Numerical analysis of the strip crown inheritance in tandem cold rolling by a novel 3D multi-stand FE model

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Abstract

The strip shape inheritance model is widely applied to improve the strip shape quality in tandem cold rolling (TCR). However, the inheritance mechanism is still currently unclear. To bridge this gap, this paper presents a new numerical method for calculating the strip crown inheritance factor. In addition, the effects of the entry strip crown on exit strip crown and flatness were quantitatively analysed at each stand in the TCR using a novel three-dimensional (3D) multi-stand elastic–plastic finite element (FE) model. The results show that the strip crown inheritance factor increases slowly from S1 (stand 1) to S3 (stand 3), while rising sharply from S3 to S5 (stand 5), reaching a peak value of 0.495 $\mu\text{m}/\mu\text{m}$ at S5. This trend coincides with that of strip plastic rigidity, which verifies that the strip crown inheritance factor is dependent on the strip plastic rigidity. Furthermore, the variation of strip crown and flatness under different entry strip crowns from S1 to S5 is jointly influenced by the pass reduction and strip plastic rigidity. Moreover, the strip crown inheritance factor increases with the deformation resistance of the strip at both S1 and S5. These findings not only offer a fresh perspective to understand the mechanism of strip crown inheritance, but also provide an important basis for optimising the strip shape control in the TCR process.

Keywords Entry strip crown · Strip crown inheritance factor · 3D multi-stand FE model · Tandem cold rolling · Strip plastic rigidity

1 Introduction

Except a portion of hot-rolled strips used directly in the manufacturing field, the rest are used as raw material for cold rolling. The quality of hot-rolled strips plays an important role in the finished cold-rolled products, especially the strip shape. This is because the cold-rolled strip inherits the

strip shape from the incoming hot-rolled strip. This phenomenon has attracted the attention of many researchers in recent decades. Guo [1] studied the cascade effect of the incoming strip crown on the exit strip crown. The results showed that the exit strip crown increases with the incoming strip crown, and the incoming strip crown has little effect on the last stand, about 2.5%. Park et al. [2] reported that the cold-strip crown values are linearly related to the hot-rolled strip values, but the degree of dependency varies with strip thickness. Zhang et al. [3] and Yu et al. [4] discovered that increasing the entry strip crown causes an increase in the exit strip crown and the edge drop in a universal crown control mill (UCM mill) and a 20-high Sendzimir mill, respectively. Ma et al. [5] carried out a numerical analysis about the influence of the hot-rolled strip profile on transverse thickness difference (TTD) of cold-rolled silicon steel, and found that the TTD of the cold-rolled strip has a nearly linear relationship with the C_{40} (crown at 40 mm from the strip edge) of the hot-rolled strip without wedge, while the TTD of cold-rolled strip increases nonlinearly with the W_{40} (wedge at 40 mm

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from the strip edge) at a certain C_{40} . Using the same method, Wang et al. [6] also found that the TTD of cold-rolled strip increases with the C_{25} (crown at 25 mm from the strip edge) of the hot-rolled strip when the C_{40} remains constant and the ratio of C_{40} to C_{25} varies. Even though the quantitative analysis of the strip crown inheritance has been extensively conducted, to our knowledge, the underlying mechanism has not been determined up to now. Therefore, additional study is essential to elucidate the mechanism behind the strip crown inheritance.

Given the fact that the five-stand TCR mill features high speed and highly continual process, it is difficult to conduct massive experiments to measure the exact entry and exit strip cross-section profiles manually, especially at the interstand. Instead, the numerical method, like the finite element method (FEM), has been widely used to tackle complicated strip rolling problems. Zhou et al. [7] predicted the strip thickness profile in a four-high mill using the 2D variable element thickness FEM. Jiang et al. [8–10] simulated the cold rolling of the thin strip by the 3D rigid-plastic FEM. Sun et al. [11] calculated the roll deformation of a six-high mill using a 3D symmetrical FE model. Aljabri et al. [12] analysed the effect of work roll crossing on the strip crown employing a 3D symmetrical FE model. Wang et al. [13–15] systematically studied the effects of work roll bending (WRB), intermediate roll bending (IRB), and intermediate roll shifting (IRS) on the strip crown and flatness in a UCM mill using a 3D elastic–plastic FE model. Cao et al. [16] applied the FEM to design new roll contours to improve the edge drop and strip crown in four-high and six-high tandem cold mills. Zamanian et al. [17] evaluated the strip crown control performance of a new uniform thickness control mill using a 3D symmetrical elastic–plastic FE model. Wang et al. [18] explored the effects of IRS on strip edge drop in a six-high tandem cold mill employing a 3D elastic–plastic FE model based on the explicit method. Wang et al. [19] calculated the strip cross-section profile and flatness under different IRS values and strip widths in a six-high SmartCrown mill using a 3D elastic–plastic FE model. However, the above FE models for cold strip rolling were limited to an individual stand, the effect of neighbouring stands on the strip shape was neglected, namely, the strip crown inheritance had not been taken into account. Linghu et al. [20] developed a 3D FE model for multi-pass rolling in a 6-high continuously variable crown (CVC) cold mill, but data transfer between the neighbouring stands like the strip crown inheritance was not clear. In our previous study [21], a novel 3D multi-stand FE model for the TCR was proposed considering the work hardening effect, thus making it available for investigating the strip crown inheritance.

