151	0.0172	0.0201	0.0376	0.0784	0.0977	0.1180	0.1455
	$m = 2, p = 1$								
	0.0239	0.0296	0.0361	0.0465	0.1496	0.6257	0.8939	1.1890	1.6157
c	0.0179	0.0211	0.0247	0.0297	0.0633	0.1615	0.2194	0.2853	0.3844
c,t	0.0140	0.0162	0.0185	0.0216	0.0401	0.0819	0.1014	0.1215	0.1495
	$m = 2, p = 2$								
	0.0217	0.0265	0.0318	0.0405	0.1232	0.5046	0.7253	0.9711	1.3463
c	0.0165	0.0194	0.0224	0.0269	0.0570	0.1479	0.2006	0.2631	0.3586
c,t	0.0131	0.0151	0.0171	0.0198	0.0362	0.0726	0.0900	0.1080	0.1334
	$m = 2, p = 3$								
	0.0202	0.0244	0.0292	0.0370	0.1085	0.4422	0.6384	0.8546	1.1769
c	0.0155	0.0182	0.0211	0.0253	0.0535	0.1408	0.1914	0.2532	0.3484
c,t	0.0125	0.0143	0.0162	0.0188	0.0341	0.0687	0.0852	0.1029	0.1278
	$m = 2, p = 4$								
	0.0189	0.0231	0.0276	0.0349	0.0995	0.4002	0.5725	0.7652	1.0554

Table 6 continued

	0.010	0.025	0.050	0.100	0.500	0.900	0.950	0.975	0.990
c	0.0149	0.0174	0.0202	0.0243	0.0515	0.1366	0.1864	0.2476	0.3391
c,t	0.0120	0.0138	0.0155	0.0181	0.0328	0.0663	0.0822	0.0997	0.1244
	$m = 3, p = 1$								
	0.0208	0.0254	0.0305	0.0386	0.1174	0.4722	0.6780	0.9003	1.2361
c	0.0158	0.0184	0.0213	0.0252	0.0508	0.1204	0.1588	0.2062	0.2770
c,t	0.0129	0.0147	0.0167	0.0193	0.0348	0.0689	0.0851	0.1022	0.1270
	$m = 3, p = 2$								
	0.0190	0.0230	0.0275	0.0345	0.0989	0.3920	0.5617	0.7589	1.0433
c	0.0146	0.0170	0.0197	0.0232	0.0459	0.1095	0.1457	0.1906	0.2593
c,t	0.0121	0.0138	0.0155	0.0179	0.0317	0.0615	0.0754	0.0905	0.1123
	$m = 3, p = 3$								
	0.0178	0.0214	0.0255	0.0317	0.0885	0.3499	0.5039	0.6809	0.9435
c	0.0140	0.0162	0.0186	0.0220	0.0433	0.1041	0.1391	0.1819	0.2477
c,t	0.0115	0.0132	0.0148	0.0170	0.0300	0.0583	0.0718	0.0864	0.1068
	$m = 3, p = 4$								
	0.0170	0.0204	0.0242	0.0300	0.0823	0.3207	0.4613	0.6185	0.8738
c	0.0134	0.0156	0.0179	0.0211	0.0417	0.1012	0.1357	0.1774	0.2425
c,t	0.0112	0.0127	0.0143	0.0164	0.0288	0.0564	0.0696	0.0838	0.1039
	$m = 4, p = 1$								
	0.0184	0.0222	0.0265	0.0333	0.0960	0.3754	0.5327	0.7093	0.9688
c	0.0143	0.0165	0.0189	0.0222	0.0422	0.0945	0.1221	0.1537	0.2027
c,t	0.0119	0.0136	0.0153	0.0176	0.0308	0.0591	0.0727	0.0877	0.1088
	$m = 4, p = 2$								
	0.0170	0.0204	0.0243	0.0302	0.0829	0.3173	0.4483	0.6050	0.8326
c	0.0133	0.0155	0.0175	0.0205	0.0386	0.0858	0.1108	0.1400	0.1876
c,t	0.0113	0.0128	0.0143	0.0164	0.0282	0.0534	0.0653	0.0785	0.0969
	$m = 4, p = 3$								
	0.0161	0.0193	0.0228	0.0280	0.0748	0.2844	0.4089	0.5494	0.7540
c	0.0128	0.0147	0.0167	0.0195	0.0365	0.0815	0.1058	0.1341	0.1802
c,t	0.0109	0.0122	0.0137	0.0157	0.0268	0.0508	0.0623	0.0745	0.0925
	$m = 4, p = 4$								
	0.0154	0.0183	0.0216	0.0265	0.0698	0.2636	0.3788	0.5055	0.6930
c	0.0123	0.0142	0.0161	0.0188	0.0352	0.0787	0.1028	0.1316	0.1762
c,t	0.0105	0.0118	0.0132	0.0151	0.0258	0.0490	0.0600	0.0718	0.0887

The symbols in the first column indicate the deterministic component: none (empty), intercept only (c) and intercept and linear trend ($c.t$), m indicates the number of integrated regressors, and p indicates the highest included power of the regressor entering with powers

