Development of similarity-based scaling criteria for creep age forming of large/extra-large panels

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Received: 25 July 2018 / Accepted: 31 October 2018 / Published online: 21 November 2018
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Abstract

A scaling method is developed for the creep age forming (CAF) process to downscale manufacturing of large/extra-large panels to lab-scale experimental trials for industrial application. Similarity theory is applied to identify both the geometrical and physical (non-geometrical) similarities between large-size prototypes and scaled-down models in all process stages of CAF, including loading, stress-relaxation and unloading (springback). A constitutive model is incorporated into the theory in order to identify the similarity in the highly non-linear stress-relaxation behaviour for aluminium alloy plates during CAF, and to obtain the effective scaling criteria for the CAFed plates after springback. The method was demonstrated by scaling down CAF manufacturing of both singly curved and doubly curved large plates under both proportional and non-proportional geometrical scaling conditions. The analytical results of the scaling method and numerical results obtained by CAF FE modelling were found to be in good agreement. Scaling diagrams linking the key deformation (springback) and structural (flexural rigidity) variables to scaling ratios under both proportional and non-proportional conditions were generated, and the developed scaling diagrams have been validated by corresponding CAF experiments. The scaling method developed in this study provides guidance on the design of scaled-down CAF experimental trials and will be used in the practical CAF process of large/extra-large panels.

Keywords Sheet metal forming · Creep age forming · Similarity theory · Scaling criteria · Extra-large panel · Dimensional analysis

Nomenclature

$D \text{ Pa} \cdot \text{m}^3$	Flexural rigidity of a plate	$t, t_f \text{ h}$	Transient time during CAF and the end time of CAF
$E \text{ GPa}$	Young's modulus	$\varepsilon_{cr}, \varepsilon_e^p -$	Creep strain and effective plastic strain, respectively
$h, L, W \text{ m}$	Thickness, length and width of the plate	$\varepsilon_{ij}^e, \varepsilon_{ij}^p, \varepsilon_{ij}^t -$	Strain components of elastic, plastic and total strain respectively
$k \text{ m}^{-1}$	Curvature of the loaded plate	$\nu -$	Poisson's ratio
$M \text{ N} \cdot \text{m}$	Moment in the loaded plate	$\sigma_A, \sigma_{dis}, \sigma_{ss}$	Strength contributions from precipitates, dislocations and solid solution respectively
$\bar{r}, \bar{\rho} -$	Normalised precipitate radius and dislocation density at ageing temperature, respectively	MPa	
$q \text{ N} \cdot \text{m}^2$	Pressure applied in the loaded plate	$\sigma_e \text{ MPa}$	Effective stress
$Sp -$	Springback percentage after CAF	$\sigma_{ij} \text{ MPa}$	Stress components
$S_{ij} \text{ MPa}$	Components of stress deviator	$\sigma_y \text{ MPa}$	Yield strength
		$\hat{\sigma}_{ij} \text{ MPa}$	Cauchy stress Jaumann rate
		$\phi -$	Airy stress function
		$\omega, \omega_0, \omega_f, \Delta\omega \text{ mm}$	Deflection, loaded deflection, formed deflection and springback deflection in the forming plate, respectively
		$X \text{ and } X' -$	Variables in the prototype (X) and in the scaled-down model (X')

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C_x –	Dimensionless similarity parameter of the variable X between the prototype and scaled-down model
$X(t)$ and $X(T)$ –	The value of variable X at the transient time t during CAF and final time t_f at the end of CAF

1 Introduction

Creep age forming (CAF) is a metal forming technology originally developed to produce large/extra-large panels with complex curvatures in the aerospace industry in order to overcome the drawbacks of conventional mechanical forming processes, such as high residual stresses and poor mechanical properties in the formed structures [1]. It has been proven as one of the most cost-effective processes for forming large skin panels of aircrafts [2].

Over the last few decades, many experimental and numerical studies have been conducted on CAF at both the lab and industrial scales. Holman [3] proved that CAF is a viable forming method to manufacture large aluminium panels for aircraft by using an autoclave and corresponding tools. As high springback usually occurs in a CAFed plate [1, 4], a number of constitutive models have been proposed to model the creep-ageing/stress-relaxation behaviour for a range of aluminium alloys [5–7], which have been implemented into FE solvers for springback prediction and tool shape optimisation [8, 9]. The CAF process has been successfully applied to manufacture aircraft skin panels, such as the EPC bulkhead of MT Aerospace Ariane V [10] and upper wing skins of the Airbus A380 [11], and it has been considered to be a promising technology for producing large/extra-large panels in other transport industries. In applying CAF to new components, forming tests using downscaled models, which significantly reduce cost compared to full-scale trials [12], are generally preferred. Hence, a method is needed to scale down large/extra-large panels for lab-scale trials of the CAF process in order to inform full-scale manufacturing.

Similarity theory identifies dimensionless parameters to quantitatively represent a complicated problem by a similar but simplified problem [13]. Although similarity theory is a viable tool for transferring experimental data from a scaled-down model to the prototype, its main applications are in the fields of computational fluid dynamics and physics [14].