Here in this paper is reported, for the first time, the study of strip crown inheritance in the TCR process employing a novel 3D multi-stand FE model. First, the effects of the

entry strip crown on exit strip crown and flatness from S1 to S5 were quantitatively studied; second, the effects of the entry strip crown on the loaded roll gap profile, deflection, and flattening deformation of work roll (WR) were analysed; third, the quantitative relationship between the strip crown inheritance factor and strip plastic rigidity was determined; next, the formation mechanism of the strip crown and flatness under different entry strip crowns was revealed; finally, the relationship between the strip crown inheritance factor and the deformation resistance of the strip at both S1 and S5 was disclosed. Not only does this work provide a new method for calculating the strip crown inheritance factor, but it also offers new insight into the strip crown inheritance.

2 3D multi-stand FE model for TCR

In this section, a novel 3D multi-stand elastic–plastic FE model for the TCR was built up based on the MSC.Marc software using the segmentation modelling strategy, data transfer technologies, and element remesh technology. Furthermore, the rolling experiment was conducted on a 2180-mm TCR production line to validate the established FE model in terms of the strip cross-section profile and rolling force.

2.1 Modelling strategy

Figure 1a shows a 2180mm TCR mill, including five six-high mills. If they are modelled in a FE model, there will be millions of elements, resulting in the tiny calculation step and high computation costs. Besides, the strip elements will distort severely because of significant elongation caused by heavy reduction. To solve the aforementioned issue, the TCR mill was divided into five distinct stands designated S1, S2, S3, S4, and S5, each of which was modelled as an individual FE model, called the segmentation modelling strategy [21], as illustrated in Fig. 1b. In order to integrate each stand into one complete model, the data transfer technologies were developed to transfer data between the neighbouring stands, including transferring the strip cross-section profile and the total equivalent plastic strain (TEPS) of the strip. Moreover, the five individual FE models were calculated one by one, and the corresponding calculation flow chart is depicted in Fig. 2.

2.2 Data transfer technologies

After one single FE model was calculated, the strip cross-section profile and TEPS need to be transferred to the next single FE model. The corresponding data transfer technologies [21] are illustrated in detail as follows.

Fig. 1 a 2180 mm TCR mill b 3D multi-stand FE model

Data transfer from model S1 to S2 is taken as an example to describe the transferring process. Figure 3 gives the calculation flow chart for data transfer of strip cross-section

Fig. 2 Calculation flow chart of the multi-stand FE model

profile. As seen in Fig. 3, first, open the last step of model S1 after it was calculated; second, extract node coordinates of the cross-section in the middle of the strip and then export these coordinates to a new data file; next, assign the node coordinates to model S2 as key nodes, which can be used to generate the 2D plane elements, i.e. the initial cross-section profile. After that, the plane elements were expanded to a 3D strip (see Fig. 4), during which the elements were remeshed to maintain regular. All the operations were carried out by Mentat commands (They are used to replay the manual operation in Mentat session, stored in a procedure file called mentat.proc).

Figure 5 shows the calculation flow chart for data transfer of the TEPS. It includes three steps. As shown in Fig. 5, the first step is to open the last step of model S1 after it has been calculated; the second step is to extract the TEPS at each integration point of strip elements and then export the TEPS to a new data file by subroutine ELEVAR; following that, the last step is to assign the TEPS to model S2 according to the element number by subroutine INITPL. In this way, the TEPS can be transferred from model S1 to S2, as displayed in Fig. 6. From Fig. 6, it can be seen that the distribution of

Fig. 3 Calculation flow chart for data transfer of strip cross-section profile

TEPS at the start of model S2 is consistent with that at the end of model S1. Figure 7 presents the TEPS contour of the rolled strip from S1 to S5. It can be observed that the TEPS

Fig. 5 Calculation flow chart for data transfer of the TEPS

increases continuously from S1 to S5; this is to say, the work hardening effect caused by plastic strain accumulation has been realised through the data transfer of the TEPS.

2.3 Material properties

The strip material used in this model was one type of high-strength steel DC01. The strip and rolls are classified as elastic–plastic body and elastic bodies, respectively, with

Fig. 4 Sketch map of data transfer of strip cross-section profile

Fig. 6 TEPS contour: **a** at the end of S1 **b** at the start of S2

Fig. 7 TEPS contour of the rolled strip: **a** S1 **b** S2 **c** S3 **d** S4 **e** S5

physical properties summarised in Table 1. Taking into account the work hardening effect, rolling-tension tests were carried out to determine the static deformation resistance of the strip, which is expressed in Eq. (1) [21]. During the TCR process, the average unit rolling pressure remains steady or even declines somewhat when the rolling speed increases [22]. This might attribute to a decrease in deformation resistance caused by a rise in strip temperature, or to a drop in external friction caused by a change in lubrication conditions

in high-speed rolling [22]. Thereby, the influence of strain rate on deformation resistance (i.e. the influence of strain rate on the rolling force) was ignored here, and the static deformation resistance was employed in this FE model.

$$\sigma_s = 365.54 + 208.1\epsilon^{0.381} \quad (1)$$

$$\epsilon = \ln \frac{H}{h} \quad (2)$$

where σ_s , ϵ , H , and h are the deformation resistance, true strain, entry thickness, and exit thickness, respectively.

