Stress analysis of double-walled pipes undergone mechanical drawing process

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Abstract

Double-walled metal pipes are important components for heat exchange and liquid transportation. They can be manufactured by mechanical drawing of two concentric pipes. In this work, a mechanical model is developed to analyze the stress state evolution during the manufacturing process, and the criterion for forming the double-walled pipe was given under the ideal elastoplastic and thick-walled assumptions. The model also encompasses the results deduced under the thin-walled assumption. Numerical simulations confirmed that the accuracy of the analytical model was within 5%. The application on actual steel materials with various parameters varied, including the wall thickness and initial gap, was analyzed. This work can provide theoretical support to industrial manufacturing procedures and help to reduce costs by eliminating required test procedures.

Keywords Double-walled pipe · Residual stress · Elastoplasticity · Numerical simulation

1 Introduction

Double-walled pipes are widely used in various applications, such as applications in the chemical industry and nuclear power plants, because of their ability to cope with extreme temperature differences or chemical environments inside and outside the pipes and effectively prevent crack propagation through the pipes [1]. Double-walled pipes consist of two layers made of the same or different kinds of materials with or without an interlayer [2–5]. Directly connected double-walled pipes can provide a sufficient heat transfer efficiency

and prevent liquid leakage caused by rupture. Thus, they are very useful in the heat exchange equipment of nuclear reactors.

The double-walled pipe can be manufactured by various methods, such as welding, explosion-based methods, or mechanical methods to press two layers together. Most previous research focused on the processing parameters for the manufacturing procedures [1, 6–10, 16]. Chen [7] summarized some existing processing techniques. Kamal and Dixit [16] selected a thermal autofrettaging method and combined it with a shrink-fitting method. Delong [8] made composite double-walled pipes by using hot extrusion first and then cold drawing. Bogatov [9] fabricated double-walled pipes under an internal pressure that could be used in the petroleum industry. Taheri-Behrooz and Pourahmadi [10] studied the effects of high temperatures on the mechanical performances of double-walled pipes.

Theoretical analyses of the stress and strain states of double-walled pipes during the manufacture procedures have been performed. Kim [11] proposed a method to measure residual stresses in double-walled pipes using neutron diffraction technology and verified the results with calculations. Wang and Li [12] discussed the stress and strain state of a pipe made using a hydraulic expansion method and provided suitable processing parameters. Lv

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[13] ignored the thickness of the inner pipe. Jia [14] considered both layers to be thin walls and presented criteria for the mechanical manufacturing of double-walled pipes. Although the thickness of the wall is generally small relative to the pipe diameter, the thick-walled assumption is worth considering because the thin-walled assumption cannot adequately represent the pipes used in real situations, and information related to the wall thickness allows us to study complex deformation and failure.

In this work, a detailed analysis on the stress state during mechanical manufacturing was performed based on the thick-walled and ideal elastoplastic assumptions. The results cover both thin- and thick-walled cases. The analytical model was validated by finite element numerical simulations. The criterion to successfully form double-walled pipes and the inner pressure exerted on the inner wall to obtain the maximum residual stress are presented. To meet the needs of actual manufacturing processes, adjustments to the theory were also presented based on the properties of some common industrial materials.

2 Analytical model of forming double-walled pipes by mechanical methods

The schematic diagram of the mechanical drawing process is shown in Fig. 1. The mechanical drawing method can be grouped into internal extrusion and external pressing methods. The analysis procedures of these two methods are similar. In our work, the internal extrusion method is selected as an example, and the internal load in the mechanical drawing procedure is simplified as a uniform internal pressure.

Fig. 1 **a** Manufacturing process of mechanical drawing method, equivalent to the internal pressure method with determined displacement [17]. **b** Schematic diagram of internal extrusion method (the spacing between the two pipes will disappear after processing)

The analysis is divided into the following steps: (1) identification of the stress and strain in a single-walled pipe under an internal pressure, (2) identification of the stress and strain in the inner shell of the two concentric pipes before they make contact, (3) the change of the stress in the double-walled pipe after the two shells make contact (unloading will start at the end of this period), (4) evolution of the stress and strain of the pipe as it undergoes the unloading procedure, and (5) evaluation of the residual pressure between the shells after unloading. The existence of residual stress is regarded as the criterion for successful manufacturing of double-walled pipes.