Limited application of scaling principles to metal-forming processes can be found in the literature. Scaling effects were investigated by Vollertsen et al. [15] in the process of micro sheet forming to produce miniature parts. Pertence and Cetlin [16] proposed a similarity procedure for ductility evaluation in models and actual materials but did not apply it to metal-forming processes. García-Rodríguez et al. [17, 18] applied dimensional analysis in scaling of fabrication of metal matrix, so as to carry out fabrication experiments in a simpler system and provide

direct guidance for the real manufacturing process. Storåkers et al. [19] applied the similarity analysis to inelastic contact, which is potential to be applied for metal forming processes. Pawelski [20] summarised the advantages and drawbacks of similarity theory for metal-forming applications and indicated that it is a useful way to assist the scaling of some simple metal-forming processes in which key process variables can be determined. But limitations of similarity theory exist once metal-forming processes become complicated, especially when highly non-linear boundary, lubrication and thermal conditions need to be considered [21, 22]. Recently, Davey et al. [23] introduced a transport equation approach to provide guidance for designing a scaled thermo-mechanical metal-forming experiment. The applicability of this scaling method was verified by simple trial cases and powder compaction process [24], but the high complexity of the method requires further investigations for its practical application to metal-forming processes.

The CAF process of large/extra-large panels is an isothermal forming process with relatively simple loading conditions; hence, thermal effects can be neglected [1, 25]. Furthermore, the non-linear creep-ageing behaviour that occurs during forming has been modelled successfully by a set of constitutive equations [5, 6, 26]. Hence, similarity theory could be an effective method for the development of scaling criteria for the CAF process.

In this paper, similarity theory is used, for the first time, to develop a scaling method for the CAF process, so as to enable downscaling of large/extra-large skin panels for CAF applications. A set of key parameters in the CAF process and a creep-ageing constitutive model have been adopted to evaluate the similarities between the prototype and scaled models based on similarity theory. The proposed scaling criteria have been applied to CAF of both singly curved and doubly curved plates of aluminium alloy 6082, and the analytical results of the scaling method have been validated by established CAF FE models and by corresponding CAF experiments.

2 Principles of similarity-based scaling criteria

According to similarity theory, dimensionless similarity parameters that characterise relations between key quantities in the prototype and scaled models need to be identified, so as to develop the criteria for scaling of the prototype problem based on dimensional analysis [27].

All quantities in the prototype for an objective process can be divided into two categories, geometrical and non-geometrical, in which non-geometrical quantities are also called physical quantities [28]. Figure 1 shows similarity parameters (C_L , C_V and C_F) for three basic independent quantities (length (L , L'), velocity (V , V') and force (F , F')) in a general system, relating the prototype and scaled model. When the similarity parameter for geometrical quantities between the prototype and scaled model (such as length $C_L = L/L'$) retains the same

value during the process, geometrical similarity exists. If the same geometrical similarity value holds for all dimensions of the object, such as length, width and thickness for a plate, proportional geometric similarity exists between the prototype and scaled models; otherwise, it is non-proportional similarity. Physical similarity is achieved when the similarity parameters for non-geometrical (physical) quantities between the prototype and scaled models (such as $C_V = V/V'$ and $C_F = F/F'$ in Fig. 1) retains the same value during the process [27].

The three basic quantities defined above (length, velocity, force) are not enough to characterise a metal forming process, and more specific geometrical and physical quantities for a particular forming process, such as the blank and tool dimensions, pressure applied from the tool, stresses and strains in the test plate and creep-ageing related variables in the CAF process, need to be considered. All of them can affect the scaling criteria for the process. Hence, dimensional analysis can be used to reveal the scaling criteria of the key process-related quantities. The prototype process and its corresponding scaled model should share the same governing equations, for example [27, 29]:

$$q_1 = f(q_2, q_3, \dots, q_n) \quad (1)$$

$$q'_1 = f(q'_2, q'_3, \dots, q'_n) \quad (2)$$

where $q_1, q_2, \dots, q_n$ are process variables in the prototype and $q'_1, q'_2, \dots, q'_n$ are the corresponding variables in the scaled model. The equations can be rewritten by replacing all process variables with their corresponding dimensionless similarity parameters, as

$$C_{q_1} = f(C_{q_2}, C_{q_3}, \dots, C_{q_n}) \quad (3)$$

where $C_{q_i} = q_i/q'_i$, where $i = 1, 2, \dots, n$. If $q_1, q_2, \dots, q_k$ are

independent variables, the dimensions of any one of the variables, e.g. $[q_i]$, can be expressed as a suitable combination of the dimensions of the others:

$$[q_i] = [q_1]^{p_1} [q_2]^{p_2} \dots [q_k]^{p_k}, (i = k + 1, \dots, n) \quad (4)$$

by appropriate choices of $p_1, p_2, \dots, p_k$. As a result, corresponding dimensionless similarity parameters can be calculated according to Eq. (4), as

$$C_{q_i} = C_{q_1}^{p_1} C_{q_2}^{p_2} \dots C_{q_k}^{p_k}, (i = k + 1, \dots, n) \quad (5)$$

Replacing C_{q_i} in Eq. (3) with that given by Eq. (5), the scaling criteria characterise the scaling relationship between the independent and dependent similarity parameters in the prototype and scaled model according to

$$F(C_{q_i}, C_{q_1}, C_{q_2}, \dots, C_{q_k}) = 1 \quad (6)$$

In the following sections, the similarity parameters of both key geometrical and physical process variables for CAF will be introduced, and corresponding scaling criteria used to characterise relationships among these parameters in the prototype and scaled models will be developed for all stages of the CAF process.