Table 7 Critical values for the $\hat{P}_u$ test for the null hypothesis of no cointegration for the case of only one regressor entering the CPR with powers

	0.010	0.025	0.050	0.100	0.500	0.900	0.950	0.975	0.990
	$m = 1, p = 1$								
	0.5845	0.8326	1.1653	1.7665	7.2292	20.5040	26.1196	31.6526	39.0403
c	1.8825	2.4752	3.1790	4.3116	11.9696	27.7789	34.0970	40.1474	48.0348
c,t	5.5163	6.8023	8.1928	10.1515	21.4332	41.1315	48.5640	55.3291	64.4630
	$m = 1, p = 2$								
	0.8033	1.1261	1.5465	2.2889	8.9577	24.5044	30.8754	36.9862	44.8619
c	2.0263	2.6460	3.4311	4.6944	13.4370	30.9825	37.8748	44.4325	52.9478
c,t	6.2801	7.7521	9.3365	11.5826	24.0819	45.2373	52.9518	60.3206	69.4706
	$m = 1, p = 3$								
	0.9509	1.3362	1.8350	2.6904	10.0724	27.0794	33.8596	40.5755	48.9771
c	2.0991	2.7549	3.5724	4.8907	14.2797	33.1538	40.4370	47.4776	56.1230
c,t	6.5576	8.1182	9.8120	12.2592	25.5527	47.9252	55.9264	63.5980	73.3074
	$m = 1, p = 4$								
	1.0791	1.5079	2.0546	3.0064	10.8663	28.8801	36.1806	43.2061	51.8041
c	2.1442	2.8181	3.6672	5.0256	14.8495	34.7177	42.4254	49.7957	58.7888
c,t	6.7354	8.3833	10.1501	12.7102	26.6210	50.0143	58.2791	66.0470	76.1264
	$m = 2, p = 1$								
	1.0623	1.6048	2.2985	3.4434	11.2203	27.2499	33.5122	39.8156	47.7341
c	2.5816	3.4984	4.5844	6.1575	15.7656	33.8495	40.9039	47.6771	55.7996
c,t	6.5444	8.1968	9.8377	12.1579	25.0446	46.3967	54.2696	61.8035	71.4986
	$m = 2, p = 2$								
	1.3404	1.9743	2.7902	4.1475	13.2952	31.1699	38.0363	44.8716	53.2264
c	2.7868	3.7925	5.0189	6.7601	17.6129	37.2768	44.7916	51.8411	60.6054
c,t	7.4636	9.2463	11.1600	13.7506	27.8002	50.4242	58.6673	66.3847	76.3675
	$m = 2, p = 3$								
	1.5755	2.2928	3.1753	4.6412	14.5958	33.8226	41.2216	48.3740	57.0102
c	2.8737	3.9536	5.2410	7.0984	18.7054	39.6763	47.7021	54.9207	63.8283
c,t	7.7986	9.7012	11.6899	14.4939	29.4515	53.3632	61.8438	69.8548	79.9388
	$m = 2, p = 4$								
	1.7339	2.4981	3.4703	5.0159	15.5252	35.8240	43.6428	51.0464	59.9510
c	2.9467	4.0430	5.3852	7.3096	19.4543	41.4571	49.6628	57.2963	66.6961
c,t	8.0506	10.0551	12.0966	15.0368	30.6521	55.3904	64.3841	72.5786	82.6272
	$m = 3, p = 1$								
	1.8991	2.8087	3.9073	5.4788	15.2663	33.3544	40.3213	47.0390	55.6948
c	3.6828	4.9143	6.2923	8.2622	19.6794	39.5247	46.8920	53.8427	63.1706
c,t	7.8556	9.7839	11.7727	14.5467	28.8012	51.7162	59.8743	67.5852	77.2638

