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Selection of materials in metal additive manufacturing via three-way decision-making

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Abstract

In this paper, an approach for selection of materials in metal additive manufacturing based on three-way decision-making is proposed. The process of this approach is divided into three stages. First, a decision matrix for a material selection problem in metal additive manufacturing is established based on the basic components of the problem and normalised via a ratio model and a unified rule. Second, the summary loss function, conditional probability, and expected losses of each alternative material are calculated according to the weighted averaging operator, grey relational analysis, and the three-way decision theory, respectively. Third, the three-way decision-making results for the problem are generated according to the developed generation rules and the best material for the problem is selected based on the generated results. The application of the approach is illustrated via a material selection example in metal additive manufacturing. The effectiveness of the approach is demonstrated via a quantitative comparison with several existing approaches. The demonstration results suggest that the proposed approach is as effective as the existing approaches and is more flexible and advantageous in solving a material selection problem in metal additive manufacturing.

1 Introduction

Metal additive manufacturing (AM) is a set of processes that join metal materials layer upon layer to build metal components directly from their three-dimensional model data [1–4]. Representative metal AM technologies include direct metal laser sintering, selective laser melting, electron beam melting, and direct metal deposition [5–7]. The research and application of metal AM processes are gaining importance and popularity because of their attractive advantages over conventional subtractive manufacturing technologies, which mainly include flexible geometric design, no extra cost to achieve geometric complexity, eliminating costs on waste material, tooling, and non-essential assembly, and reducing the production time [8].

Application of a metal AM technology to manufacture a component includes a set of activities, where material selection is a critical one [9, 10]. In this activity, a material that can simultaneously optimise certain key factors for a specific application, such as the quality, property, manufacturability, and cost of the component, is selected from a certain number of available materials. Currently, nearly 1500 materials for metal AM have been developed and sold on the market [11]. How to select appropriate materials to build a component satisfying certain requirements is not easy [12]. There are two main reasons for this. First, the selection needs a comprehensive understanding of the utilities, performance, strengths, and limitations of all alternative materials and the influence of materials on the property and quality of the component, which is difficult to achieve. Second, different materials might belong to the same material type and have considerable overlap in utilities, performance, properties, metallurgical incompatibilities, and limitations [13, 14], which increases the difficulty in actual selection.

To assist selection of materials in AM, many approaches have been developed in the literature [15]. For example, a knowledge-based system for AM material selection

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integrated with an existing computer-aided design (CAD) environment was developed in [16]; an AM process and material selection approach based on CAD feature analysis was presented in [17]; a targeted material selection process for polymers in laser sintering was introduced in [18]; An integrated decision-making model for multi-criterion decision-making (MCDM) problems in design for AM based on an aggregation of deviation and similarity models was developed in [19]; an integrated AM material and process selection method based on analytical hierarchy process and simple additive weighting was presented in [20]; A heuristic and analytical algorithm for AM material selection based on analytical hierarchy process was developed in [21]; an integrated AM material-design-process selection methodology based on analytical hierarchy process was developed in [22]; a multi-criterion evaluation system for AM process and resource selection based on analytical hierarchy process was developed in [23]; an AM machine and material selection approach based on best-worst method was presented in [24]; a generic method for MCDM problems in design for AM based on a fuzzy power weighted Maclaurin symmetric mean operator was developed in [25]; four MCDM methods including SAW (simple additive weighting), MOORA (multi-objective optimisation based on ratio analysis), TOPSIS (technique for order performance by similarity to ideal solution), and VIKOR (abbreviation of a term in Serbian that means multi-criterion optimisation and compromise solution) were applied to sustainable AM material selection in [26]; a material selection approach for biomedical AM based on TOPSIS was developed in [27]; a metal AM material selection approach based on information entropy method and CODAS (combinative distance-based assessment) was presented in [28].

The selection of AM materials is usually associated with the selection of AM processes/machines, since the materials supported by different processes/machines are usually different. In practice, to fabricate a part or a group of parts, an AM process/machine to be used is generally determined first, and then a material satisfying certain requirements is selected from among those supported by the process/machine and also available on the market [20]. An AM process/machine selection approach might also be applied to AM material selection. Currently, there are also many AM process/machine selection approaches in the literature [29]. For example, an AM process selection method based on graph theory and matrix approach was developed in [30]; an approach for selection of AM techniques based on an adaptive analytical hierarchy process was presented in [31]; six MCDM methods including TOPSIS, graph theory and matrix approach, analytical hierarchy process, multiplicative analytical hierarchy process, simple pair analysis, and Verein Deutscher Ingenieure were applied to