3 Identifying geometrical similarity for scaling of the CAF process

3.1 Geometrical similarity of the test plate during CAF

The test plate is the only deformable component during CAF and its shape remains the same within the tool surface during the main stage of CAF. Figure 2 shows the dimensions of the full-scale prototype and scaled-down model of an aluminium alloy plate after loading in CAF. The main geometrical similarity parameters defined between the prototype and the model include

$$\frac{L}{L'} = C_L \quad (7)$$

$$\frac{W}{W'} = C_W$$

$$\frac{h}{h'} = C_h$$

$$\frac{\omega}{\omega'} = C_\omega$$

where L, W and h are respectively the length, width and thickness of the prototype plate, and ω is the largest deflection in the loaded plate, as illustrated in Fig. 2. L', W', h' and ω' are

Fig. 1 Schematic of similarities between the prototype and scaled model for a general system

Fig. 2 Dimensions of the full-scale prototype and scaled-down model of the test plate after loading in CAF

corresponding variables in the scaled-down model. When $C_L = C_W = C_h = C_\omega$, proportional scaling applies for the plate in CAF. Otherwise, non-proportional scaling applies.

3.2 Geometrical similarity of springback after CAF

Springback plays a decisive role in the CAF process, as significant springback generally occurs in the CAFed plate, which affects the final shape. In order to demonstrate the forming capability of CAF of a large panel using its scaled-down model, the similarity of the springback between the prototype and the scaled-down model is the key factor to be captured.

Springback percentage (Sp) of the plate after unloading in CAF is defined as

$$Sp = \frac{\omega_0 - \omega_f}{\omega_0} \times 100\% \quad (8)$$

where ω_0 is the maximum deflection in the plate after loading and ω_f is the deflection value at the same point after unloading (springback), as shown in Fig. 2.

A dimensionless parameter to represent the similarity of springback between the prototype and the scaled-down model is defined as (Fig. 2)

$$C_{sp} = \frac{Sp}{Sp'} = \frac{\omega_0 - \omega_f}{\omega_0} \frac{\omega_0'}{\omega_0' - \omega_f'} \quad (9)$$

$C_{sp} = 1$ indicates that the same springback percentage can be observed in the prototype and the scaled-down model.

$C_{sp} > 1$ represents that more springback occurs in the prototype after CAF.

4 Identification of physical similarity for scaling of the CAF process

The evolution of physical variables determines the creep-ageing behaviour that occurs in the plate during CAF and controls the final springback of the CAFed plate. In order to obtain the scaling criteria for the CAF process, the evolution of similarities of key physical variables needs to be determined throughout the entire CAF process, including loading, stress-relaxation and unloading (springback).

4.1 Stage I: loading

Föppl–von Kármán plate equations have been used to study the scaling criteria of test plates loaded with large deflection ω in CAF, in response to the loading conditions (including internal forces and pressures) defined per unit area, as shown in Fig. 3 [30]. The relationship between the internal force F_T , deflection ω and external pressure q in the loaded plate can be presented by the following governing equation:

$$D\nabla^4\omega - \left(F_{T11} \frac{\partial^2\omega}{\partial x^2} + 2F_{T12} \frac{\partial^2\omega}{\partial x\partial y} + F_{T22} \frac{\partial^2\omega}{\partial y^2} \right) = q \quad (10)$$

where $D = Eh^3/[12(1-\nu^2)]$ is the flexural rigidity of the test plate, ν is Poisson's ratio, and E is Young's Modulus; x and y denote two mutually perpendicular in-plane directions in the

Fig. 3 Internal force and external pressure of a plate with large deflection

plate, as shown in Fig. 3. F_{T11} and F_{T22} are stress components along the x axis and y axis respectively, and F_{T12} is the shear stress in the plate; ∇^4 is the biharmonic operator:

$$\nabla^4 = \nabla^2 \nabla^2 = \frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \quad (11)$$

The strain compatibility equation (the biharmonic equation) can be re-expressed in terms of the stress components ($\sigma = [\sigma_{11}, \sigma_{22}, \tau_{12}]$) according to the Hooke's law to obtain strains in terms of stresses, as

$$\begin{aligned} \frac{\partial^2 \sigma_{11}}{\partial y^2} + \frac{\partial^2 \sigma_{22}}{\partial x^2} - \nu \frac{\partial^2 \sigma_{11}}{\partial x^2} - \nu \frac{\partial^2 \sigma_{22}}{\partial y^2} - 2(1 + \nu) \frac{\partial^2 \tau_{12}}{\partial x \partial y} \\ = E \left[\left(\frac{\partial^2 \omega}{\partial x \partial y} \right)^2 - \frac{\partial^2 \omega}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} \right] \end{aligned} \quad (12)$$

The airy stress function Φ is defined as

$$\begin{aligned} \sigma_{11} &= \frac{\partial^2 \Phi}{\partial y^2} \\ \sigma_{22} &= \frac{\partial^2 \Phi}{\partial x^2} \\ \tau_{12} &= -\frac{\partial^2 \Phi}{\partial x \partial y} \end{aligned} \quad (13)$$