Table 7 continued

	0.010	0.025	0.050	0.100	0.500	0.900	0.950	0.975	0.990
	$m = 3, p = 2$								
	2.2619	3.3407	4.5744	6.3891	17.5405	37.3604	44.7409	51.6503	60.9317
c	3.9874	5.3412	6.8811	9.1055	21.7137	43.0863	50.9164	58.1100	67.8342
c,t	8.8561	11.0187	13.1809	16.2311	31.6690	55.6368	64.2565	72.2536	82.4774
	$m = 3, p = 3$								
	2.5320	3.7201	5.0262	6.9525	18.9865	40.0945	47.8203	55.2260	64.8798
c	4.1438	5.5758	7.1904	9.5542	23.0131	45.5700	53.6396	61.3098	71.1478
c,t	9.2529	11.5671	13.9194	17.1643	33.4200	58.4419	67.3272	75.5342	86.6035
	$m = 3, p = 4$								
	2.7677	4.0083	5.3600	7.3944	20.0353	42.1077	50.3066	57.8614	67.7364
c	4.2703	5.7288	7.3922	9.8555	23.9210	47.4662	55.8692	63.8357	73.9377
c,t	9.5352	11.9249	14.4225	17.7965	34.7057	60.6408	69.7676	78.1590	89.6387
	$m = 4, p = 1$								
	3.0653	4.3193	5.7271	7.8107	19.1964	39.0550	46.5991	53.6258	62.5284
c	4.9186	6.4937	8.2245	10.6248	23.5729	45.1669	52.9950	60.2570	69.9523
c,t	9.1915	11.4011	13.7004	16.8383	32.4819	56.8046	65.3240	73.2446	83.5502
	$m = 4, p = 2$								
	3.5018	4.9869	6.6427	8.9719	21.6929	43.1144	51.0096	58.7039	68.0398
c	5.3188	7.1237	9.0606	11.6876	25.8405	48.7391	56.9244	64.9450	74.3894
c,t	10.3519	12.7393	15.2253	18.6448	35.3487	60.8919	69.8928	78.2459	89.0233
	$m = 4, p = 3$								
	3.8165	5.4458	7.1799	9.6354	23.3204	45.9577	54.1944	61.9963	71.7996
c	5.5579	7.4697	9.4988	12.2695	27.3318	51.3781	59.9915	68.0369	78.3567
c,t	10.8577	13.4067	16.0711	19.6715	37.2752	63.9113	73.0392	81.6191	92.4438
	$m = 4, p = 4$								
	4.1029	5.7503	7.5856	10.1474	24.4741	48.0333	56.6295	64.6451	74.7573
c	5.6837	7.6545	9.7979	12.6560	28.3573	53.3702	62.1743	70.4037	81.1153
c,t	11.2134	13.8634	16.6048	20.3738	38.6532	66.1506	75.5562	84.4401	95.6502

See caption of Table 6 for further explanations

with the equation defining $\tilde{Z}_t$ and $\hat{\tau}$ and with $\hat{\theta}$ denoting the OLS coefficient estimate in (1), which is now a spurious regression with an integrated error process $\{u_t\}$. The fact that we are in the spurious regression case implies that:

$$T^{-1}G^{-1}\hat{\theta} = \left(\sum_{t=1}^{T} GZ_tZ_t'G\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T} T^{1/2}GZ_t\frac{y_t}{\sqrt{T}}\right) \tag{18}$$

$$\Rightarrow \left(\int_0^1 J(r)J(r)'dr\right)^{-1}\int_0^1 J(r)B_w(r)dr, \tag{19}$$

using again the notation $\Delta y_t = w_t$ and $\frac{1}{\sqrt{T}} y_{[rT]} = \frac{1}{\sqrt{T}} \sum_{t=1}^{[rT]} w_t \Rightarrow B_w(r)$. This implies that $\frac{1}{T^2}\sum_{t=1}^{T} \hat{u}_t^2 = \frac{1}{T^2}\hat{\tau}'\tilde{Z}_t\tilde{Z}_t'\hat{\tau} =$

$$(1,\ -\hat{\theta}')\begin{pmatrix}1 & 0\\ 0 & T^{-1}G^{-1}\end{pmatrix}\left\{\sum_{t=1}^{T}\begin{pmatrix}T^{-1} & 0\\ 0 & G\end{pmatrix}\tilde{Z}_t\tilde{Z}_t'\begin{pmatrix}T^{-1} & 0\\ 0 & G\end{pmatrix}\right\}\begin{pmatrix}1 & 0\\ 0 & T^{-1}G^{-1}\end{pmatrix}\begin{pmatrix}1\\ -\hat{\theta}\end{pmatrix} \tag{20}$$

converges to:

$$\tau'\int_0^1 J^*(r)J^*(r)'dr\tau, \tag{21}$$

with τ and $J^*(r)$ as defined in the main text. Consistency of the numerator, i.e., of $\tilde{\Omega}_{w\cdot v}$ follows as in the proof of Phillips and Ouliaris (1990, Theorem 4.1), which relies upon the consistency of $\hat{\xi}_t \to \xi_t$ and thus of $\tilde{\Omega} \to \Omega$ under usual assumptions on kernel and bandwidth. □

Proof of Proposition 3 The main issue is the existence of a bijective transformation between $\mathbf{B}_v(r)$ and a vector of functions of standard Brownian motions. Denote with $\tilde{\mathbf{B}}_v(r) = [B_1(r), \ldots, B_m(r)]'$ and let $\Omega_{vv}^{1/2}$ be an upper triangular matrix such that $\tilde{\mathbf{B}}_v(r) = \Omega_{vv}^{1/2}\mathbf{W}(r)$, with $\mathbf{W}(r)$ a vector of standard Brownian motions. Furthermore, denote the (m, m) element of Ω_{vv} by $\Omega_{vv}(m, m)$. This yields:

$$\mathbf{B}_v(r) = \begin{pmatrix}\tilde{\mathbf{B}}_v(r)\\ B_m^2(r)\\ \vdots\\ B_m^{p_m}(r)\end{pmatrix} = \begin{bmatrix}\Omega_{vv}^{1/2} & 0 & \ldots & 0\\ 0 & \Omega_{vv}(m,m) & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ 0 & \ldots & 0 & \Omega_{vv}(m,m)^{p_m/2}\end{bmatrix}\begin{pmatrix}\mathbf{W}(r)\\ W_m^2(r)\\ \vdots\\ W_m^{p_m}(r)\end{pmatrix}.$$

Inserting this expression into (9) and multiplying the terms then leads to (13) and thus to the result for the CT test.