2.4 Governing equation

Marc is a stiffness-based programme that focuses on force–displacement interactions. In a linear elastic system,

Table 1 Material physical parameters of rolls and strip

Item	Density (kg/m ³)	Poisson's ratio	Young's modulus (GPa)
Rolls	7860	0.3	210
Strip	7850	0.3	211

force and displacement are connected through the constant stiffness; the governing equation can be written as Eq. (3) [23].

$$Ku=F \quad (3)$$

where K denotes the stiffness matrix, which is expressed as Eq. (4); u denotes the nodal displacement vector; and F denotes the force vector.

$$K = \int_0^{\Omega} B^T DB \, d\Omega \quad (4)$$

where B is the strain–displacement matrix, and D is the elastic matrix.

In a nonlinear stress analysis, Marc analyses incrementally and describes the governing equation as Eq. (5) in terms of the incremental displacement vector δu and the incremental force vector δF [23].

$$K\delta u = \delta F \quad (5)$$

For the cold rolling analysis, boundary conditions can divide into two types: the displacement boundary conditions and force boundary conditions. Equation (6) can be used to describe the displacement boundary conditions [24]. The force boundary conditions are comprised of the point force, surface force, and load surface; hence, the F can be defined as Eq. (7) [23].

$$u = \bar{u}(x, t) \quad (6)$$

where $\bar{u}$ denotes the specified quantity, x denotes the loaded boundary, and t denotes the time.

$$F = F_{point} + F_{surface} + F_{body} + F^* \quad (7)$$

where F_{point} , $F_{surface}$, and F_{body} represent the point load vector, surface load vector, and body load vector, respectively; F^* denotes the other load vectors like initial stress.

2.5 Simulation conditions

In this model, the roll contours of WR, intermediate roll (IMR), and back-up roll (BR) were modelled by discrete points, which were conventional contour, CVC [25], and Anti-CVC, respectively. Moreover, in order to ensure the accuracy of calculation, the full Newton–Raphson method for the implicit algorithm was applied to solve the nonlinear equilibrium equations. The Coulomb bilinear friction model was adapted to define the rolling contact behaviour. The entry and exit tension stresses (with opposite directions) were exerted on the strip head and tail surfaces, respectively, as shown in Fig. 8. Besides, the entry strip cross-section profile at S1 was assumed using a quadratic curve, with the crown of 0.05 mm, as displayed in Fig. 9. In addition, the

Fig. 8 Entry and exit tension stresses in a single FE model

elements used in the model were eight-node hexahedral elements, with a total element number of 214,808 for a single FE model. The other parameters are listed in Table 2. The calculation was performed on a workstation equipped with an Intel Xeon E5-2667 v3 processor and 64 GB of RAM.

2.6 Validation of the 3D multi-stand FE model

Industrial experiments in a 2180 mm TCR mill were conducted to validate the accuracy of the developed 3D multi-stand FE model. The experimental rolling force is compared with the simulated one for each stand from S1 to S5 in Fig. 10. As illustrated in Fig. 10, the simulated rolling force shows a declining trend from S1 to S5 due to a decrease in pass reduction [26]. Besides, the simulated rolling force is close to the experimental value, with a relative error of less than 5%. Figure 11 shows the simulated and experimental exit strip cross-section profile from S1 to S5 [18]. As shown in Fig. 11, it is obvious that the simulated strip crown (the difference between the strip centre and edge thickness) decreases significantly from S1 to S5, which is consistent with the practical experience. Moreover, the simulated values agree with the experimental ones in the majority of

Fig. 9 Entry strip cross-section profile

Table 2 Simulation conditions

Parameters	S1	S2	S3	S4	S5
Diameter of WR (mm)	520	520	520	520	520
Barrel length of WR (mm)	2,180	2,180	2,180	2,180	2,180
Distance between bearings of WR (mm)	3,400	3,400	3,400	3,400	3,400
Diameter of IMR (mm)	610	610	610	610	610
Barrel length of IMR (mm)	2,580	2,580	2,580	2,580	2,580
Distance between bearings of IMR (mm)	3,440	3,440	3,440	3,440	3,440
Diameter of BR (mm)	1,380	1,380	1,380	1,380	1,380
Barrel length of BR (mm)	2,180	2,180	2,180	2,180	2,180
Distance between bearings of BR (mm)	3,400	3,400	3,400	3,400	3,400
Rolling velocity (m/s)	3.5	5.0	7.5	10.5	14.5
Intermediate roll shifting (mm)	17	18	8	46	25
Intermediate roll bending (kN)	280	273	302	323	225
Work roll bending (kN)	271	270	309	328	220
Entry thickness (mm)	4.409	3.013	1.927	1.343	1.057
Exit thickness (mm)	3.013	1.927	1.343	1.057	1.000
Reduction ratio (%)	31.6	36.0	30.3	21.3	5.4
Entry tension (kN)	255	465	330	245	195
Exit tension (kN)	465	330	245	195	40
Friction coefficient	0.05	0.05	0.05	0.05	0.05
Entry strip width (mm)	1,285	-	-	-	-

regions at S5. This demonstrates the accuracy of the developed multi-stand FE model.