2.1 Stress and strain analysis of single-walled pipe

When a single-walled pipe is subjected to an increasing internal or external radial pressure, it will gradually enter the plastic state from the innermost surface to the outmost surface. A plastic interface will emerge in the pipe wall, inside which there is a plastic zone, and outside there is an elastic zone. It will expand outwards. When it reaches the outermost surface, the entire pipe enters the plastic regime [3]. The evolution of the stress and strain of a double-walled pipe will be similar based on the expansion of the plastic interface.

Based on plane strain model, the stress and strain analysis of the mechanical internal extrusion method is carried out using the Tresca stress condition and the ideal elastic–plastic hypothesis. For a pipe with inner and outer radii a and b , internal pressure q_1 , and external pressure $q_2 = 0$, there is a plastic interface at radius r_p , and the radial stress at this interface is q_p , as shown in Fig. 2a. The interior zone of the interface is plastic, while the exterior zone is elastic. The radial and tangential stress distribution can be expressed as

Fig. 2 **a** Schematic diagram of a single-walled pipe. **b** Schematic diagram of a double-walled pipe

$$\sigma_r = \begin{cases} \sigma_s \ln \frac{r}{a} - q_1 & a \leq r \leq r_p \\ \left(1 - \frac{b^2}{r^2}\right) \frac{r_p^2 q_p}{b^2 - r_p^2} & r_p \leq r \leq b \end{cases} \quad (1)$$

$$\sigma_\theta = \begin{cases} \sigma_s \left(\ln \frac{r}{a} + 1\right) - q_1 & a \leq r \leq r_p \\ \left(1 + \frac{b^2}{r^2}\right) \frac{r_p^2 q_p}{b^2 - r_p^2} & r_p \leq r \leq b \end{cases} \quad (2)$$

where σ_s is the yield stress of the pipe.

The distribution of the displacement along the radius can be expressed as follows:

$$u = \begin{cases} \frac{1}{2} \frac{(1+\nu)\sigma_s r_p^2}{Eb^2 r} \left[(1-2\nu)r_p^2 + b^2\right] & a \leq r \leq r_p \\ \frac{1}{2} \frac{(1+\nu)\sigma_s r_p^2}{Eb^2 r} \left[(1-2\nu)r^2 + b^2\right] & r_p \leq r \leq b \end{cases} \quad (3)$$

If the whole pipe is still in the elastic region, then

$$u = \frac{(1+\nu)q_1 a^2}{E(b^2 - a^2)} \left[(1-2\nu)r + \frac{b^2}{r}\right]. \quad (4)$$

Similarly, the stress distribution under an external pressure of q_2 (while $q_1 = 0$) can be obtained:

$$\sigma_r = \begin{cases} \frac{r_p^2 b^2 (q_2 - q_p)}{b^2 - r_p^2} \frac{1}{r^2} + \frac{r_p^2 p_p - b^2 q_2}{b^2 - r_p^2} & a \leq r \leq r_p \\ \sigma_s \ln \frac{r}{a} & r_p \leq r \leq b \end{cases} \quad (5)$$

$$\sigma_\theta = \begin{cases} -\frac{r_p^2 b^2 (q_2 - p_p)}{b^2 - r_p^2} \frac{1}{r^2} + \frac{r_p^2 p_p - b^2 q_2}{b^2 - r_p^2} & a \leq r \leq r_p \\ \sigma_s \left(\ln \frac{r}{a} + 1\right) & r_p \leq r \leq b \end{cases} \quad (6)$$

It is worth noting that, as can be seen from here, even if the pipe is under an external pressure, it will also enter the plastic state from the inner surface.