selection of AM technologies in [32]; an AM process selection approach based on analytical hierarchy process and the technical specifications of a part was presented in [33]; a decision-making methodology to facilitate AM process selection and assist product/part design based on analytical hierarchy process was developed in [34]; an AM process selection method based on fuzzy logic was proposed in [35]; An experimental design approach for the selection of AM processes based on TOPSIS was developed in [36]; a decision support method for AM process selection based on knowledge value measuring was proposed in [37]; a decision support system for AM process selection based on fuzzy set was developed in [38]; a weighted rough set based fuzzy axiomatic design approach for AM process selection was presented in [39]; an integration of fuzzy analytical hierarchy process and fuzzy TOPSIS was applied to prioritise AM processes for micro-fabrication in [40]; a decision support system for AM process selection based on a hybrid of analytical hierarchy process and a modified TOPSIS was developed in [41]; an approach to evaluate the AM machine selection problem for healthcare applications based on an integration of fuzzy analytic hierarchy process and TOPSIS was presented in [42]; an AM machine selection approach based on fuzzy power weighted Bonferroni aggregation operators was proposed in [43]; an MCDM method for AM process selection based on stepwise weight assessment ratio analysis and complex proportional assessment was presented in [44].

Each of the AM material/process/machine selection approaches above can work well in its specific context, but they lack flexibility and could generate undesirable results when prior knowledge for selection is insufficient or the cost of improper decisions is high, because they are all based on two-way decision model (2WDM). 2WDM is a granular computing technique commonly used in MCDM. As the name suggests, there are two decisions on an alternative in 2WDM. They are acceptance and rejection, which mean that an alternative is either accepted or rejected. 2WDM is simple and direct and has certain advantages when prior knowledge for decision-making is sufficient or replacement of improper decisions does not require a high cost. However, there is yet no evidence that a comprehensive knowledge base for selection of materials in metal AM has been established. Further, many metal AM processes are widely known for their high machine, material, and processing costs [45, 46].

In this paper, three-way decision model (3WDM) is introduced to propose a new approach for selection of materials in metal AM. 3WDM is another granular computing technique used in MCDM [47]. Compared to 2WDM, 3WDM has an additional decision called abstaining. This makes the model more flexible and advantageous than 2WDM since it effectively prevents

Fig. 1 A comparison of 2WDM and 3WDM using a real-life example

Decision	The last egg is good	The last egg is bad
Beating the last egg to the bowl	Six eggs will not be wasted	The five eggs beaten in the bowl will be wasted
Discarding the last egg	The last egg will be wasted	The five eggs beaten in the bowl will not be wasted
Beating the last egg to a new bowl to have a check	Six eggs will not be wasted and he will need to wash the new bowl	The five eggs beaten in the bowl will not be wasted and he will need to wash the new bowl

premature classification of the alternatives at the edge of acceptance and rejection. Because of such feature, 3WDM is more suitable for the decision-making problems where there is not enough prior knowledge or replacement of inappropriate decisions is costly [48]. Here a real-life decision-making example cited from [49] is introduced to intuitively compare the two models. A woman was going to make an omelette with only six eggs left in her kitchen. After she had beaten five eggs in a bowl, her husband arrived to assist with the last egg. He appreciates decision-making theory. Two thoughts crossed his mind as he picked up the last egg: Was this egg a good one? Which decision-making model was most suitable for the current circumstance? If he adopts 2WDM, there will be two options for him. One option is beating the egg to the bowl and the other is discarding it. The benefits and possible costs associated with these options are shown in Fig. 1. Naturally, if he is confident that the last egg is a good egg, then the first option is more suitable; Otherwise, the second option is better. However, if it is difficult for him to judge whether the last egg is good or bad, then neither option seems to be suitable. In other words, it appears that 2WDM is not suited for this circumstance. If the man adopts 3WDM, an additional option, beating the egg to a new bowl to have a check, will be available for him. The benefits and possible costs associated with this option are also shown in Fig. 1. It can be seen that 3WDM is more suitable than 2WDM under the circumstance.

The remainder of the paper is organised as follows: Section 2 describes the details of the proposed approach. An illustration of the application of the approach and a demonstration of the effectiveness of the approach are documented in Section 3. Section 4 draws a conclusion of the paper.