3 Evaluation index of strip shape

In practice, strip shape includes two parts: strip cross-section profile and strip flatness. The cross-section profile reflects the geometric features along the strip width direction, while

the flatness reflects the geometric characteristics along the strip rolling direction. They are evaluated by various indexes below.

3.1 Strip cross-section profile

The indexes of strip cross-section profile (see Fig. 12) consist of crown, edge drop, wedge, and local high-spot. Specifically, strip crown C_{40} , edge drop E_{40} , and wedge W_{40} , expressed as Eqs. (8)–(10) below, are generally used as feature values to evaluate the strip cross-section profile.

Fig. 10 Comparison between the experimental and simulated rolling forces from S1 to S5

Fig. 11 Comparison among exit strip cross-section profiles from S1 to S5

Fig. 12 Sketch map of strip cross-section profile

$$C_{40} = \frac{(h_c - h_d') + (h_c - h_d'')}{2} \quad (8)$$

$$E_{40} = \frac{(h_d' - h_e') + (h_d'' - h_e'')}{2} \quad (9)$$

$$W_{40} = h_d'' - h_d' \quad (10)$$

where C_{40} , E_{40} , and W_{40} denote the strip crown, edge drop, and wedge at 40 mm from the strip edge, respectively; h_c denotes the centre thickness; h_d' and h_d'' denote the thickness at 40 mm from the edge (DS side and OS side separately); and h_e' and h_e'' denote the edge thickness at the DS side and OS side separately.

3.2 Strip flatness

The flatness defect occurs during strip rolling as a result of the strip fibres' non-uniform elongation, which includes centre wave, edge wave, quarter wave, and edge-centre coupled wave [27], as shown in Fig. 13. When the rolled strip sample is cut into lots of narrow fibres longitudinally, the fibres contract or stretch elastically as the residual stress is released. In this way,

Fig. 13 The flatness types: **a** centre wave **b** edge wave **c** quarter wave **d** edge-centre coupled wave

the flatness can be expressed by the length difference of these fibres as Eq. (11) [27].

$$I(x) = \frac{L(x) - \bar{L}}{\bar{L}} \times 10^5 \quad (11)$$

where $I(x)$ is the flatness at location x (the unit is IU); $L(x)$ is the length of strip fibre at location x ; and $\bar{L}$ is the average length of the strip fibres.

A quartic polynomial can be used to fit the flatness curve as Eq. (12), which can also be rewritten using Chebyshev polynomials [28, 29] as Eq. (13).

$$I(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^5 \quad (12)$$

where a_0 , a_1 , a_2 , a_3 , and a_4 are the coefficients; x is the normalised width, from -1 to 1 .

$$I(x) = -\frac{1}{2}xFlt_1 - \frac{1}{2}(2x^2 - 1)Flt_2 - \frac{1}{2}(4x^3 - 3x)Flt_3 - \frac{1}{2}(8x^4 - 8x^2 + 1)Flt_4 + \Delta \quad (13)$$

where Flt_1 , Flt_2 , Flt_3 , and Flt_4 denote the first flatness, quadratic flatness, cubic flatness, and quartic flatness, respectively, which can be expressed by Eqs. (14)–(17); Δ denotes the error.

$$Flt_1 = -2a_1 - \frac{3}{2}a_3 \quad (14)$$

$$Flt_2 = -a_2 - a_4 \quad (15)$$

$$Flt_3 = -\frac{1}{2}a_3 \quad (16)$$

$$Flt_4 = -\frac{1}{4}a_4 \quad (17)$$

The asymmetrical shape control means (like roll tilting) are not considered in this work, and thus the Flt_1 and Flt_3 are neglected, while the Flt_2 and Flt_4 are focused on evaluating the flatness. It can be deduced from Eqs. (13), (15), and (17) that: when $Flt_2 > 0$, it is the centre wave; whereas when $Flt_2 < 0$, it is the edge wave; when $Flt_4 > 0$, it is the quarter

wave; whereas when $Flt_4 < 0$, it is the edge-centre coupled wave. Therefore, the wave types can be classified into four different assemblies according to the value of Flt_2 and Flt_4 , as displayed in Fig. 14, which makes it easy to determine the wave type.

4 Entry strip crown

To study the effects of entry strip crown on exit strip crown and flatness, five different entry strip cross-section profiles were selected to develop the FE model from S1 to S4, while seven different entry strip cross-section profiles were chosen for S5, as shown in Fig. 15. The corresponding entry strip crown C_{40}^{En} are summarised in Table 3. During the FE modelling, the other parameters were kept the same in Table 2 except the entry strip cross-section profile.

5 Results and discussions

5.1 Effects of entry strip crown on exit strip crown

Figure 16 presents the relative thickness deviation (RTD) of the strip under different entry strip crowns from S1 to S5. It can be seen from Fig. 16 that the distribution of RTD along the strip width direction is non-homogeneous; that is, the RTD at the strip edge is much larger than that in the central strip region, forming the so-called edge drop. The RTD along the strip width direction shows a ‘hill’ shape

Fig. 14 Four different wave assemblies

from S1 to S3, and the shape changes a little when the entry strip crown varies. By contrast, as for S4 and S5, the shape of RTD changes from a ‘hill’ to a ‘valley’ when the entry strip crown varies from positive to negative. Moreover, the RTD shows a decreasing trend on the whole from S1 to S5.