The displacement along the radius is

$$u = \begin{cases} -\frac{1}{2} \frac{(1+\nu)\sigma_s r_p^2}{Ea^2 r} \left[(1-2\nu)r^2 + a^2\right] & a \leq r \leq r_p \\ -\frac{1}{2} \frac{(1+\nu)\sigma_s r_p^2}{Ea^2 r} \left[(1-2\nu)r_p^2 + a^2\right] & r_p \leq r \leq b \end{cases} \quad (7)$$

2.2 Stress and strain analysis of double-walled pipe

2.2.1 Contact of two layers of a double-walled pipe

The inner radius of the inner pipe is a , the outer radius of the outer pipe is c , and the initial gap between the inner and outer pipe is d_0 , as shown in Fig. 2b. Young's modulus, Poisson's ratio, and ultimate stress of the material are denoted as E , ν , and σ , respectively, and the inner and outer pipes are denoted by the subscripts 1 and 2, respectively. Since the spacing between the pipes is much smaller than the radius of the pipes, the outer radius of the inner pipe and the inner radius of the outer pipe both can be regarded as b .

According to the study of a single pipe (Eq. 7), the stress and strain states of the inner pipe when the pipes make contact will be determined by the initial gap between two layers. The different states are the elastic-deformed state, completely plastic-deformed state, or a state in between.

1. When the pipes make contact, $r_p = a$, and $u|_{r=b} = u_e$.

The second formula of Eq. (7) can be written as

$$u_e = \frac{1}{2} \frac{(1+\nu_1)\sigma_{s1} a^2}{E_1 b^3} \left[(1-2\nu_1)b^2 + b^2\right] = \frac{(1-\nu_1^2)\sigma_{s1} a^2}{bE_1}. \quad (8)$$

If $d_0 \leq u_e$, the inner pipe is still in an elastic state when two layers make contact.

- When the pipes make contact, $r_p = b$, and $u|_{r=b} = u_l$. The first formula of Eq. (8) can be written as

$$u_l = \frac{1}{2} \frac{(1 + \nu_1) \sigma_{s1} b^2}{E_1 b^3} [(1 - 2\nu_1) b^2 + b^2] = \frac{(1 - \nu_1^2) \sigma_{s1} b}{E_1}. \quad (9)$$

If $d_0 \geq u_e$, the inner pipe is completely in the plastic regime when two pipes make contact.

- When d_0 is between u_e and u_l , there will be a plastic interface in the inner pipe as the pipes contact. The radius of the plastic interface is r_{p1} .

2.2.2 After two layers contact

The internal pressure of the surface where the inner and outer pipes contact is denoted as q_r . If the inner pipe is still in the elastic stage when two pipes make contact (Eq. 9), the conditions for the inner surface of the pipes to enter the plastic regime can be written as

$$q_1 - q_r = \frac{b^2 - a^2}{2b^2} \sigma_{s1} (\text{inner pipe}), \quad (10)$$

$$q_r = \frac{c^2 - b^2}{2c^2} \sigma_{s2} (\text{outer pipe}) \quad (11)$$

If Eq. (10) is satisfied before Eq. (11), the inner pipe will enter the plastic regime earlier than the outer pipe. On the contrary, the outer pipe will enter the plastic regime earlier.

The condition for when the outer pipe enters the full plastic regime is as follows:

$$q_r = \sigma_{s2} \ln \frac{c}{b}, \quad (12)$$

and condition for when the inner pipe enters the full plastic regime is as follows:

$$q_1 - q'_1 = \sigma_{s1} \ln \frac{b}{a}, \quad (13)$$

where q'_1 indicates the increment of q_1 that occurs as the pressure q_r is applied on the interfaces between inner and outer pipes. q_1 cannot increase after the inner pipe completely enters the plastic stage. However, when the outer pipe obstructs the deformation process of the inner pipe, q'_1 and q_r will appear on the inner and outer surfaces of the inner pipe. q_r undergoes reverse elastic loading on the inner pipe, so q'_1 offsets q_r if the effects of the wall thickness are ignored.

In the actual production process, the gap between the inner and outer pipes is sufficiently large that the inner pipe

is already in the full plastic regime when contact is made. This will be the default assumption in later sections.