The argument for the $\hat{P}_u$ also rests upon the above transformation with two differences: First, the considered vector is $J^*(r)$, i.e., also the first component corresponding to y_t is considered. Second, the block structure of τ, with the first element equal to one, is used to simplify the limit of $\hat{P}_u$ from (10) to the expression (14) involving only (functions) of standard Brownian motions. The calculations are analogous, with the difference being the terms with powers (and also the deterministic components), to the calculations in the proofs of Phillips and Ouliaris 1990, Lemma 2.2 and Theorem 4.1). □

Table 8 Test results for CO_2 emissions for the sample range 1870–2016 (for New Zealand 1878–2016)

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Australia	0.068	15.079	−2.670	0.068	15.095	−2.606
Austria	0.058	**69.253**	**−4.740**	0.049	**70.224**	**−4.828**
Belgium	0.048	**96.424**	**−6.085**	0.047	**104.213**	**−6.677**
Canada	*0.090*	13.250	−3.340	0.050	28.482	**−4.682**
Denmark	0.045	38.912	**−4.740**	0.047	38.915	**−4.736**
Finland	0.041	**96.490**	**−5.805**	0.033	**106.786**	**−6.493**
France	0.055	45.066	**−4.931**	0.053	45.361	**−4.863**
Germany	**0.123**	**67.160**	**−7.285**	**0.104**	**67.244**	**−7.365**
Italy	*0.095*	30.320	**−4.445**	0.078	45.018	**−5.523**
Japan	*0.094*	9.590	**−5.987**	0.057	14.294	**−5.994**
New Zealand	0.074	12.087	**−5.588**	0.071	12.533	**−5.965**
Norway	0.074	28.158	−3.381	0.073	35.655	−3.612
Portugal	**0.122**	18.278	**−9.863**	**0.127**	18.635	**−10.279**
Spain	0.072	38.083	−3.448	0.068	38.156	−3.496
Sweden	0.069	35.247	**−4.180**	0.071	36.260	*−4.262*
Switzerland	*0.097*	**85.899**	**−6.242**	0.057	**106.725**	**−6.769**
UK	0.072	**83.345**	**−6.449**	0.069	**83.569**	**−6.387**
USA	0.077	8.302	−2.456	0.059	15.479	−3.428

For further explanations see caption of Table 2 with the difference that the results in this table are based on the Bartlett kernel with bandwidth according to Andrews (1991)

Appendix B: Tables with critical values

Supplementary Appendix C: Robustness of results

Robustness: Kernel and bandwidth

Table 9 Test results for CO_2 emissions for the sample range 1870–2016 (for New Zealand 1878–2016)

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Australia	**0.108**	10.022	−2.634	**0.108**	10.033	−2.546
Austria	0.050	*50.805*	−3.797	0.037	*51.517*	−3.800
Belgium	0.059	36.581	**−5.758**	0.056	39.536	**−6.020**
Canada	**0.144**	9.496	−3.404	0.053	20.413	**−4.962**
Denmark	0.043	26.667	**−4.667**	0.045	26.669	**−4.661**
Finland	0.046	**64.688**	**−5.618**	0.030	**71.591**	**−6.090**
France	0.066	22.267	**−4.911**	0.060	22.412	**−4.798**
Germany	*0.105*	**57.400**	**−8.129**	*0.087*	**57.472**	**−8.218**
Italy	**0.137**	31.127	*−4.004*	*0.085*	46.217	**−5.245**
Japan	**0.148**	8.742	**−5.828**	0.062	13.031	**−5.999**
New Zealand	**0.111**	14.237	**−5.419**	*0.097*	14.762	**−5.814**
Norway	**0.116**	19.163	−3.285	*0.090*	24.266	−3.646
Portugal	**0.106**	20.157	**−9.129**	**0.110**	20.550	**−9.364**
Spain	*0.088*	35.162	−3.380	*0.087*	35.229	−3.466
Sweden	0.082	25.377	**−4.403**	*0.082*	26.106	*−4.469*
Switzerland	*0.091*	**86.834**	**−6.078**	0.050	**107.887**	**−6.657**
UK	0.066	**92.738**	**−7.055**	0.063	**92.987**	**−7.004**
USA	**0.152**	6.582	−2.395	0.075	12.272	−3.659

For further explanations see caption of Table 2 with the difference that the results in this table are based on the Quadratic Spectral kernel with bandwidth according to Newey and West (1994)

Table 10 Test results for CO_2 emissions for the sample range 1870–2016 (for New Zealand 1878–2016)