Figure 17 exhibits the efficiency curves of the entry strip crown on the exit strip crown from S1 to S5. It is apparent from Fig. 17 that all the curves show a linearly decreasing trend when the entry strip crown decreases from positive to negative, but with different slopes. To be specific, when the entry strip crown C_{40}^{En} decreases from 116.4 to $-145.7 \mu\text{m}$, the exit strip crown C_{40}^{Ex} of S1 decreases from 96.8 to $89.8 \mu\text{m}$, and the C_{40}^{Ex} of S2 declines gradually from 78.0 to $70.1 \mu\text{m}$. Similarly, the C_{40}^{Ex} of S3 shows a steady decrease from 48.7 to $32.7 \mu\text{m}$ when the C_{40}^{En} decreases from 119.7 to $-115.1 \mu\text{m}$. In comparison, when it comes to S4 and S5, the C_{40}^{Ex} evolves from positive to negative with decreasing the C_{40}^{En} . More precisely, the C_{40}^{Ex} of S4 sees a big decline from 27.2 to $-12.9 \mu\text{m}$ when the C_{40}^{En} decreases from 106.7 to $-112.7 \mu\text{m}$, whereas the C_{40}^{Ex} of S5 undergoes the steep drop from 25.6 to $-41.5 \mu\text{m}$ when the C_{40}^{En} decreases from 59.9 to $-75.2 \mu\text{m}$.

In order to quantitatively study the effect of entry strip crown on exit strip crown, the strip crown inheritance factor η is introduced by Eq. (18).

$$\eta = \frac{\Delta C_{40}^{Ex}}{\Delta C_{40}^{En}} \quad (18)$$

where ΔC_{40}^{Ex} is the variation of C_{40}^{Ex} , and ΔC_{40}^{En} is the variation of C_{40}^{En} .

According to Eq. (18), η can be obtained by fitting the efficiency curves from Fig. 17, namely the slope, as displayed in Fig. 18. It can be found from Fig. 18 that from S1 to S3, the η increases slowly from 0.027 to 0.067 $\mu\text{m}/\mu\text{m}$, and then rises sharply to 0.183 $\mu\text{m}/\mu\text{m}$ at S4, finally peaking to 0.495 $\mu\text{m}/\mu\text{m}$ at S5. This indicates that the effect of the entry strip crown on the exit strip crown increases slowly in the first three stands, but surges in the last two stands.

5.2 Effects of entry strip crown on strip flatness

Figure 19 shows the strip flatness under various entry strip crowns from S1 to S5. From Fig. 19, it is clear that the flatness along the strip width direction changes dramatically with the variation of the entry strip crown. To quantify the variation of flatness, the flatness curves can be decomposed into Flt_2 and Flt_4 using Eq. (13), as shown in Fig. 20. From Figs. 19 and 20, it can be observed that when the C_{40}^{En} decreases from 116.4 to $-145.7 \mu\text{m}$ at S1, the Flt_2 decreases linearly from 0.08 to -6.43 IU , while the Flt_4 declines linearly from 0.54 to 0.27 IU. This means that the quadratic flatness changes from centre wave to edge wave, but the

Fig. 15 Entry strip cross-section profiles: **a** S1 **b** S2 **c** S3 **d** S4 **e** S5

edge wave dominates. Meanwhile, the quarter wave occurs, with the decreasing intensity. The same trend can also be found at S2. As for S3, when the C_{40}^{En} decreases from 119.7 to $-115.1 \mu\text{m}$, the Flt_2 sees a linear decrease from 3.09

to -9.35 IU , while the Flt_4 remains stable at 0.07 IU . This suggests that the quadratic flatness changes from centre wave to edge wave, while the quarter wave appears and its magnitude changes little. By contrast, when the C_{40}^{En} decreases from

Table 3 Entry strip crowns

Cases	Entry strip crown C_{40}^{En} (μm)				
	S1	S2	S3	S4	S5
#1	116.4	116.4	119.7	106.7	59.9
#2	50.9	50.9	58.5	47.6	35.1
#3	−14.6	−14.6	0.6	−6.9	12.1
#4	−80.2	−80.2	−57.2	−59.8	−10.4
#5	−145.7	−145.7	−115.1	−112.7	−32.0
#6	-	-	-	-	−53.6
#7	-	-	-	-	−75.2

106.7 to $-112.7 \mu\text{m}$ at S4, there is a linear drop in Flt_2 from 6.78 to -8.39 IU , as well as another linear drop in Flt_4 from 0.17 to -0.32 IU . This indicates that the quadratic flatness changes from centre wave to edge wave, while the quartic flatness evolves from quarter wave to edge-centre coupled wave. Furthermore, the flatness of S5 shows a similar trend as that of S4.

To quantify the effect of entry strip crown on flatness, the efficiency factor K_F is introduced by Eq. (19), which is determined by fitting the efficiency curves from Fig. 20.

$$K_F = \frac{\Delta Flt}{\Delta C_{40}^{En}} \quad (19)$$

where ΔFlt is the variation of flatness.