Inserting the airy stress function Φ into Eqs. (10) and (12), the governing equation and biharmonic equation of the loaded plate can be expressed as

$$D \nabla^4 \omega = h \left(\frac{\partial^2 \Phi}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \Phi}{\partial y^2} \frac{\partial^2 \omega}{\partial x^2} - 2 \frac{\partial^2 \Phi}{\partial x \partial y} \frac{\partial^2 \omega}{\partial x \partial y} \right) + q \quad (14)$$

$$\nabla^4 \Phi = E \left[\left(\frac{\partial^2 \omega}{\partial x \partial y} \right)^2 - \frac{\partial^2 \omega}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} \right] \quad (15)$$

The governing equations given by Eqs. (14) and (15) can be used for the loading stage of CAF in both the prototype and

scaled-down model. For scaling of the test plate in CAF, the same alloy is used in both the prototype and scaled-down model since each aluminium alloy has specific creep-ageing behaviour. Hence, they have the same material properties of $E = E'$ and $\nu = \nu'$.

The dimensionless similarity parameter of flexural rigidity between the prototype and scaled-down model is represented by the plate thickness and it can be calculated according to Eq. (5):

$$C_D = \frac{D}{D'} = \frac{h^3}{h'^3} = C_h^3 \quad (16)$$

In addition, the similarity parameters for the pressure and the airy function are defined as

$$C_q = \frac{q}{q'} \quad (17)$$

$$C_\Phi = \frac{\Phi}{\Phi'}$$

Hence, based on the dimensional analysis introduced by Eqs. (1) to (6) in Sect. 2, the scaling criteria for the loading stage of CAF can be obtained by substituting all similarity parameters in Eqs. (7), (16) and (17) into the process equations given by Eqs. (14) and (15). The resulting formula is

$$\frac{C_\Phi^2}{C_\omega^2 C_h^2} = C_q \frac{C_l^4}{C_h^3 C_\omega} = 1 \quad (18)$$

For CAF of large/extra-large plates, the plane stress assumption is used and the Cauchy stress tensor components are $\sigma_i = [\sigma_{11}, \sigma_{22}, \tau_{12}]$, expressed in a vector format (Voigt notation). According to Eq. (13), the airy stress function Φ is dependent on the stress tensor components σ_i and dimensions (length or width) of the loaded plates. Hence, based on the dimensional analysis from Eqs. (1) to (6), the scaling criterion relates the airy

stress function to stress components and dimensions of loaded plates can be presented as

$$\frac{C_\phi}{C_{\sigma_i} C_l^2} = 1, (i = 1, 2, 3) \quad (19)$$

where $C_{\sigma_i} = \sigma_i / \sigma'_i$. Combining Eqs. (18) and (19) together, the scaling criteria between key physical parameters (σ_i , q) and geometrical parameters (l , h , ω) after the loading stage of CAF can be updated as

$$C_{\sigma_i}^2 = \frac{C_\omega^2 C_h^2}{C_l^4} \quad (20)$$

$$C_q = \frac{C_h^3 C_\omega}{C_l^4} \quad (21)$$

4.2 Stage II: stress-relaxation

During the stress-relaxation stage of CAF, the shape of the plate is fixed by the tool, where creep strain occurs and leads to the relaxation of internal stresses of the loaded plate. As a result, geometrical similarities remain the same during stress-relaxation, but physical similarities, such as stresses, are evolving according to the highly non-linear stress-relaxation behaviour of the material. Hence, the scaling criteria obtained from the loading stage of CAF need to be updated for the stress-relaxation stage.

Stress-relaxation behaviour under multi-axial loading conditions of the test plate can be modelled as [31]

$$\dot{\varepsilon}_{ij}^p = \left(\frac{3S_{ij}}{2\sigma_e} \right) \dot{\varepsilon}_e^p \quad (22)$$

$$\dot{\sigma}_{ij} = 2G\dot{\varepsilon}_{ij}^e + \lambda\dot{\varepsilon}_{kk}^e \quad (23)$$

$$\dot{\varepsilon}_{ij}^e = \dot{\varepsilon}_{ij}^t - \dot{\varepsilon}_{ij}^p \quad (24)$$

where $\dot{\varepsilon}_{ij}^p$ is the plastic strain rate tensor during stress-relaxation, $\dot{\varepsilon}_{ij}^e$ and $\dot{\varepsilon}_{ij}^t$ are respectively the rate of the elastic and total strain, $\dot{\sigma}_{ij}$ is the Cauchy stress Jaumann rate, $G = E/[2(1 + \nu)]$ and $\lambda = E\nu/[(1 + \nu)(1 - 2\nu)]$ are the Lamé elasticity constants, and ε_e^p is the effective plastic strain that comes from the creep strain (ε_{cr}) generated during stress-relaxation. In addition, the evolution of the stress and strain in the plate during stress-relaxation can be calculated by using the constitutive model proposed for CAF, as listed below [26]:

$$\dot{\varepsilon}_e^p = \dot{\varepsilon}_{cr} = A_1 \sinh \left\{ B_1 \left[\sigma_e (1 - \bar{\rho}) - k_0 \sigma_y \right] \right\} \text{sign}(\sigma) \quad (25)$$

$$\dot{\bar{r}} = C_r (Q - \bar{r})^{m_3} \left(1 + \gamma_0 \bar{\rho}^{m_4} \right) \quad (26)$$