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Australia	0.083	14.723	−2.670	*0.086*	14.739	−2.609
Austria	0.058	**75.487**	**−4.846**	0.050	**76.545**	**−4.933**
Belgium	0.040	**98.242**	**−6.135**	0.044	**106.178**	**−6.718**
Canada	0.076	10.237	−3.339	0.043	22.005	**−4.614**
Denmark	0.038	38.968	**−4.726**	0.041	38.971	**−4.721**
Finland	0.035	**95.854**	**−5.874**	0.030	**106.083**	**−6.558**
France	0.047	39.043	**−4.929**	0.045	39.298	**−4.865**
Germany	**0.116**	**58.498**	**−7.247**	*0.097*	**58.571**	**−7.328**
Italy	0.081	25.955	**−4.484**	0.067	38.537	**−5.603**
Japan	0.078	8.913	**−5.960**	0.050	13.286	**−5.997**
New Zealand	0.073	11.792	**−5.549**	0.069	12.227	**−5.932**
Norway	0.060	26.703	−3.383	0.062	33.813	−3.609
Portugal	**0.113**	16.698	**−9.870**	**0.119**	17.024	**−10.277**
Spain	0.063	35.456	−3.445	0.058	35.524	−3.499
Sweden	0.060	35.546	*−4.150*	0.062	36.568	*−4.220*
Switzerland	*0.090*	**87.613**	**−6.213**	0.051	**108.855**	**−6.751**
UK	0.064	**73.503**	**−6.377**	0.061	**73.700**	**−6.316**
USA	0.062	7.072	−2.266	0.051	13.187	−3.108

For further explanations see caption of Table 2 with the difference that the results in this table are based on the Quadratic Spectral kernel with bandwidth according to Andrews (1991)

Table 11 Test results for SO_2 emissions for the sample range 1870–2016 (for New Zealand 1878–2016)

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Australia	0.050	38.851	−3.275	0.047	40.471	−3.384
Austria	0.072	28.834	**−4.191**	0.051	39.378	−4.073
Belgium	0.081	19.571	*−4.037*	0.043	30.249	−3.776
Canada	0.055	25.583	−3.599	0.053	25.741	−3.649
Denmark	0.061	11.836	−3.244	0.045	24.701	−4.009
Finland	0.063	28.260	*−4.100*	0.065	28.717	−3.910
France	*0.095*	6.345	−2.346	*0.096*	13.468	−2.255
Germany	0.082	5.941	−2.665	0.074	11.303	−2.590
Italy	**0.109**	3.572	−2.276	**0.209**	3.810	−1.559
Japan	0.084	15.155	−2.957	0.049	19.087	−4.118
New Zealand	0.072	9.859	**−4.862**	0.049	13.441	**−7.054**
Norway	0.064	26.826	−3.155	0.066	27.077	−3.231
Portugal	0.074	13.692	−2.968	0.076	13.752	−3.073
Spain	**0.117**	12.100	−1.699	**0.163**	13.089	−1.487
Sweden	0.053	13.118	−3.436	0.058	14.438	−2.967
Switzerland	*0.105*	9.960	−2.217	0.060	*55.467*	**−5.056**
UK	*0.087*	10.208	−1.629	0.063	21.014	−2.703
USA	**0.218**	9.985	−0.826	**0.219**	10.039	−0.811

For further explanations see caption of Table 2 with the difference that the results in this table are based on the Bartlett kernel with bandwidth according to Andrews (1991)

Table 12 Test results for SO_2 emissions for the sample range 1870–2016 (for New Zealand 1878–2016)

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Australia	0.049	37.798	−3.399	0.045	39.374	−3.519
Austria	0.075	25.453	−3.633	0.052	34.759	−3.608
Belgium	*0.096*	15.796	−3.701	0.040	24.414	−3.872
Canada	0.051	21.662	*−3.994*	0.048	21.796	−4.018
Denmark	*0.098*	9.939	−3.203	0.054	20.743	−4.036
Finland	0.076	23.523	*−3.977*	0.073	23.903	−3.905
France	**0.107**	5.214	−2.342	0.063	11.069	−2.566
Germany	*0.096*	5.821	−2.534	0.060	11.075	−2.861
Italy	0.085	6.081	−2.299	*0.086*	6.486	−1.809
Japan	**0.111**	15.898	−3.284	0.031	20.022	**−4.534**
New Zealand	*0.089*	12.421	**−4.867**	0.051	16.934	**−6.968**

Table 12 continued

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Norway	0.066	21.564	−3.220	0.064	21.766	−3.276
Portugal	*0.099*	11.272	−3.207	*0.100*	11.322	−3.303
Spain	0.071	11.803	−1.865	0.077	12.768	−1.857
Sweden	0.063	10.275	−3.364	0.065	11.310	−2.907
Switzerland	**0.207**	5.453	−1.655	0.055	30.369	**−5.156**
UK	0.072	2.175	−1.723	0.060	4.477	−2.747
USA	0.081	7.554	−1.351	0.071	7.595	−1.324

For further explanations see caption of Table 2 with the difference that the results in this table are based on the Quadratic Spectral kernel with bandwidth according to Newey and West (1994)

Table 13 Test results for SO_2 emissions for the sample range 1870–2016 (for New Zealand 1878–2016)