Figure 21 compares the efficiency factor K_F of Flt_2 and Flt_4 from S1 to S5. As shown in Fig. 21, the K_F of Flt_2 is much larger than that of Flt_4 . This demonstrates that the quadratic flatness dominates compared with the quartic flatness. What is more, from S1 to S4, the K_F of Flt_2 increases steadily from 0.025 to $0.069 \text{ IU}/\mu\text{m}$, and then decreases to $0.062 \text{ IU}/\mu\text{m}$ at S5. In comparison, from S1 to S5, the K_F of Flt_4 shows an increase from 0.001 to $0.008 \text{ IU}/\mu\text{m}$, except S3 where the K_F of Flt_4 is close to zero. This indicates that the effect of entry strip crown on quadratic flatness increases continuously from S1 to S4, and then decreases at S5. Comparatively, the effect of entry strip crown on quartic flatness exhibits an increasing trend from S1 to S5, except S3 in which the entry strip crown has little influence on quartic flatness.

5.3 Effects of entry strip crown on the loaded roll gap profile

In the strip rolling process, the strip shape is determined by the loaded roll gap profile. Specifically, the non-uniform loaded roll gap profile leads to the uneven deformation and residual stress of the strip, resulting in the non-desirable

crown and flatness [27, 30]. Therefore, strip shape control methods, including various roll stack layouts, roll contours, and flatness actuators such as roll bending and roll shifting, are all used to regulate the loaded roll gap profile to be more uniform. Figure 22 compares the loaded roll gap profiles under various entry strip crowns from S1 to S5. It can be seen from Fig. 22 that the influence on the loaded roll gap is different from S1 to S5. Specifically, as for S1, S2, and S3, when the entry strip crown decreases from positive to negative, the crown of the loaded roll gap profile decreases slightly. In contrast, when the entry strip crown decreases from positive to negative at S4 and S5, the crown of the loaded roll gap profile shows an obvious decrease. More precisely, the C_{40} of loaded roll gap profile at S4 drops significantly from 55.3 to $-19.5 \mu\text{m}$ when the C_{40}^{En} decreases from 106.7 to $-112.7 \mu\text{m}$, whereas the C_{40} of loaded roll gap profile at S5 falls dramatically from 58.3 to $-75.0 \mu\text{m}$ when the C_{40}^{En} decreases from 59.9 to $-75.2 \mu\text{m}$. This indicates that the loaded roll gap profile shows high similarity with the entry strip cross-section profile at S4 and S5. Furthermore, the loaded roll gap profile agrees well with the exit strip cross-section profile (see Fig. 16) except for a little elastic recovery of the strip.

5.4 Effects of entry strip crown on the elastic deformation of WR

During the strip rolling process, the loaded roll gap profile is determined by the elastic deflection and flattening deformation of the roll stack. Figure 23 shows the deflection of WR axis under different entry strip crowns from S1 to S5. It can be observed that the deflection of WR axis shows an S-like shape, which accords with the CVC contour of IMR. Furthermore, the deflection of WR axis increases with the decrease in entry strip crowns from S1 to S5. This is because the average entry strip thickness increases with the decrease of the entry strip crown, leading to the larger plastic deformation of the strip, especially at the strip edge, consequently resulting in the larger elastic deflection of WR axis. More specifically, from S1 to S3, the deflection of WR axis is hill-like in shape along the strip width direction, thus forming the large crown of the loaded roll gap, causing the large exit strip crown (see Fig. 16a–c). In comparison, when it comes to S4 and S5, the deflection of WR axis distributes more uniformly along the strip width direction, so the crown of the loaded roll gap profile is relatively small, resulting in a small exit strip crown (see Fig. 16d, e).

Figure 24 compares the flattening deformation of WR between WR and strip under different entry strip crowns from S1 to S5. It can be seen from Fig. 24 that, from S1

Fig. 16 RTD of strip under different entry strip crowns: **a** S1 **b** S2 **c** S3 **d** S4 **e** S5

Fig. 17 The efficiency curves of entry strip crown on exit strip crown

to S3, the flattening deformation of WR changes little in the central strip region, whereas it increases slightly with the decrease in entry strip crown at the strip edge. This can be explained by that: on the one hand, the strip is easy to deform due to the low plastic rigidity of the strip [21]; on the other hand, the pass reduction ratio is large enough during the upstream stands (more than 30%), so the influence of the entry strip crown on the total plastic deformation of the strip is comparatively small, and the plastic deformation only increases a little at the strip edge owing to an increase in strip thickness. These two aspects affect the flattening deformation of WR at the same time, so the entry strip crown exerts a slight effect on the flattening deformation of WR in the upstream stands. By contrast,

Fig. 18 The strip crown inheritance factor from S1 to S5

regarding S4 and S5, when the entry strip crown decreases from positive to negative, the flattening deformation of WR decreases in the central strip region, but increases significantly at the strip edge. This result can be explained by the fact that: first, the strip plastic rigidity is too large [21], causing the strip hard to deform; second, the pass reduction during the downstream stands is relatively small, especially for S5 only with the reduction of 0.057 mm, and any small fluctuation of entry strip crown will have a significant influence on the plastic deformation of the strip. The above two aspects have a joint effect on the flattening deformation of WR. As a result, the entry strip crown has a significant effect on the flattening deformation of WR in the downstream stands, which is similar to the entry cross-section profile of the strip.