2.2.3 Unloading

After contact, the unloading from any of the stress states mentioned in Sect. 1.2.2 can be calculated by the following method. Some materials have a lag in deformation, and the load must be held for a period of time to ensure that sufficient deformation occurs. Considering that some materials have a lag in deformation because of viscoelasticity, it is necessary to maintain the load for a period to ensure sufficient deformation [15]. This is not considered in this article.

The unloading process is an elastic process. For unloading stresses of Δq_1 at $r = a$, and Δq_r at $r = b$,

$$\Delta \sigma_r = \begin{cases} \frac{a^2 b^2 (\Delta q_r - \Delta q_1)}{(b^2 - a^2) r^2} + \frac{a^2 \Delta q_1 - b^2 \Delta q_r}{b^2 - a^2} & a \leq r \leq b \\ -\frac{b^2 \Delta q_r}{c^2 - b^2} \left(\frac{c^2}{r^2} - 1 \right) & b \leq r \leq c \end{cases}, \quad (14)$$

$$\Delta \sigma_\theta = \begin{cases} -\frac{a^2 b^2 (\Delta q_r - \Delta q_1)}{(b^2 - a^2) r^2} + \frac{a^2 \Delta q_1 - b^2 \Delta q_r}{b^2 - a^2} & a \leq r \leq b \\ \frac{b^2 \Delta q_r}{c^2 - b^2} \left(\frac{c^2}{r^2} + 1 \right) & b \leq r \leq c \end{cases}. \quad (15)$$

The radial strain condition is

$$\Delta \varepsilon_{r1}|_{r=b} = \Delta \varepsilon_{r2}|_{r=b}. \quad (16)$$

Thus,

$$\frac{\nu_1 (2a^2 \Delta q_1 - (a^2 + b^2) \Delta q_r)}{(b^2 - a^2) E_1} + \frac{\Delta q_r}{E_1} = \frac{\nu_2 (c^2 + b^2)}{c^2 - b^2} \frac{\Delta q_r}{E_2} + \frac{\Delta q_r}{E_2}. \quad (17)$$

Fig. 3 Schematic diagram of the internal pressure loading process: **a** the inner and outer pipes are not in contact during loading (the dotted line is the elastoplastic boundary), **b** after loading and both the inner and outer pipes are completely plastic, and **c** after unloading, residual compressive stress exists between the two pipes

When $\Delta q_1 = q_1$, the load is completely removed. If $\Delta q_r \leq q_r$, the pipes can be pressed together; otherwise, they will be disconnected. The residual compressive stress between the two pipes is $q_{re} = q_r - \Delta q_r$. When $\Delta q_r = q_r$, the inner and outer pipes just fit together, but there is no pressure on the contact surface.

2.3 Criterion for successful manufacturing of double-walled pipe

The inner pipe is already full in the plastic regime when contact is made:

$$q_1 = \sigma_{s1} \ln \frac{b}{a} + q_r. \quad (18)$$

When $\Delta q_1 = q_1$, we obtain

$$\Delta q_1 = q_r \left[\frac{1}{E_2} \left(\nu_2 \frac{c^2 + b^2}{c^2 - b^2} + 1 \right) + \frac{1}{E_1} \left(\nu_1 \frac{a^2 + b^2}{b^2 - a^2} - 1 \right) \right] \frac{b^2 - a^2}{2a^2 \nu_1} E_1. \quad (19)$$

We let

$$B \equiv \left[\frac{1}{E_2} \left(\nu_2 \frac{c^2 + b^2}{c^2 - b^2} + 1 \right) + \frac{1}{E_1} \left(\nu_1 \frac{a^2 + b^2}{b^2 - a^2} - 1 \right) \right] \frac{b^2 - a^2}{2a^2 \nu_1} E_1, \quad (20)$$

Fig. 4 **a** Model used for numerical simulation. **b** Numerical simulation results of the relationship between the displacement of the inner surface and the pressure on the innermost surface of the inner pipe.