2 The proposed approach

In this section, the proposed approach for selection of materials in metal AM is described in detail. A general flow of this approach is delineated in Fig. 2. The proposed approach takes as input a set of alternative materials,

certain selection criteria, value of each selection criterion of each alternative material, and weight of each selection criterion and outputs three-way decision-making results together with the best material for a material selection problem in metal AM. Its main process is divided into three stages: initialisation, calculation, and determination. In the initialisation stage, a decision matrix for the problem is first established and then normalised. In the calculation stage, the summary loss function, conditional probability, and expected losses of each alternative material are successively calculated. In the determination stage, the three-way decision-making results for the problem are generated and the best material for the problem is selected. The details of these three stages are explained below.

2.1 Initialisation stage

There are two steps in the initialisation stage. The first step is to establish a decision matrix for a material selection problem in metal AM. Formally, the basic components of this problem include m alternative materials $M_1, M_2, ..., M_m$, n selection criteria $C_1, C_2, ..., C_n$, value of each selection criterion (C_j) of each alternative material (M_i) $x_{i,j}$ ($i = 1, 2, ..., m$; $j = 1, 2, ..., n$), and weight of each selection criterion (C_j) w_j ($0 \leq w_j \leq 1$ and $\Sigma_{j=1}^{n} w_j = 1$). The alternative materials can be screened from current material types available for metal AM, which mainly include stainless steel, tool steel, titanium alloy, aluminium alloy, inconel, nickel alloy, and copper alloy [11]. The selection criteria are generally identified according to the specific requirements of the component to be built, in which the properties of alternative materials (e.g. density, melting point, tensile strength, hardness, thermal conductivity), the predicted properties of the built component (e.g. predicted tensile strength, predicted hardness, predicted elongation, predicted surface roughness), and the predicted cost to build the component may be considered [50]. The values of selection criteria of alternative materials can be acquired from vendor documents and benchmark data, prediction based on theoretical models, simulations, or experiments, or evaluation based on expert experience [29]. Based

Fig. 2 A general flow
of the proposed approach
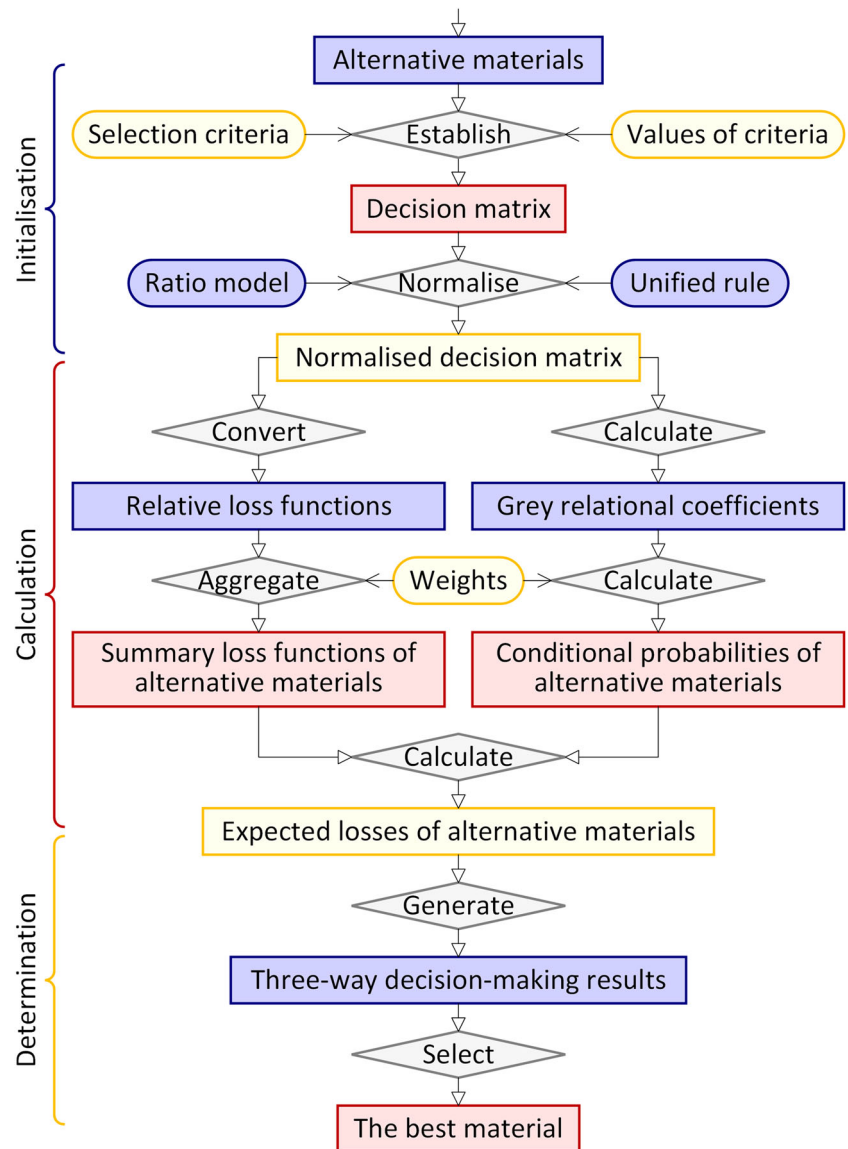
on these components, a decision matrix is established as $A = [x_{i,j}]_{m \times n}$.