5.5 The relationship between the strip crown inheritance factor and strip plastic rigidity

From Fig. 18, it is clear that the strip crown inheritance factor shows an increasing trend from S1 to S5, which is consistent with the prior research [31]. This trend is similar to that of the strip plastic rigidity Q calculated in our previous study [21], and the two variations (i.e. Q and η) from S1 to S5 are compared in Fig. 25a. It can be seen from Fig. 25a that the strip crown inheritance factor agrees with the strip plastic rigidity. Given the stands from S1 to S5 are equipped with the same equipment, the mill rigidity at each stand remains almost the same. This indicates that the strip crown inheritance factor depends on the strip plastic rigidity that is influenced by the work hardening effect. Figure 25b displays the strip crown inheritance factor versus the strip plastic rigidity. It can be found from Fig. 25b that the strip crown inheritance factor increases nonlinearly with the strip plastic rigidity, which can be well fitted by an exponential function curve, expressed as Eq. (20). This affords the underlying mechanism of strip crown inheritance, and the qualitative relationship offers a valuable reference for optimising the strip crown inheritance mathematical model in the TCR process.

$$\eta = 0.88855 - 0.91753 \cdot \exp(-1.49848E - 5 \cdot Q) \quad (20)$$

where Q denotes the strip plastic rigidity.

5.6 The formation mechanism of strip shape under different entry strip crowns

According to the analysis earlier, the formation mechanism of strip shape under different entry strip crowns can be obtained, as shown in Fig. 26. The whole tandem mill

Fig. 19 Strip flatness under different rolling forces: **a** S1 **b** S2 **c** S3 **d** S4 **e** S5

Fig. 20 The efficiency curves of entry strip crown on FIt_2 **a** and FIt_4 **b**

can be divided into two parts: upstream stands (S1, S2, S3) and downstream stands (S4, S5). For the upstream stands featured with heavy reduction and low plastic rigidity, the loaded roll gap profile changes slightly under different entry strip crowns (Fig. 26, left panel), and thereby the exit strips all show a similar cross-section profile. This can be explained by the following factors: on the one hand, the range of the entry strip crown (see Table 3) is much smaller compared with the heavy pass reduction (see Table 2), which means that the deformation caused by the variation of entry strip crown accounts for a little in comparison to the total deformation; on the other hand, it is easy for the strip to deform due to its low plastic rigidity, implying that the thickness difference (see Fig. 15) caused by the variation of entry strip crown is easy to

eliminate. Therefore, the entry strip crown has a limited effect on the exit strip crown (see Fig. 18). In other words, the upstream stands have a small strip inheritance factor. By contrast, the entry strip crown has a significant effect on the flatness due to the uneven elongation caused by the various compressed deformation. As shown in Fig. 26 (left panel), when the entry strip crown is negative, the edge wave dominates compared with the weak quarter wave, which is attributed to the large elongation of fibres at the strip edge due to the large compression at the strip edge. Moreover, the edge wave weakens while the quarter wave strengthens when the entry strip crown increases from negative to positive. This is owing to the larger elongation of fibres at the one-quarter position compared with that at the strip edge.

As for the downstream stands with light reduction and high plastic rigidity, it can be observed from Fig. 26 (right panel) that the loaded roll gap shows the similar profile to the entry strip cross-section. This can be explained by the fact that the plastic deformation introduced by the variation of the entry strip crown is close to the light reduction amount, especially for S5 (generally less than 0.1 mm), and the strip is hard to deform owing to the high plastic rigidity, so the exit strip inherits much crown from the entry strip. For S5, the similarity between the entry strip crown and exit strip crown (namely the strip inheritance factor) is close to 0.5 (see Fig. 18). Furthermore, the flatness shows different characteristics under various entry strip crowns. Specifically, the edge-centre coupled wave appears when it comes to the negative entry strip crown, owing to the large elongation of fibres located at both the strip centre and edge. By contrast, the positive entry strip crown introduces the centre wave and quarter wave

Fig. 21 The flatness efficiency factors of entry strip crown

Fig. 22 The loaded roll gap profiles under different entry strip crowns: **a** S1 **b** S2 **c** S3 **d** S4 **e** S5

Fig. 23 Deflection of the WR axis: **a** S1 **b** S2 **c** S3 **d** S4 **e** S5

Fig. 24 Flattening deformation of WR between WR and strip: **a** S1 **b** S2 **c** S3 **d** S4 **e** S5

Fig. 25 a The values of Q and η from S1 to S5 b The curve of η - Q

because of the large elongation of fibres located at the strip centre and one-quarter position, while the flat entry strip brings in good flatness.

From the above mechanism analysis, the guidelines for strip shape control in the industry production line can be obtained below: first, the strip crown control should be mainly arranged within the upstream stands due to their small strip crown inheritance factor; second, to maintain a good flatness during the upstream stands, the incoming hot-rolled strip crown should be as small as possible, even close to the zero crown; third, to realise a good flatness in the downstream stands, the nearly-flat entry strip should be acquired from the upstream stands, so the flatness actuators like roll bending and roll shifting should be operated with the capacity to maintain the

uniform loaded roll gap profile in the upstream stands, providing favourable strip crown condition to the downstream stands.