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Australia	0.056	38.767	−3.276	0.054	40.383	−3.385
Austria	0.065	31.234	**−4.260**	0.047	42.655	−4.135
Belgium	0.072	17.997	*−4.085*	0.046	27.816	−3.757
Canada	0.050	21.331	−3.567	0.049	21.463	−3.606
Denmark	0.050	11.538	−3.240	0.040	24.078	−4.055
Finland	0.056	28.437	**−4.168**	0.059	28.897	−3.972
France	*0.100*	5.998	−2.370	**0.266**	12.732	−2.364
Germany	*0.087*	5.983	−2.650	**0.110**	11.383	−2.611
Italy	**0.176**	2.954	−2.309	**0.953**	3.151	−1.554
Japan	0.073	15.490	−2.907	0.057	19.508	−4.048
New Zealand	*0.090*	9.577	**−4.853**	0.048	13.056	**−7.041**
Norway	0.066	27.005	−3.185	0.068	27.257	−3.242
Portugal	0.078	13.992	−2.962	*0.081*	14.053	−3.075
Spain	**0.339**	12.107	−1.733	**0.768**	13.097	−1.551
Sweden	0.046	11.847	−3.292	0.059	13.040	−2.848
Switzerland	*0.097*	10.241	−2.176	0.058	**57.034**	**−5.106**
UK	**0.161**	10.714	−1.758	0.066	22.056	−2.660
USA	**6.276**	8.172	−0.801	**8.516**	8.216	−0.766

For further explanations see caption of Table 2 with the difference that the results in this table are based on the Quadratic Spectral kernel with bandwidth according to Andrews (1991)

Robustness: Sample range until 2000

Table 14 Test results for CO_2 emissions for the sample range 1870–2000 (for New Zealand 1878–2000)

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Australia	**0.107**	11.294	−2.623	**0.106**	11.306	−2.530
Austria	0.055	**57.186**	−3.816	0.042	**57.988**	−3.822
Belgium	0.061	*51.074*	**−5.710**	0.057	*55.199*	**−5.667**
Canada	**0.144**	11.882	−3.355	0.056	25.542	**−4.878**
Denmark	0.047	36.027	**−4.673**	0.049	36.029	**−4.670**
Finland	0.048	**75.487**	**−5.719**	0.033	**83.542**	**−6.136**
France	0.066	27.859	**−4.929**	0.062	28.041	**−4.863**
Germany	**0.108**	**69.262**	**−8.001**	*0.090*	**69.349**	**−8.089**
Italy	**0.138**	34.120	**−4.174**	*0.088*	*50.661*	**−5.520**
Japan	**0.149**	8.718	**−5.889**	0.065	12.995	**−6.009**
New Zealand	**0.112**	13.698	**−5.422**	*0.098*	14.203	**−5.809**
Norway	**0.116**	20.583	−3.334	*0.092*	26.063	−3.617
Portugal	**0.108**	20.919	**−9.181**	**0.111**	21.327	**−9.412**
Spain	*0.091*	42.031	−3.315	*0.089*	42.112	−3.384
Sweden	0.084	29.150	**−4.312**	*0.084*	29.988	*−4.323*
Switzerland	*0.097*	**86.476**	**−6.291**	0.056	**107.442**	**−6.763**
UK	0.070	**91.459**	**−6.894**	0.067	**91.704**	**−6.848**
USA	**0.148**	12.620	−2.337	0.077	23.531	−3.558

For further explanations see caption of Table 2

Table 15 Test results for SO_2 emissions for the sample range 1870–2000 (for New Zealand 1878–2000)

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Australia	0.051	38.885	−3.402	0.048	40.506	−3.521
Austria	0.080	28.695	−3.776	0.055	39.188	−3.695
Belgium	*0.100*	17.752	−3.738	0.043	27.436	−3.556
Canada	0.055	22.976	*−3.865*	0.053	23.118	−3.888
Denmark	*0.101*	10.010	−3.160	0.057	20.890	−4.006
Finland	0.078	25.292	*−4.082*	0.077	25.701	−3.905
France	**0.112**	5.524	−2.353	0.067	11.725	−2.455
Germany	*0.098*	6.057	−2.584	0.062	11.524	−2.782

Table 15 continued

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Italy	*0.089*	3.835	−2.296	*0.090*	4.090	−1.729
Japan	**0.113**	15.237	−3.219	0.036	19.190	*−4.447*
New Zealand	*0.090*	11.831	**−4.849**	0.054	16.130	**−6.941**
Norway	0.069	22.719	−3.216	0.067	22.931	−3.283
Portugal	*0.100*	11.880	−3.175	**0.102**	11.933	−3.271
Spain	0.074	12.060	−1.799	0.080	13.046	−1.761
Sweden	0.066	11.155	−3.709	0.067	12.278	−2.908
Switzerland	**0.203**	6.766	−1.703	0.060	37.684	**−5.067**
UK	0.075	0.841	−1.510	0.066	1.731	−2.647
USA	0.085	10.517	−1.230	0.074	10.574	−1.200

For further explanations see caption of Table 2

Robustness: Old data vintage (sample range until 2000)

Table 16 Test results for CO_2 emissions for the sample range 1870–2000 (for New Zealand 1878–2000) using the earlier data vintage