$$\dot{\bar{\rho}} = A_3 (1 - \bar{\rho}) \left| \dot{\varepsilon}_{cr} \right| - C_p \bar{\rho}^{m_5} \quad (27)$$

$$\dot{\sigma}_A = C_A \dot{\bar{r}}^{m_1} (1 - \bar{r}) \quad (28)$$

$$\dot{\sigma}_{ss} = C_{ss} \dot{\bar{r}}^{m_2} (\bar{r} - 1) \quad (29)$$

$$\dot{\sigma}_{dis} = A_2 n \bar{\rho}^{n-1} \dot{\bar{\rho}} \quad (30)$$

$$\sigma_y = \sigma_{ss} + \sqrt{(\sigma_A^2 + \sigma_{dis}^2)} \quad (31)$$

Normalised microstructural parameters, including the normalised dislocation density ($\bar{\rho}$) and precipitate radius ($\bar{r}$), are considered in the CAF constitutive model, which control the evolution of creep strain rate ($\dot{\varepsilon}_{cr}$) and yield strength (σ_y) during CAF. Yield strength of the alloy during CAF was composed of three strength components: age hardening (σ_A), solid solutes hardening (σ_{ss}) and dislocation hardening (σ_{dis}).

Hence, during the stress-relaxation stage of CAF, the stresses remaining (σ_i^f) in the plate are changing according to the processing time t , and it can be updated according to the constitutive model, as

$$\begin{aligned} \sigma_i^f(t) &= \sigma_i(0) - \int_0^t \dot{\sigma}_i dt \\ &= \sigma_i(0) - \int_0^t \left(2G \left(-\frac{3S_{ij}}{2\sigma_e} \right) \dot{\varepsilon}_e^p + \lambda \dot{\varepsilon}_{kk}^e \right) dt \end{aligned} \quad (32)$$

In order to assist the analysis of springback after stress-relaxation, the stress moment resultants through thickness of the plate during stress-relaxation were calculated, which are defined as [32]

$$M_i(t) = \int_{-h/2}^{h/2} \sigma_i^f(t) z dz \quad (33)$$

where z is the distance to the neutral axis through thickness of the test plate, ranging from $-h/2$ to $h/2$.

The scaling criteria for stress obtained in the loading stage (Eq. (20)) will be updated with the process time t as

$$C_{\sigma_i}(t) = \frac{\sigma_i^f(t)}{\sigma_i^{f'}(t)} \quad (34)$$

4.3 Stage III: unloading (springback)

After the stress-relaxation stage with a total time of t_f , the plate is unloaded. It is assumed that the equivalent stress moments $M_i(t_f)$ remaining in the plate will be fully released during unloading, leading to the final springback of the CAFed plate [33]. Hence, the final stress moment in the plate can be treated as $M_f = 0$ and the change of moment is

$$\Delta M_i = M_f - M_i(t_f) = -M_i(t_f) \quad (35)$$

The change of the deflection in the loaded plate because of ΔM_i during unloading (springback) can be calculated according to the Föppl–von Kármán plate [32]:

$$\begin{cases} \Delta M_{11} = -D \left(\frac{\partial^2 \Delta \omega}{\partial x^2} + \nu \frac{\partial^2 \Delta \omega}{\partial y^2} \right) \\ \Delta M_{22} = -D \left(\nu \frac{\partial^2 \Delta \omega}{\partial x^2} + \frac{\partial^2 \Delta \omega}{\partial y^2} \right) \\ \Delta M_{12} = -D(1-\nu) \left(\frac{\partial^2 \Delta \omega}{\partial x \partial y} \right) \end{cases} \quad (36)$$

Then, the final formed deflection is $\omega_f = \omega - \Delta \omega$ and the springback similarity (C_{sp}) between the prototype and scaled-down model for the CAFed plate can be obtained according to Eq. (9), as

$$C_{sp}(t_f) = \frac{\Delta \omega}{\omega} \frac{\omega'}{\Delta \omega'} \quad (37)$$

During loading in CAF process, the plate is positioned in the tool freely without any clamps and hence, the geometric boundary conditions in the forming plate during CAF are applied through loading stresses from tools, whose effect has been analysed and scaled in Sect. 4.1. However, it should be noted that some approximations have been assumed for other boundary conditions of CAF process in the scaling criteria developed in this study, including

1. The same alloy is used in both the prototype and scaled-down model, as each aluminium alloy has specific creep-ageing behaviour and cannot be scaled down if different alloys are applied
2. Thermal effects during forming are not considered in CAF process and scaling, since the main stage of CAF is carried out in an isothermal condition
3. Effects of friction on loading and stress-relaxation of CAF process and related scaling criteria can be neglected, because: (a) the same materials are used in prototype and scaled models for CAF; (b) quasi-static loading speed is used in both prototype and scaled models and (c) loaded plates are fixed firmly in the tool during stress-relaxation
4. Both the prototype and scaled-down model are assumed to be elastically loaded in CAF in this study, as CAF is mainly applied to large/extra-large panels with slight deformations, such as wing skin panels, and elastic loading is reasonable for most CAF cases currently.

5 Case study and discussion

In this section, the scaling criteria developed for the CAF process is applied to scale down both singly

curved and doubly curved large plates during CAF, so as to illustrate the application procedures of the scaling criteria and instruct further applications to CAF for more complex-shaped structures.