The region in the circle corresponds to Fig. 4c. **c** Agreement between the theory and the simulation results

and thus,

$$(B - 1) \left(q_1 - \sigma_{s1} \ln \frac{b}{a} \right) > \sigma_{s1} \ln \frac{b}{a}. \quad (21)$$

If Eq. (25) is not satisfied, the compacted double-walled pipe cannot be obtained. In this case, the following can be used as a criterion:

$$(B - 1) \frac{\sigma_{s2} \ln \frac{c}{b}}{\sigma_{s1} \ln \frac{b}{a}} > 1. \quad (22)$$

The residual stress is

$$q_{re} = q_r - \frac{q_1}{B}. \quad (23)$$

When $q_1 = q_{1max} = \sigma_{s1} \ln \frac{b}{a} + \sigma_{s2} \ln \frac{c}{b}$ (both pipes enter fully plastic regimes before unloading), q_{re} will reach a maximum:

$$q_{re-max} = \sigma_{s2} \ln \frac{c}{b} - \frac{1}{B} \left(\sigma_{s1} \ln \frac{b}{a} + \sigma_{s2} \ln \frac{c}{b} \right). \quad (24)$$

If the pipes are formed by an external pressure, the criterion can be obtained by a similar method (the outer pipe has entered the fully plastic regime before unloading):

$$\frac{\sigma_{s1} \ln \frac{b}{a}}{\sigma_{s2} \ln \frac{c}{b} + \sigma_{s1} \ln \frac{b}{a}} B_2 > 1, \quad (25)$$

where

$$B_2 \equiv \left[\frac{1}{E_2} \left(\nu_2 \frac{c^2 + b^2}{c^2 - b^2} - 1 \right) + \frac{1}{E_1} \left(\nu_1 \frac{a^2 + b^2}{b^2 - a^2} + 1 \right) \right] \frac{c^2 - b^2}{2c^2 \nu_2} E_2. \quad (26)$$

The residual stress is related to the thicknesses of the pipes. The final stress–strain distribution is shown in Fig. 3c. If there is a requirement on the minimum value of the residual stress in a real application, the initial thicknesses of the pipes can be specially selected using Eq. (28).

Under the thin-walled assumption, $a \approx b \approx c$ and $\ln \frac{b}{a} \approx \ln \frac{c}{b} \neq 0$. Therefore, $B \approx \frac{E_1 \nu_2}{E_2 \nu_1} + 1$, and the criterion to press them together is $\frac{\sigma_{s2} \nu_2}{E_2} \geq \frac{\sigma_{s1} \nu_1}{E_1}$ (or $\varepsilon_{s2} \nu_2 \geq \varepsilon_{s1} \nu_1$). Similarly, $B_2 \approx \frac{E_2 \nu_1}{E_1 \nu_2} + 1$ in the external pressure method, and the condition becomes $\frac{\sigma_{s2} \nu_2}{E_2} \leq \frac{\sigma_{s1} \nu_1}{E_1}$. When Poisson's ratios are considered to be the same for the inner and outer pipes, the results show agreement with previous work [6]. In this case, the inner and outer pipes made of the same material can only be pressed together with no pressure on the contact surface, and a sufficiently tight double-walled pipe cannot be successfully fabricated. The result deduced from the thick-walled assumption shows that residual stress can exist in this case, which is more realistic.

2.4 Application in fabrication of stainless-steel double-walled pipe

Data were chosen from the most popular commercial stainless-steel pipes, with $\sigma_s = 200$ MPa, $E = 200$ GPa, and $\nu = 0.25$. The material is considered an ideal elastic–plastic model. The diameters of the outer and inner pipes were 27 and 22 mm, while both wall thicknesses were 2 mm. Referring to Fig. 2b, the parameters can be set as $a = 9$ mm, $b = 11$ mm, and $d_0 = 0.5$ mm for the inner pipe before

(a)

(b)

Fig. 5 **a** Radial stress on the interface for different q_1 . **b** Radial stress from displacement of the inner surface

Table 1 Stresses of inner tube

	Inner wall of inner pipe	Outer wall of inner pipe
σ'_r	$-q'_1$	$-q_r$
σ'_θ	$\frac{(a^2+b^2)q'_1-2b^2q_r}{b^2-a^2}$	$\frac{2a^2q'_1-(a^2+b^2)q_r}{b^2-a^2}$

the contact happens, and $a = 9.5$ mm, $b = 11.5$ mm, and $c = 13.5$ mm after contact.