The second step is to normalise the established decision matrix. The values of different criteria usually have different ranges, which cause inconvenience to subsequent calculations. To this end, each criterion value is converted into a number in [0, 1] using the following ratio model [51]:

$$y_{i,j} = \frac{x_{i,j}}{\sqrt{\sum_{i=1}^{m} x_{i,j}^2}} \qquad (1)$$

Further, there are generally two types of criteria in an MCDM problem. They are positive and negative criteria. The value of a positive criterion is positively correlated with the decision-making results, while the value of a negative criterion is the opposite. For example, the hardness and price of a material respectively belong to a positive criterion and

a negative criterion in a material selection problem in metal AM, since the greater the hardness and the lower the price, the better the decision-making results. To unify the effect of different types of criteria, the following unified rule is applied to each converted criterion value:

$$z_{i,j} = \begin{cases} y_{i,j} & \text{if } C_j \text{ is a positive criterion} \\ 1 - y_{i,j} & \text{if } C_j \text{ is a negative criterion} \end{cases} \qquad (2)$$

Through the conversion and unification, a normalised decision matrix is obtained as $B = [z_{i,j}]_{m \times n}$.

2.2 Calculation stage

The calculation stage consists of two parallel steps and a subsequent step. The first step is to calculate the summary loss function of each alternative material. Let S_j and $\neg S_j$ be two states that respectively stand for the alternative material

Table 1 Relative loss function derived from $z_{i,j}$

Decision	S_j	$\neg S_j$
D_{ACC}	0	$1 - z_{i,j}$
D_{ABS}	$\lambda_j z_{i,j}$	$\lambda_j(1 - z_{i,j})$
D_{REJ}	$z_{i,j}$	0

M_i meets the selection criterion C_j and M_i does not meet C_j. Each state corresponds to the three decisions acceptance (denoted as D_{ACC}), abstaining (denoted as D_{ABS}), and rejection (denoted as D_{REJ}) in 3WDM. Inspired by the relative loss functions developed in [52–54], a relative loss function derived from the normalised criterion value $z_{i,j}$ is established in Table 1, where $0 \leq \lambda_j \leq 1$ is a risk avoidance coefficient that corresponds to C_j.

This relative loss function is explained as follows: The cost of correct decisions is 0; The cost of rejection is $z_{i,j}$ when M_i meet C_j; The cost of acceptance is $1 - z_{i,j}$ when M_i does not meet C_j; The cost of abstaining in each state lies between the cost of acceptance and the cost of rejection in this state. According to the relative loss function in Table 1, each normalised criterion value $z_{i,j}$ is converted into a relative loss function:

$$f(z_{i,j}) = \begin{bmatrix} 0 & 1 - z_{i,j} \\ \lambda_j z_{i,j} & \lambda_j(1 - z_{i,j}) \\ z_{i,j} & 0 \end{bmatrix} \tag{3}$$

Using the weighted averaging operator, the relative loss functions of each alternative material (i.e. the n relative loss functions $f(z_{i,1})$, $f(z_{i,2})$, ..., $f(z_{i,n})$) are aggregated into a summary loss function:

$$f(z_i) = \begin{bmatrix} 0 & \Sigma_{j=1}^n w_j(1 - z_{i,j}) \\ \Sigma_{j=1}^n w_j \lambda_j z_{i,j} & \Sigma_{j=1}^n w_j \lambda_j(1 - z_{i,j}) \\ \Sigma_{j=1}^n w_j z_{i,j} & 0 \end{bmatrix} \tag{4}$$