5.1 Application of the theory

5.1.1 Singly curved case

Figure 4a shows the dimensions of the singly curved plate after loading in the prototype and scaled-down model. The governing equations for large deflection of loaded plates in Eqs. (14) and (15) can be degenerated to the singly curved condition, as

$$\frac{d^4 \omega}{dx^4} = \frac{q}{D} \quad (38)$$

The degenerated problem then can be described by the standard beam theory, as [34],

$$\begin{cases} M = -D \frac{d^2 \omega}{dx^2} \\ q = \frac{d^2 M}{dx^2} \\ k = \frac{1}{R} = \frac{d\theta}{dx} = \frac{d^2 \omega}{dx^2} \end{cases} \quad (39)$$

where M is the moment in the loaded plate, k is the curvature of the loaded plate and R is the corresponding radius of curvature value. Hence, the scaling criteria obtained for the loading stage of CAF (Eqs. (20) and (21)) can be updated for the singly curved case, as

$$C_\sigma = \frac{C_h}{C_R} \quad (40)$$

$$C_M = \frac{C_h^3}{C_R} \quad (41)$$

It is noted that the length and width dimensions of the test plate have no effects on the stress and moment similarities according to Eqs. (40) and (41), and thus, they can be removed in the scaling criteria for CAF of the singly curved plate. Both C_h and C_R are independent geometrical parameters which affect the similarity parameters of stress and moment for the singly curved plate after loading during CAF. These two parameters will be used to describe the scaling ratio of this case in the following study.

Fig. 4 Dimensions of **a** loaded shape and **b** springback properties of the singly curved plate in prototype and scaled-down CAF model

During the stress-relaxation stage, C_σ and C_M can be updated according to Eqs. (33) and (34). For the singly curved plate, the results vary during the stress-relaxation time t , as

$$C_\sigma(t) = \frac{\sigma(t)}{\sigma'(t)} = \frac{\sigma_0 - \int_0^t (-E\dot{\varepsilon}_{cr}) dt}{\sigma'_0 - \int_0^t (-E\dot{\varepsilon}'_{cr}) dt} \quad (42)$$

$$C_M(t) = \frac{\int_{-h/2}^{h/2} \sigma(t) z dz}{\int_{-h/2}^{h/2} \sigma'(t) z dz} \quad (43)$$

Figure 4b illustrates the shape of the test plate during unloading in CAF. During unloading, the curvature change ($\Delta k = 1/R_0 - 1/R_f$) from the release of the residual moment $M(t_f)$ can be calculated by Eq. (39):

$$\Delta k = \frac{-M(t_f)}{D} \quad (44)$$

where t_f is the total stress-relaxation time. The final radius of the CAFed plate after springback is $R_f = 1/(k + \Delta k)$, and the similarity parameter for springback can be presented with radius data of the plate as

$$C_{sp} = \frac{\Delta\omega}{\omega} \frac{\omega'}{\Delta\omega'} = \frac{\Delta R}{R_0} \frac{R'_0}{\Delta R'} \quad (45)$$

where $\Delta R = (R_f - R_0)$.

In summary, in the case of scaling of CAF with the large singly curved plate, the scaling criteria, which represent the similarities of stress evolution during CAF and springback after CAF between the prototype and the scaled-down model with different scaling ratios (represented by C_h and C_R), can be analytically obtained through Eqs. (42) and (45). The scaling results of the CAF process with various scaling ratios, including both proportional and non-proportional scaling conditions, will be discussed in the following sections.

5.1.2 Doubly curved case

Figure 2 illustrates the scaling down of a doubly curved case for CAF process, with constant curvature radius of R_x (R'_x) and R_y (R'_y) in x and y directions. The airy stress function (Eq. (13)) of two principal stresses in both directions then can be updated as

$$\begin{aligned} \sigma_{11} &= \frac{\partial^2 \Phi}{\partial y^2} = \frac{Eh}{1-\nu^2} \left(\frac{1}{R_x} + \nu \frac{1}{R_y} \right) \\ \sigma_{22} &= \frac{\partial^2 \Phi}{\partial x^2} = \frac{Eh}{1-\nu^2} \left(\frac{1}{R_y} + \nu \frac{1}{R_x} \right) \end{aligned} \quad (46)$$

Replacing Eq. (46) into Sect. 4.1, the scaling criteria obtained for the loading stage of CAF (Eqs. (20) and (21)) can be updated, for the doubly curved case, as

$$C_{\sigma_i} = \frac{C_h}{C_{R_i} + C_\nu C_{R_j}} = \frac{C_h}{C_{R_i} + C_{R_j}}; \quad i, j \in (x, y) \quad (47)$$

$$C_{M_i} = \frac{C_h^3}{C_{R_i} + C_\nu C_{R_j}} = \frac{C_h^3}{C_{R_i} + C_{R_j}}; \quad i, j \in (x, y) \quad (48)$$

i and j respectively represent x and y or y and x . The scaling criteria for stress-relaxation and unloading stages in CAF of the doubly curved case then can be updated according to Eqs. (22) to (37) in Sect. 4.

In Eqs. (47) and (48), C_h , C_{R_x} and C_{R_y} are the three independent geometrical parameters in determining similarity parameters of stress and moment for the doubly curved case during CAF. In this study, the scaling case with $C_{R_x} = C_{R_y}$ is selected for further demonstration and discussion.