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Australia	**0.296**	11.560	−2.886	**0.259**	12.868	−2.498
Austria	0.083	**63.516**	**−4.504**	*0.083*	**64.114**	**−4.547**
Belgium	*0.095*	**66.359**	**−5.661**	0.069	**68.694**	**−5.994**
Canada	**0.286**	11.110	−3.231	*0.098*	25.475	**−4.776**
Denmark	**0.119**	**49.891**	**−4.901**	*0.100*	*51.498*	**−4.943**
Finland	0.075	**73.254**	**−5.390**	0.042	**79.318**	**−5.906**
France	**0.107**	36.151	**−4.822**	**0.113**	36.193	**−4.857**
Germany	**0.257**	**61.717**	**−6.699**	**0.212**	**66.902**	**−7.610**
Italy	**0.158**	39.011	*−3.952*	**0.164**	39.024	−3.952
Japan	**0.293**	6.635	**−5.629**	**0.130**	10.144	**−5.650**
New Zealand	**0.270**	11.111	**−5.198**	**0.132**	13.851	**−7.175**
Norway	**0.191**	25.439	*−4.080*	**0.164**	29.202	−3.516
Portugal	**0.222**	18.394	**−9.126**	**0.207**	18.922	**−9.537**

Table 16 continued

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Spain	**0.183**	36.268	−3.449	**0.169**	36.768	−3.604
Sweden	**0.170**	29.349	−3.604	**0.131**	32.206	−3.666
Switzerland	**0.268**	31.297	−3.764	**0.105**	46.214	**−5.357**
UK	**0.125**	**130.958**	**−8.997**	**0.111**	**136.550**	**−9.118**
USA	**0.357**	12.448	−2.151	**0.152**	25.426	−3.869

For further explanations see caption of Table 2

Table 17 Test results for SO_2 emissions for the sample range 1870–2000 (for New Zealand 1878–2000) using the earlier data vintage

	Quadratic			Cubic		
	CT	$\hat{P}_u$	PO_t	CT	$\hat{P}_u$	PO_t
Australia	**0.215**	20.375	−2.746	**0.214**	20.404	−2.708
Austria	**0.165**	15.311	−3.478	**0.175**	20.231	−3.645
Belgium	**0.188**	26.919	*−4.068*	*0.088*	42.594	−4.129
Canada	0.074	21.233	*−4.879*	0.068	21.362	**−4.949**
Denmark	**0.266**	24.030	−1.577	**0.153**	44.356	−3.758
Finland	**0.120**	17.851	−2.967	**0.131**	19.342	−2.982
France	**0.159**	11.392	−2.410	**0.107**	18.982	−3.186
Germany	**0.131**	17.167	−2.237	**0.121**	21.299	−2.672
Italy	**0.129**	23.783	−2.554	**0.147**	31.723	−3.065
Japan	**0.255**	5.853	−3.538	**0.150**	7.442	−3.919
New Zealand	**0.226**	20.312	−2.785	**0.166**	22.332	−3.322
Norway	**0.151**	22.654	−3.680	**0.153**	22.990	−3.765
Portugal	**0.217**	*46.728*	**−4.589**	**0.206**	47.775	**−4.611**
Spain	**0.180**	13.689	−2.559	**0.221**	14.580	−2.349
Sweden	**0.130**	16.067	−3.128	**0.146**	17.231	−3.100
Switzerland	**0.119**	*46.846*	**−5.564**	**0.112**	*49.448*	**−5.559**
UK	**0.236**	28.799	−2.899	0.075	**58.572**	**−5.817**
USA	**0.315**	12.041	−2.369	**0.142**	18.025	−3.226

For further explanations see caption of Table 2

References

Andrews DWK (1991) Heteroskedasticity and autocorrelation consistent covariance matrix estimation. Econometrica 59:817–858

Boden TA, Marland G, Andres RJ (2018) Global, regional, and national fossil-fuel CO_2 emissions. Carbon Dioxide Information Analysis Center at Appalachian State University, Boone North Carolina

Bolt J, Inklaar R, de Jong H, van Zanden JL (2018). Rebasing Maddison: new income comparisons and the shape of long-run economic development. Maddison Project Working Paper, 10. Maddison Project Database, version 2018

Bradford D, Fender R, Shore SH, Wagner M (2005) The environmental Kuznets curve: exploring a fresh specification. Contrib Econ Anal Policy 4(1):5

Chang Y, Park JY, Phillips PCB (2001) Nonlinear econometric models with cointegrated and deterministically trending regressors. Econom J 4:1–36

Choi I, Saikkonen P (2010) Tests for nonlinear cointegration. Econom Theory 26:682–709

Darvas Z (2008) Estimation bias and inference in overlapping autoregressions: implications for the target-zone literature. Oxf Bull Econ Stat 70:1–22

de Jong R (2002) Nonlinear estimators with integrated regressors but without exogeneity. Mimeo

de Jong R, Wagner M (2022) Panel cointegrating polynomial regression analysis and an illustration with the environmental Kuznets curve. Forthcoming in Econom Stat. https://doi.org/10.1016/j.ecosta.2022.03.005