The second step is to calculate the conditional probability of each alternative material. One of the crucial elements of 3WDM is the conditional probability. In some applications of 3WDM, the conditional probabilities are assigned an identical fixed value. However, the conditional probabilities of the alternatives in an MCDM problem are typically different and generally need to be determined according to the values of criteria of alternatives. In [55], TOPSIS is used to determine the conditional probabilities. The two states of a criterion are referred to as the positive and negative ideal solutions. The conditional probabilities of alternatives are implied by the final relative closeness degrees. This method, which uses distance as a scale, can only reflect how the data curves are positioned in relation to one another and cannot convey the significance of alternatives through the trend in a data sequence. Grey relational analysis [56], a technique for assessing the degree of relationship between data sequences, was found to provide an effective measure for the trend difference of data sequences and the similarity between data curves [53]. To this end, grey relational analysis is applied to calculate the conditional probability of each alternative material. Firstly, the grey relational coefficients between the alternative material M_i and $\max_{j=1}^n\{z_{i,j}\}$ and between M_i and $\min_{j=1}^n\{z_{i,j}\}$ are calculated as

$$\delta_{i,j}^+ = \frac{\min_{i=1}^m \min_{j=1}^n \left\{\left|z_{i,j} - \max_{j=1}^n\{z_{i,j}\}\right|\right\} + 0.5 \max_{i=1}^m \max_{j=1}^n \left\{\left|z_{i,j} - \max_{j=1}^n\{z_{i,j}\}\right|\right\}}{\left|z_{i,j} - \max_{j=1}^n\{z_{i,j}\}\right| + 0.5 \max_{i=1}^m \max_{j=1}^n \left\{\left|z_{i,j} - \max_{j=1}^n\{z_{i,j}\}\right|\right\}} \tag{5}$$

$$\delta_{i,j}^- = \frac{\min_{i=1}^m \min_{j=1}^n \left\{\left|z_{i,j} - \min_{j=1}^n\{z_{i,j}\}\right|\right\} + 0.5 \max_{i=1}^m \max_{j=1}^n \left\{\left|z_{i,j} - \min_{j=1}^n\{z_{i,j}\}\right|\right\}}{\left|z_{i,j} - \min_{j=1}^n\{z_{i,j}\}\right| + 0.5 \max_{i=1}^m \max_{j=1}^n \left\{\left|z_{i,j} - \min_{j=1}^n\{z_{i,j}\}\right|\right\}} \tag{6}$$

Then, the grey relational degrees of M_i with respect to $\max_{j=1}^n\{z_{i,j}\}$ and of M_i with respect to $\min_{j=1}^n\{z_{i,j}\}$ are calculated as

$$\delta_i^+ = \Sigma_{j=1}^n w_j \delta_{i,j}^+ \tag{7}$$

$$\delta_i^- = \Sigma_{j=1}^n w_j \delta_{i,j}^- \tag{8}$$

Finally, the conditional probability of M_i is computed as the relative closeness of the grey relation of M_i

$$P(S|[M_i]) = \frac{\delta_i^+}{\delta_i^+ + \delta_i^-} \tag{9}$$

where S is a state that stands for the alternative material M_i meets an aggregation of all n criteria.

The third step is to calculate the expected losses of each alternative material. Let $\neg S$ be a state that stands for the

alternative material M_i does not meet an aggregation of all n criteria. If $P(S|[M_i]) + P(\neg S|[M_i]) = 1$, according to the three-way decision theory [47], the summary loss function in Eq. (4), and the conditional probability in Eq. (9), the expected losses of M_i when making the decision D_* ($* = $ ACC, ABS, REJ) are calculated as

$$L(D_{ACC}|[M_i]) = \left(1 - P(S|[M_i])\right)\left(\Sigma_{j=1}^n w_j(1 - z_{i,j})\right) \quad (10)$$

$$L(D_{ABS}|[M_i]) = P(S|[M_i])\left(\Sigma_{j=1}^n w_j \lambda_j z_{i,j}\right) \\ + \left(1 - P(S|[M_i])\right)\left(\Sigma_{j=1}^n w_j \lambda_j (1 - z_{i,j})\right) \quad (11)$$

$$L(D_{REJ}|[M_i]) = P(S|[M_i])\left(\Sigma_{j=1}^n w_j z_{i,j}\right) \quad (12)$$

2.3 Determination stage

There are two steps in the determination stage. The first step is to generate three-way decision-making results for the material selection problem. According to the Bayesian decision theory, the decision with the lowest cost is the best one [47]. Based on this, three decision rules are developed as follows:

(1) If $L(D_{ACC}|[M_i]) \le L(D_{ABS}|[M_i])$ and $L(D_{ACC}|[M_i]) \le L(D_{REJ}|[M_i])$, then make the decision D_{ACC} for M_i;
(2) If $L(D_{ABS}|[M_i]) \le L(D_{ACC}|[M_i])$ and $L(D_{ABS}|[M_i]) \le L(D_{REJ}|[M_i])$, then make the decision D_{ABS} for M_i;
(3) If $L(D_{REJ}|[M_i]) \le L(D_{ACC}|[M_i])$ and $L(D_{REJ}|[M_i]) \le L(D_{ABS}|[M_i])$, then make the decision D_{REJ} for M_i.