5.2 FE simulation

CAF FE models have been developed that effectively predict the creep-ageing behaviour and springback properties of test plates for the CAF process [26, 31]. CAF FE models for AA6082-T6 based on the stress-relaxation constitutive model proposed in [7] have been developed in PAM-STAMP. FE models for CAF of the singly curved plate with prototype size and corresponding scaled size have been carried out, in order to demonstrate the effectiveness of the analytical scaling criteria developed in this study.

Figure 5 shows the FE models of CAF of AA6082-T6 for both singly curved and doubly curved cases. The plates were modelled in PAM-STAMP using a four-node quadrilateral shell element with Batoz Q4 gamma formulation, while the tool surfaces were modelled using rigid shell elements. The global mesh sizes in the plates and tools were set at 3 and 5 mm, respectively.

The creep-ageing constitutive model listed in Eqs. (25) to (31) was implemented into PAM-STAMP through a material

subroutine [31] to characterise corresponding behaviour in the plate during CAF. Other basic material properties were defined as yield strength of 290 MPa, Young's modulus of 72 GPa and Poisson's ratio of 0.3.

The FE model simulates the CAF process through three stages, namely loading, stress-relaxation and unloading. The plate is elastically loaded to have full contact with the tool surface by the pressure initially applied along the z -direction. Stress-relaxation calculation of AA6082-T6 at 160 °C then starts in the loaded plate for a certain process time t_f and in the cases studied here $t_f = 5$ h. Finally, the pressure is released and springback occurs in the CAFed plate.

For the singly curved case, according to Eqs. (40) and (41), thickness (h) and loading radius (R) are the two independent parameters to be considered in scaling. A prototype model with blank size of 3000 mm in length and 1000 mm in width is selected for CAF FE simulations, and scaling-related dimensions of the test case are defined as 8 mm in thickness and 2200 mm in curvature radius of the singly curved tool. CAF numerical simulations for the scaled models with scaling ratios of $C_R = C_h = 1, 2, 4$ and 8 under the proportional scaling condition, and $C_R = 4$, $C_h = 2.5, 3, 3.5$ and 4 under the non-proportional scaling condition have been performed for comparison.

While, for the doubly curved case, thickness (h) and loading radius in both directions (R_x and R_y) are the three parameters to be investigated. The prototype model is selected with blank size of 3000 mm in both length and width, and scaling-related parameters are defined as $h = 8$ mm, $R_x = 4000$ mm and $R_y = 3000$ mm. CAF simulations for non-proportional scaled models with $C_{R_x} = C_{R_y} = 4$, $C_h = 2.5, 3, 3.5$ and 4 have been performed for further analysis.

5.3 Establishment of scaling diagrams for CAF of AA6082

5.3.1 Singly curved case: proportional scaling condition

Under the proportional scaling condition, similarity parameters of all dimensions remain the same ($C_h = C_R$ in this case).

Fig. 5 FE models for CAF of the a singly curved plate and b doubly curved plate

Fig. 6 Distributions of effective stress in the test plate after loading and 8 h CAF of AA6082-T6 from FE models with prototype size and proportional scaled models (Prop. Model) with scaling ratios of $C_h = C_R = 2$ and 4

Hence, after loading, the stress similarity (C_σ) between the prototype and scaled models is 1 according to Eq. (40). Similar stress distributions can be observed in the test plate after loading with different scaling ratios from the FE modelling results, as shown in Fig. 6, which agree well with the analytical results. With the same initial stress distributions in both prototype and scaled models, the same stress-relaxation behaviour occurs during CAF, as indicated by the constitutive model in Eqs. (25)–(31). Hence, $C_\sigma(t) = 1$ can be obtained throughout the CAF process. The same behaviour was obtained from the CAF FE model. Similar stress distributions were observed in plates with different scaling ratios, as shown in Fig. 6.

A scaling diagram that quantitatively correlates key physical similarity parameters to geometrical scaling ratios can be obtained using the scaling criteria derived in this study, in order to guide the design of scaled-down CAF models. Figure 7 shows the scaling diagram for CAF of a singly curved plate under proportional scaling conditions. As indicated by both analytical and numerical results, the stress

similarity equals to 1 during all stages of CAF with different scaling ratios. As a result, the same springback percentage can be observed in both the prototype and scaled models under all proportional conditions, where $C_{sp} = 1$ as indicated in Fig. 7.

In addition to the final shape of the formed plate, some structural properties are important for CAFed structures and need to be considered during scaling. The flexural rigidity, which is the key factor to characterise the stability of panels, is taken into account. The change of the flexural rigidity similarity of plates (C_D) with different scaling ratios is shown as the dashed line in Fig. 7. The results show that the flexural rigidity similarity of the plate increases dramatically, as the cube of the increasing scaling ratio (Eq. (16)), which largely limits the maximum scaling ratio that can be used for scaled-down CAF experimental trials when the lowest flexural rigidity is required for the actual and scaled structures.

5.3.2 Singly curved case: non-proportional scaling condition

Under the non-proportional scaling condition, C_h/C_R varies with different scaling conditions, which affects the stress similarity in the test plate after loading of CAF, as indicated by Eq. (40). Figure 8 shows the numerical results of stress distributions in the test plates with different scaling ratios of C_h and C_R values after loading and CAF. Unlike the results in Fig. 6, with a lower value of C_h/C_R , larger stress exhibits in the loaded plate and more stress has been relaxed after CAF.