Friedl B, Getzner M (2003) Determinants of CO_2 emissions in a small open economy. Ecol Econ 45:133–148

Galeotti M, Manera M, Lanza A (2009) On the robustness of robustness checks of the EKC hypothesis. Environ Resour Econ 42:551–574

Grabarczyk P, Wagner M, Frondel M, Sommer S (2018) A cointegrating polynomial regression analysis of the material Kuznets curve. Resour Policy 57:236–245

Grossman GM, Krueger AB (1993) Environmental impacts of a North American free trade agreement. In: Garber P (ed) The Mexico-US free trade agreement. MIT Press, Cambridge, pp 13–56

Holtz-Eakin D, Selden TM (1995) Stoking the fires? CO_2 emissions and economic growth. J Public Econ 57:85–101

Hong SH, Phillips PCB (2010) Testing linearity in cointegrating relations with an application to purchasing power parity. J Bus Econ Stat 28:96–114

Ibragimov R, Phillips PCB (2008) Regression asymptotics using martingale convergence methods. Economet Theor 24:888–947

Jansson M (2002) Consistent covariance matrix estimation for linear processes. Econom Theor 18:1449–1459

Kuznets S (1955) Economic growth and income inequality. Am Econ Rev 45:1–28

Kwiatkowski D, Phillips PCB, Schmidt P, Shin Y (1992) Testing the null hypothesis of stationarity against the alternative of a unit root: how sure are we that economic time series have a unit root? J Econom 54:159–178

Labson BS, Crompton PL (1993) Common trends in economic activity and metals demand: cointegration and the intensity of use debate. J Environ Econ Manag 25:147–161

National Research Council (1975) Air quality and stationary source emission control. The National Academies Press, Washington, DC

Newey W, West K (1994) Automatic lag selection in covariance matrix estimation. Rev Econ Stud 61:631–654

OECD (2020) Air and GHG emissions (indicator). https://doi.org/10.1787/93d10cf7-en (Accessed on 05 March 2020)

Park JY, Phillips PCB (1999) Asymptotics for nonlinear transformations of integrated time series. Econom Theor 15:269–298

Park JY, Phillips PCB (2001) Nonlinear regressions with integrated time series. Econometrica 69:117–161

Perman R, Stern DI (2003) Evidence from panel unit root and cointegration tests that the environmental Kuznets curve does not exist. Aust J Agric Resour Econ 47:325–347

Piketty T (2014) Capital in the twenty-first century. Harvard University Press, Cambridge

Phillips PCB, Hansen BE (1990) Statistical inference in instrumental variables regression with I(1) processes. Rev Econ Stud 57:99–125

Phillips PCB, Ouliaris S (1990) Asymptotic properties of residual based tests for cointegration. Econometrica 58:165–193

Shin Y (1994) A residual-based test for the null of cointegration against the alternative of no cointegration. Econom Theor 10:91–115

Smith SJ, van Aardenne J, Klimont Z, Andres RJ, Volke A, Delgado Arias S (2011) Anthropogenic sulfur dioxide emissions, 1850–2005: national and regional data set by source category, Version 2.86. Data distributed by the NASA Socioeconomic Data and Applications Center (SEDAC), CIESIN, Columbia University, Palisades, New York

Stern DI (2004) The rise and fall of the environmental Kuznets curve. World Dev 32:1419–1439

Stypka O, Wagner M (2019) The Phillips unit root tests for polynomials of integrated processes revisited. Econ Lett 176:109–113

Stypka O, Wagner M (2020) Cointegrating multivariate polynomial regressions: fully modified OLS estimation and inference. Mimeo

Stypka O, Wagner M, Grabarczyk P, Kawka R (2019) On the asymptotic validity of “standard” FM-OLS estimation and inference in cointegrating polynomial regression models. Revised and resubmitted to Econom Theor

Svensson LEO (1992) An interpretation of recent research on exchange rate target zones. J Econ Perspect 6:119–144

Vogelsang TJ, Wagner M (2016) An integrated modified OLS RESET test for cointegrating regressions. Mimeo

Wagner M (2008) The carbon Kuznets curve: a cloudy picture emitted by bad econometrics? Energy Resour Econ 30:388–408

Wagner M (2012) The Phillips unit root tests for polynomials of integrated processes. Econ Lett 114:299–303

Wagner M (2015) The environmental Kuznets curve, cointegration and nonlinearity. J Appl Economet 30:948–967

Wagner M, Hong SH (2016) Cointegrating polynomial regressions: fully modified OLS estimation and inference. Econom Theor 32:1289–1315

Wagner M, Reichold K (2022) Cointegrating polynomial regressions: fully modified OLS estimation and inference. Revised and resubmitted to Econom Rev

Wagner M, Grabarczyk P, Hong SH (2020) Fully modified OLS estimation and inference for seemingly unrelated polynomial cointegrating regressions and the environmental kuznets curve for carbon dioxide emissions. J Econom 214:216–255

Yandle B, Bjattarai M, Vijayaraghavan M (2004) Environmental Kuznets curves: a review of findings, methods, and policy implications. Research Study 02.1 update, PERC