On the basis of these decision rules, the three-way decision-making results can be generated according to the following rules: If the decision D_{ACC} is made for M_i, then classify M_i to an acceptance set M_{ACC}; If the decision D_{ABS} is made for M_i, then classify M_i to an abstaining set M_{ABS}; If the decision D_{REJ} is made for M_i, then classify M_i to a rejection set M_{REJ}.

The second step is to select the best material. According to the three-way decision theory [47] and the generated three-way decision-making results, the best material is selected from the acceptance set M_{ACC} if this set is non-empty. Otherwise, it is selected from the abstaining set M_{ABS}.

3 Application and demonstration

In this section, the application of the proposed approach is first illustrated via a material selection example in metal AM [28]. Then a quantitative comparison between the proposed approach and several existing approaches is carried out to demonstrate the effectiveness of the approach.

3.1 Application of the proposed approach

A user needs to select a material from eight alternative materials to build a component using a metal AM process. The eight alternative materials are 316L stainless steel (M_1), H13 tool steel (M_2), Ti6Al4V titanium alloy (M_3), AlSi10Mg aluminium alloy (M_4), AlSi12Cu2Fe aluminium alloy (M_5), Inconel 718 (M_6), Hastelloy X nickel alloy (M_7), and CuSn10 copper alloy (M_8). The selection criteria of these materials are identified as price (C_1), density (C_2), tensile strength (yield) (C_3), elastic modulus (C_4), Brinell hardness (C_5), and electrical resistivity (C_6). The values of these criteria of the eight alternative materials are cited from [28] and listed in Table 2. The weights of the selection criteria are assumed to be 0.3, 0.1, 0.3, 0.1, 0.1, and 0.1, respectively.

Using the proposed approach, the material selection problem above can be solved according to the following steps:

(1) Establish a decision matrix. Based on the eight alternative materials, six selection criteria, and values of

Table 2 Values of six selection criteria of eight alternative materials

Material	Price (/kg)	Density (g/cm^3)	Tensile strength (MPa)	Elastic modulus (GPa)	Brinell hardness	Electrical resistivity ($\mu\Omega\cdot$m)
316L	380	8.000	170	193.0	217	0.74000
H13	190	7.800	1380	215.0	207	0.52000
Ti6Al4V	1990	4.420	910	114.0	334	1.78000
AlSi10Mg	280	2.580	190	70.0	75	0.05943
AlSi12Cu2Fe	650	2.670	140	74.5	100	0.07500
Inconel 718	3000	8.192	550	204.9	363	1.18200
Hastelloy X	4000	8.220	385	205.0	230	1.18000
CuSn10	600	8.800	180	130.0	70	0.16000

Table 3 Calculated summary loss functions

Material	Function	Decision	S	$\neg S$
M_1	$f(z_1)$	D_{ACC}	0	0.5464
		D_{ABS}	0.1587	0.1913
		D_{REJ}	0.4536	0
M_2	$f(z_2)$	D_{ACC}	0	0.3424
		D_{ABS}	0.2301	0.1199
		D_{REJ}	0.6576	0
M_3	$f(z_3)$	D_{ACC}	0	0.4902
		D_{ABS}	0.1784	0.1716
		D_{REJ}	0.5098	0
M_4	$f(z_4)$	D_{ACC}	0	0.6412
		D_{ABS}	0.1256	0.2244
		D_{REJ}	0.3588	0
M_5	$f(z_5)$	D_{ACC}	0	0.6637
		D_{ABS}	0.1177	0.2323
		D_{REJ}	0.3363	0
M_6	$f(z_6)$	D_{ACC}	0	0.5838
		D_{ABS}	0.1457	0.2043
		D_{REJ}	0.4162	0
M_7	$f(z_7)$	D_{ACC}	0	0.6865
		D_{ABS}	0.1097	0.2403
		D_{REJ}	0.3135	0
M_8	$f(z_8)$	D_{ACC}	0	0.6118
		D_{ABS}	0.1359	0.2141
		D_{REJ}	0.3882	0

the six selection criteria of the eight alternative materials, a decision matrix for the material selection problem is established as $A = [x_{i,j}]_{8\times 6}$, where the values of $x_{i,j}$ are listed in Table 2.