If the inner pipe is still fully elastic when contact is made, the following must be satisfied:

$$d_0 \leq \frac{(1-\nu^2)\sigma_s a^2}{bE} = 0.0069(\text{mm}).$$

This requirement is almost impossible to achieve in practice. If the inner pipe is already fully plastic when in contact, it requires.

$$d_0 \geq \frac{(1-\nu^2)\sigma_s b}{E} = 0.0103(\text{mm}),$$

which means it is reasonable that the inner pipe is already fully plastic when contact is made. Furthermore, if

$$(B-1)\frac{\sigma_{s2}\ln\frac{c}{b}}{\sigma_{s1}\ln\frac{b}{a}} \approx 1.43 > 1,$$

then the conditions for double-walled pipe fabrication are satisfied.

When $r_p = c, q_r = \sigma_{s2}\ln\frac{c}{b} = 32.07\text{MPa}$ and $q_1 = \sigma_{s1}\ln\frac{b}{a} + \sigma_{s2}\ln\frac{c}{b} = 70.28\text{MPa}$, the maximum residual stress of 6.00 MPa can be achieved.

3 Model validation

3.1 Finite element analysis simulations

We used the solid mechanics module in the COMSOL Multiphysics 5.2 software with a triangular mesh to simulate quasi-static conditions. The size and material parameters of the pipes are the same as those in Sect. 1.4.

The model used for the numerical simulation is shown in Fig. 4a, and the parameters are indicated in Fig. 4a. As in the theoretical part, the pipe is considered an ideal elastic–plastic material. The upper and lower boundaries of the pipe are set as symmetric boundary conditions, and external force comes from the inner side. Stress changes at the contact of the two pipes will be considered.

During loading, the inner wall of the inner pipe moved 0.66 mm outward, and when unloading, the inner wall of the inner pipe moved back 0.011 mm. Figure 4a shows the pressure on inner surface and the displacement of the inner pipe. ① to ② is the external deformation stage of the internal pipe until it enters full plasticity. ② to ③ is the stage after the inner pipe enters full plasticity and before the pipes make contact, showing that in the simulation, the inner pipe reached full plasticity before the two pipes made contact. During the period from ② to ③, the pressure on the inner surface of the inner pipe did not increase but slightly decreased due to the increase in the radius (this decrease was ignored in the theoretical calculations). ③ to ④ is the stage after the pipes made contact and the outer pipe entered the fully plastic regime, corresponding to the stage in which q_1 generated the increment q'_1 to offset the increase in q_r in the theoretical calculations. The increase in the pressure in this stage in Fig. 4a was q'_1 . The outer pipe reached full plasticity at ④, and unloading began at ⑤. ⑥ represents the end of unloading. Both pipes in this simulation entered the fully plastic regime before unloading, and the outer pipe had just entered it. The obtained residual stress will be approximately the maximum value if the compression is successful. In Fig. 4b, the simulation results are compared with the theoretical results derived from Eqs. (6), (7), and (8). The inner pipe began to yield when the displacement of the inner surface was 0.7 mm.

Figure 5a shows the theoretical and simulated radial stresses on the interface for different q_1 . The results of the previous section are also included. The maximum value is at the right end of the curve. In theory, the maximum internal pressure is maintained and cannot be increased. On the Y-axis, the results were similar, and on the X-axis, there were slight differences. The reasons for these differences will be explained in the next section. Figure 5b shows the radial stress on the interface obtained for different u versus the displacement of the inner surface.