5.7 The strip crown inheritance factor at stands S1 and S5

To further verify the mechanism of strip crown inheritance proposed in Sect. 5.5, three different steel grades (780X, H420LA, and DC01) were selected for the incoming hot-rolled strip. The corresponding static deformation resistance curves are shown in Fig. 27. It can be found that the deformation resistance of 780X is much larger than that of H420LA, which is much larger than that of DC01. The corresponding strip crown inheritance factors

Fig. 26 The formation mechanism of strip shape under different entry strip crowns

Fig. 27 The static deformation resistance curves of different steel grades

versus the strip plastic rigidity at S1 and S5 are presented in Fig. 28. It can be observed from Fig. 28 that the strip plastic rigidity and strip crown inheritance factor of 780X are significantly greater than those of H420LA, which are significantly greater than those of DC01. Furthermore, the strip crown inheritance factor shows a nonlinear increasing trend with the strip plastic rigidity, which can be well described by an exponential function curve, as expressed in Eqs. (21) and (22). This again demonstrates that the strip crown inheritance factor depends on the strip plastic rigidity, which depends on the strip deformation resistance. This suggests that the strip crown inheritance factor should be determined according to the strength levels of the strip.

$$\eta = 0.07628 - 0.08749 \cdot \exp(-1.39592E - 4 \cdot Q) \quad (21)$$

$$\eta = 0.70695 - 0.30054 \cdot \exp(6.18331E - 6 \cdot Q) \quad (22)$$

From Fig. 28a, it can also be found that the strip crown inheritance factor is small at S1, less than 0.1, even for high-strength steel 780X. This means that the stand S1 can be used to overcome the influence of the fluctuation for the incoming hot-rolled strip's crown and hardness on the crown and flatness of finished products [32]. By contrast, it can be observed from Fig. 28b that the strip crown inheritance factor is relatively large at S5, especially for 780X, with the largest value of 0.671. This suggests that the strip crown during stands S2–S4 should be kept in a good state, providing an appropriate condition for stand S5.

The small strip crown inheritance factor at S1 can be explained by that: it is known that the mill rigidity is determined by the whole mill, including the roll stack, mill house, and other auxiliary facilities; however, only the roll stack is taken into account in the present FE model, which leads to an increase in mill rigidity, as well as the roll gap rigidity. Consequently, this results in a decrease in the strip crown inheritance factor.

In this work, we mainly discuss the effects of entry strip crown on strip flatness. However, the entry strip edge drop or even wedge also exerts an effect on strip flatness. Abdelkhalik et al. [33] reported that the edge drop causes a slight decrease in wavelength during the cold thin strip rolling. The transverse thickness difference of the cold-rolled strip increases nonlinearly with the wedge of the hot-rolled strip [5]. Therefore, the quantitative study about the effects of entry strip edge drop or even wedge on strip flatness will be carried out in future work.

Fig. 28 The curve of η - Q : a S1 b S5

6 Conclusions

In this paper, on the basis of a 3D multi-stand elastic–plastic FE model for the TCR process considering the work hardening effect, the effects of the entry strip crown on exit strip crown and flatness at each stand were quantitatively analysed. The main conclusions can be drawn as follows:

1. The pass reduction and strip plastic rigidity have a combined effect on the evolution of strip crown and flatness under different entry strip crowns from S1 to S5. When the entry strip crown decreases from positive to negative, the exit strip crown decreases slightly in the upstream stands; while the exit strip crown undergoes a great decrease in the downstream stands. This is in accordance with the crown of the loaded roll gap profile.
2. With the variation of the entry strip crown, the quadratic flatness dominates compared with the quartic flatness from S1 to S5. Specifically, the influence of the entry strip crown on the quadratic flatness shows a steadily increasing trend from S1 to S4, although there is a little drop at S5, whereas the influence of the entry strip crown on the quartic flatness sees a gradual increase from S1 to S5, except S3 where the effect is close to zero.
3. When the entry strip crown decreases from positive to negative, the deflection of WR axis increases gradually; the flattening deformation of WR between WR and strip decreases in the strip centre, but increases at the strip edge.
4. Strip crown inheritance factor increases slowly from S1 to S3, but surges from S3 to S5. This is consistent with the trend of strip plastic rigidity. It demonstrates that the strip crown inheritance factor depends on the strip plastic rigidity. Furthermore, the strip crown inheritance factor increases with the strip's deformation resistance at both S1 and S5.

This work not only lays a solid foundation for future studies on strip shape inheritance, but also provides useful guidelines for improving the strip shape inheritance preset model and online strip shape control.

Author contribution Lianjie Li: conceptualization, formal analysis, investigation, methodology, writing – original draft. Haibo Xie: writing – review and editing. Tianwu Liu: validation, investigation. Mingshuai Huo: data processing. Xing sheng Li: offer assistance for subroutine development and calculation. Xu Liu: validation. Enrui Wang: data curation, validation. Jianxin Li: investigation, supervision. Hongqiang Liu: investigation. Li Sun: resources, investigation. Zhengyi Jiang: writing – review and editing, supervision, project administration.

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Code availability The calculation was performed based on the MSC Marc software.

Declarations

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