(2) Normalise the decision matrix. According to Eq. (1), each criterion value in A (i.e. $x_{i,j}$) is converted into a number in [0, 1] (i.e. $y_{i,j}$). Further, among the six selection criteria, price is a negative criterion, while the remaining ones are positive criteria. Based on this, the converted criterion values (i.e. $[y_{i,j}]_{8\times 6}$) are normalised according to Eq. (2) and a normalised decision matrix is obtained as

$$B = [z_{i,j}]_{8\times 6} = \begin{bmatrix} 0.9306 & 0.4157 & 0.0936 & 0.4234 & 0.3403 & 0.2836 \\ 0.9653 & 0.4053 & 0.7596 & 0.4716 & 0.3246 & 0.1993 \\ 0.6367 & 0.2297 & 0.5009 & 0.2501 & 0.5237 & 0.6821 \\ 0.9489 & 0.1341 & 0.1046 & 0.1535 & 0.1176 & 0.0228 \\ 0.8813 & 0.1388 & 0.0771 & 0.1634 & 0.1568 & 0.0287 \\ 0.4523 & 0.4257 & 0.3028 & 0.4495 & 0.5692 & 0.4529 \\ 0.2697 & 0.4272 & 0.2119 & 0.4497 & 0.3607 & 0.4522 \\ 0.8905 & 0.4573 & 0.0991 & 0.2852 & 0.1098 & 0.0613 \end{bmatrix}$$

(3) Calculate the summary loss functions. According to Eq. (3), each normalised criterion value in B (i.e. $z_{i,j}$) is converted into a relative loss function (i.e. $f(z_{i,j})$), in which the risk avoidance coefficients of the six selection criteria take the same value 0.35. Then according to Eq. (4) and the given weights, the relative loss functions of each alternative material (i.e. $f(z_{i,1}), f(z_{i,2}), ..., f(z_{i,9})$) are aggregated into a summary loss function (i.e. $f(z_i)$), as listed in Table 3.

(4) Calculate the conditional probabilities. According to Eqs. (5)–(9), the conditional probability of each alternative material (i.e. $P(S|[M_i])$) is respectively calculated, as listed in Table 4.

(5) Calculate the expected losses. According to Eqs. (10)–(12), the expected losses of each alternative material when making the decision D_* ($* =$ ACC, ABS, REJ) are respectively calculated, as listed in Table 5.

(6) Generate three-way decision-making results. According to the developed decision and classification rules, the three-way decision-making results for the material selection problem are obtained as $M_{ACC} = \{M_2\}$, $M_{ABS} = \{M_1, M_3, M_6\}$, and $M_{REJ} = \{M_4, M_5, M_7, M_8\}$.

(7) Select the best material. Based on the generated three-way decision-making results, M_2 (i.e. H13 tool steel) is selected as the best material for the material selection problem.

3.2 Demonstration of effectiveness

As reviewed in the introduction, most of the existing AM material/process/machine selection approaches are based on certain MCDM methods, which mainly include analytical hierarchy process, best-worst method, fuzzy power weighted Bonferroni aggregation operators (FPWBAOs), fuzzy power weighted Maclaurin symmetric mean operator (FPWMSMO), SAW, MOORA, TOPSIS, VIKOR, and CODAS. To demonstrate the effectiveness of the proposed approach, a quantitative comparison between it and the existing approaches based on FPWBAOs [43], FPWMSMO [25], SAW [26], MOORA [26], TOPSIS [26], VIKOR [26], and CODAS [28] is carried out. In this comparison, each approach is applied to solve the material selection problem above. Each value in Table 2 is converted into a number in [0, 1] using the ratio model in Eq. (1). The converted results together with the given criterion weights are taken as the input of the existing approaches. It is worth noting that the existing approaches based on analytical hierarchy

Table 4 Calculated conditional probabilities

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
$P(S|[M_i])$	0.4708	0.5489	0.5384	0.4405	0.4402	0.4826	0.4841	0.4538

Table 5 Calculated expected losses

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
$L(D_{ACC}|[M_i])$	0.2892	0.1545	0.2262	0.3587	0.3715	0.3020	0.3542	0.3342
$L(D_{ABS}|[M_i])$	0.1759	0.1804	0.1753	0.1809	0.1818	0.1760	0.1771	0.1786
$L(D_{REJ}|[M_i])$	0.2136	0.3610	0.2745	0.1581	0.1480	0.2009	0.1517	0.1762

process and best-worst method are not included in the comparison, because these two methods are based on pairwise comparison of criteria and require specific pairwise comparison matrices as input, while there are not such matrices in the material selection problem. When adapting the FPWMSMO, both of the controlling parameters of Maclaurin symmetric mean and Hamacher t-norm and t-conorm are set to 3. When adapting the FPWBAOs, both of the controlling parameters of Bonferroni mean and geometric Bonferroni mean are set to 1 and 2; The controlling parameter of Hamacher t-norm and t-conorm is set to 3; The risk attitude factor is set to 0.5. The results of the comparison are listed in Table 6.