Table 2 Value of stress at critical position and time of inner tube

Time point	σ_r/MPa		σ_θ/MPa	
	Inner wall of inner pipe	Outer wall of inner pipe	Inner wall of inner pipe	Outer wall of inner pipe
Outer pipe began to be pressed	−35.765	0.23091	164.24	200.23
Outer pipe entered full plasticity	−67.652	−31.805	132.35	168.19

Table 3 Value of stress at critical position of inner pipe

	Inner wall of inner pipe /10 ⁶ Pa	Outer wall of inner pipe /10 ⁶ Pa
σ'_r /Pa	-31.887	-31.574
σ'_θ /Pa	-31.89	-32.04

3.2 Verification of approximation in theoretical analysis

In the theoretical calculation section, the increment q'_1 of q_1 caused by q_r can be taken as $q'_1 = q_r$. The rationality of this approximation is as follows. The elastic effect of q_r and q'_1 on the inner tube caused increments of the stresses in the inner pipe σ'_θ and σ'_r , respectively, and the distributions are

$$\sigma'_r = \frac{a^2 b^2 (q_r - q'_1)}{b^2 - a^2} \frac{1}{r^2} - \frac{b^2 q_r - a^2 q'_1}{b^2 - a^2}, \quad (27)$$

$$\sigma'_\theta = \frac{a^2 b^2 (q'_1 - q_r)}{b^2 - a^2} \frac{1}{r^2} - \frac{b^2 q_r - a^2 q'_1}{b^2 - a^2}. \quad (28)$$

The values at the inner and outer walls are shown in Table 1.

The variations of σ_θ and σ_r of the inner and outer walls of the inner pipe during the simulation from contact to the outer pipe entering the fully plastic regime are shown in Table 2. The data in Table 3 are the simulation results corresponding to Table 1.

At $\frac{(q'_1 - q_r)}{q'_1} = \frac{(\sigma'_{rin} - \sigma'_{rout})}{\sigma'_{rin}} = \frac{-3.1887 \times 10^7 - (-3.1574 \times 10^7)}{-3.1887 \times 10^7} = 0.0098$, the difference between q'_1 and q_r is less than 1%. If $q'_1 = q_r$ is obtained, the following can also be obtained:

$$\sigma'_{\theta in} = \sigma'_{\theta out} = \frac{(a^2 + b^2)q'_1 - 2b^2 q_r}{b^2 - a^2} = \frac{2a^2 q'_1 - (a^2 + b^2)q_r}{b^2 - a^2} = -q_r. \quad (29)$$

For $\frac{(\sigma''_{\theta in} - \sigma''_{\theta out})}{\sigma''_{\theta in}} = \frac{-3.189 \times 10^7 - (-3.204 \times 10^7)}{-3.189 \times 10^7} = 0.0047$ (as shown in Table 3). Therefore, it is reasonable that $q'_1 = q_r$ in the theoretical calculation.

In addition, the theoretical value of q_r for the outer pipe entering full plasticity was $q_r = \sigma_{s2} \ln \frac{c}{b} = 3.207 \times 10^7$ Pa, which was 1.6% different from the simulation value (3.157×10^7 Pa). The value of q_1 for the outer pipe to enter the fully plastic regime was $q_1 = \sigma_{s1} \ln \frac{b}{a} = 4.013 \times 10^7$ Pa, which was 0.2% different from the simulation value (4.004×10^7 Pa). Considering the numerical calculation got a discrete result, the time node corresponding to the selected stress value deviated slightly from the time node that we are interested in; the error was within the acceptable range.

3.3 Influence of parameters on results of internal pressure method

Setting $\sigma_{s1} = \sigma_{s2} = \sigma_s$, $E_1 = E_2$, $\nu_1 = \nu_2$, $b = c - t$, $a = c - 2t$, $c^* = c/t$, (t is the thickness of the pipe wall) in Eq. (28), we obtain the following:

Fig. 6 Consistency between theory and numerical simulations when **a** σ_s was fixed at 210 MPa and **b** c^* was fixed at 6.5

$$\frac{q_{re-max}}{\sigma_s} = \ln \frac{c^*}{c^* - 1} - \frac{1}{B} \ln \frac{c^*}{c^* - 2}, \quad (30)$$

where

$$B \equiv \left[\frac{2c^{*2} - 2c^* + 1}{2c^* - 1} + \frac{2c^{*2} - 6c^* + 5}{2c^* - 3} \right] \frac{2c^* - 3}{2c^{*2} - 8c^* + 8}. \quad (31)$$