As can be seen from Table 6, M_2 is the alternative material in the first position of each order, M_3 and M_1 or M_3 and M_6 are the alternative materials in the second and third positions, and the remaining alternative materials are in the following positions. Such results are fully consistent with the three-way decision-making results generated by the proposed approach: M_2 is recommended, M_4, M_5, M_7, and M_8 are not recommended, and the remaining alternative materials require more prior knowledge to make further decisions. This validates the proposed approach in solving a material selection problem in metal AM.

The quantitative comparison above has demonstrated that the proposed approach is as effective as several existing approaches from the perspective of generated decision-making results. As the proposed approach comes from a combination of 3WDM and MCDM, it has a significant advantage over these existing approaches. The existing approaches are based on conventional 2WDM, while the proposed approach is based on 3WDM. As illustrated in the introduction, 3WDM is more flexible than 2WDM since it

can effectively avoid hasty classification of the alternatives in a state of hesitation. Benefiting from this feature, 3WDM is more suitable for a decision-making problem in which there is not sufficient prior knowledge or replacement of improper decisions requires a high cost. There is not a comprehensive knowledge base for material selection in metal AM and many metal AM processes are widely known for their high costs. From these two aspects, the proposed approach may be more advantageous than the existing approaches.

4 Conclusion

In this paper, an approach for selection of materials in metal AM based on three-way decision-making is presented. The main process of this approach consists of an initialisation stage, a calculation stage, and a determination stage. In the initialisation stage, a decision matrix for a material selection problem in metal AM is established and normalised. The summary loss function, conditional probability, and expected losses of each alternative material are successively calculated in the calculation stage. In the determination stage, the three-way decision-making results for the problem are produced and the best material for the problem is determined. The paper also introduces a material selection example in metal AM to illustrate the application of the presented approach and demonstrates the effectiveness of the approach via a quantitative comparison with several existing approaches. The demonstration results suggest that the presented approach is as effective as the existing approaches and is more flexible than them in solving a material selection problem in metal AM.

Table 6 Results of the quantitative comparison with several existing approaches

Selection approach	Reference	Order or partition of alternative materials
FPWBAOs-based approach	Ref. [43]	$M_2 \succ M_3 \succ M_1 \succ M_6 \succ M_8 \succ M_7 \succ M_4 \succ M_5$
FPWMSMO-based approach	Ref. [25]	$M_2 \succ M_3 \succ M_6 \succ M_1 \succ M_7 \succ M_8 \succ M_4 \succ M_5$
SAW-based approach	Ref. [26]	$M_2 \succ M_3 \succ M_1 \succ M_6 \succ M_8 \succ M_4 \succ M_5 \succ M_7$
MOORA-based approach	Ref. [26]	$M_2 \succ M_3 \succ M_1 \succ M_6 \succ M_8 \succ M_4 \succ M_5 \succ M_7$
TOPSIS-based approach	Ref. [26]	$M_2 \succ M_3 \succ M_1 \succ M_8 \succ M_4 \succ M_5 \succ M_6 \succ M_7$
VIKOR-based approach	Ref. [26]	$M_2 \succ M_3 \succ M_6 \succ M_1 \succ M_8 \succ M_7 \succ M_4 \succ M_5$
CODAS-based approach	Ref. [28]	$M_2 \succ M_3 \succ M_6 \succ M_1 \succ M_4 \succ M_7 \succ M_8 \succ M_5$
The proposed approach	This paper	$\{\{M_2\}, \{M_1, M_3, M_6\}, \{M_4, M_5, M_7, M_8\}\}$

Future work will aim especially at studying the application of three-way decision-making in solving other MCDM problems in the field of AM. For example, three-way decision-making would be applied to determine the optimal AM part design variant from a set of candidate design variants, choose the best AM machine to fabricate a specific component from several available machines, find the optimal orientation to build an AM part from a certain number of alternative orientations, select the best combination of process parameters to build an AM part or a group of AM parts from some possible combinations, and schedule AM component batches to a network of AM machines.

Declarations

Ethical approval This paper does not contain any studies with human participants or animals performed by any of the authors.

Consent to participate Not applicable

Consent for publication Not applicable

Conflict of interest The authors declare no competing interests.

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