Similarly, the inner wall pressure leading to the maximum residual stress is

$$\frac{q_{1max}}{\sigma_s} = \ln \frac{c^*}{c^* - 2}. \quad (32)$$

Figure 6a shows the relationships between $\frac{q_{re-max}}{\sigma_s}$, $\frac{q_{1max}}{\sigma_s}$, and c^* at fixed σ_s , and Fig. 6b shows the relationships diagrams of q_{re-max} , q_{1max} , and σ_s at fixed c^* . The theoretical results are compared with the numerical simulation results. An increase in c^* caused decreases in $\frac{q_{re-max}}{\sigma_s}$ and $\frac{q_{1max}}{\sigma_s}$ when σ_s was fixed, and the increase in σ_s caused increases in q_{re-max} and q_{1max} when c^* was fixed. Furthermore, the theory and numerical simulations were consistent. Thus, the theory can be used to predict the results of double-walled pipes manufactured by the internal pressure method.

Fig 7 **a** Results of a simple tensile test of a real steel tube. Residual stress when the **b** inner radius of the inner pipe, **c** outer radius of the inner pipe, and **d** both the inner and outer radii of the inner pipe were varied

4 Adjustment for material with hardening effect

The stress–strain curve of 316L stainless steel obtained by a simple tensile test is shown in Fig. 7a. For materials with a hardening function $\sigma_h = \sigma_h(\epsilon_p)$, Eq. (22) is written as

$$q_1 = q_p + q_r + q_h. \quad (33)$$

When the internal pressure $q_p = \sigma_s \ln \frac{b}{a}$ and the inner pipe was fully in the plastic regime, q_h appeared during the period from the inner pipe entering the complete plastic regime to making contact with the outer pipe. q_h is related to the hardening function and the distance between the inner and outer pipes, which is written as $q_h = q_h(\sigma_h, d)$. Figures 7b, c, and d show that the results of the simulation verified the influence of the spacing d given by Eq. (35). The trend was roughly consistent with previously reported results [11].

Inequality (26) can be changed to the following:

$$(B - 1)q_r > q_p + q_h(\sigma_h, d). \quad (34)$$

For a material with a hardening function, the success of the internal expansion method will be influenced by the distance between the inner and outer pipes. As q_h increases with d , inequality (38) becomes difficult to satisfy, so it will become more difficult to compress the two pipes with a larger gap. In the design and manufacturing process, this factor should be fully considered.

5 Conclusion

In this work, the mechanical extrusion process for manufacturing double-walled pipes was analyzed using the thick-walled assumption, ideal elastoplastic relationship, and Tresca yield condition. An analytical model of the double-walled pipe was derived, and the criterion for manufacturing double-walled pipes was obtained. The residual stress between the inner and outer pipes is necessary for successful manufacturing, which can be predicted by theory. Numerical simulations were used to verify the accuracy of the theory.

In the manufacturing process using actual materials with hardening effects, the gap between the inner and outer layers needs to be considered because its influence on the residual stress is larger than that of other parameters. This work can provide a theoretical basis for the industrial manufacturing of double-walled pipes. The conclusions from the mold processing method analysis were similar to those of the theoretical analysis.

Based on actual production conditions, the assumption that for a wide enough space, the inner pipe is completely plastic when it contacts the outer pipe was proposed, and the

reliability was verified. However, the theoretical calculations and numerical simulations in this work were based on quasi-static processes, so they are not valid for dynamic processes, such as in explosion-based methods. Furthermore, the deformation and phase transformations of steel crystals in the processing were not discussed in this paper, which will be a subject of future work.

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Author contribution The complete theoretical analysis and numerical simulation of the process of preparing double-walled tube by internal pressure method are presented.

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Data availability statement Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

Declarations

Ethics approval and consent to participate Not applicable.

Conflict of interest The authors declare no competing interests.

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