Figure 9 shows the evolutions of stress and springback similarities along the CAF time t with different non-proportional scaling ratios of C_h/C_R . When the scaling ratio $C_h/C_R < 1$, higher stress can be observed in the scaled-down model after loading and more stress-relaxation occurs during the stress-relaxation stage, leading to higher stress and springback similarity parameters with longer CAF time, as shown in Fig. 9. When $C_h/C_R > 1$, lower loaded stresses and smaller relaxed stresses can be observed in the scaled model, and both similarity parameters tend to decrease with CAF time.

Fig. 7 Scaling diagram of similarities of stress (C_σ), springback (C_{sp}) and flexural rigidity (C_D) of the CAFed plates under proportional scaling conditions with different scaling ratios ($C_h = C_R$) (symbols represent FE results)

Fig. 8 Distributions of effective stress in the test plate after loading and 8 h CAF of AA6082-T6 from FE models with prototype size and non-proportional scaled models (non-prop. model) with scaling ratios of $C_R = 4$, $C_h = 2.5$ and 3

The scaling diagrams for CAF of a singly curved plate under non-proportional scaling conditions, based on the developed scaling criteria, are shown in Fig. 10. With a particular small scaling ratio of C_h/C_R , the stress in the loaded scaled-down plate

exceeds yield ($C_h/C_R = 0.44$ in this case), which is beyond the scope of this study. Hence, scaling diagrams were divided into plastic and elastic regions based on the C_h/C_R value and only the elastic area is considered and discussed in this work.

Fig. 9 Analytical (solid lines) and FE results (symbols) of evolution of similarities of **a** stress ($C_\sigma(t)$) and **b** springback ($C_{sp}(t)$) during the stress-relaxation stage under non-proportional scaling condition with different scaling ratio of C_h/C_R

Fig. 10 Scaling diagrams of **a** stress similarity after loading ($C_\sigma(0)$) and stress-relaxation ($C_\sigma(t_f)$) and **b** springback ($C_{sp}(t_f)$) and flexural rigidity (C_D) similarity of CAFed singly curved plates under non-proportional scaling with different C_h/C_R values (symbols represent FE results)

Figure 10a illustrates the changing of stress similarity between the prototype and scaled models along non-proportional scaling ratio of C_h/C_R after loading ($C_\sigma(0)$) and stress-relaxation ($C_\sigma(t_f)$, t_f represents the total stress-relaxation time). The evolution of non-linear stress-relaxation behaviour with different scaling ratios has been well captured. Figure 10b shows the scaling diagram of springback similarity and structural (flexural rigidity) similarity for CAF of the singly curved plate. Good agreement has been achieved between numerical results, and analytical results of the final springback similarity after CAF in both the prototype and scaled models for a set of scaling ratios.

The scaling diagram proposed in Fig. 10b can be used as a look-up table to design advanced experimental trials with scaled-down models for CAF of the prototypal large/extra-large singly curved panels under their specific scaling constraints. With any geometrical scaling ratios (C_R and C_h) determined by particular scaling constraints, the results from the scaled model can be used to predict the prototype according to Fig. 10b. For example, if a tool with $R = 1000$ mm is available and the flexural rigidity of the scaled-down model is expected to be greater than 1/5 of the prototype (i.e. $C_D \leq 5$), a scaling ratio of $C_R = 2200/1000 = 2.2$ can be determined for scaling and $C_D = 5$ can be used as another scaling constraint. A $C_D - (C_h/C_R)$ curve with $C_R = 2.2$ can be plotted in the scaling diagram according to Eq. (16). Then, the non-proportional scaling ratio for the scaled-down model, $C_h/C_R = 0.776$, can be determined through the scaling diagram, as shown in Fig. 10b. Under this scaling condition, after 5 h CAF, the springback percentage (Sp') obtained from the experimental trials for AA6082 with the selected scaled-down model can be directly used to predict the springback property of the prototype (Sp) after CAF. In this case, the springback similarity parameter between them can be directly obtained as $C_{sp} = Sp/Sp' = 1.09$ through the scaling diagram in Fig. 10b.

5.3.3 Doubly curved case

Figure 11 shows the scaling diagram of springback and structural (flexural rigidity) similarities for CAF of the doubly curved plates, with the scaling strategy that curvatures in both directions are scaled with the same ratio ($C_{R_x} = C_{R_y} = C_R$). Numerical and analytical results of CAFed springback similarities in both the prototype and scaled models have achieved good agreements. This figure can also act as a look-up table for scaled-down models designing for CAF process of large/extra-large doubly curved plates. Considering the same scaling constraints of singly curved plates mentioned in Sect. 5.3.2, $C_{R_x} = C_{R_y} = C_R = 2.2$ and $C_D = 5$, the scaled-down model for CAF of the doubly curved prototype plate can be designed with a scaling ratio of $C_h/C_R = 0.762$, as shown in Fig. 11. After 5 h CAF, the springback similarity parameter

Fig. 11 Scaling diagram of springback ($C_{sp}(t_f)$) and flexural rigidity (C_D) similarity of CAFed doubly curved plates under non-proportional scaling with different C_h/C_R ($C_{R_x} = C_{R_y} = C_R$) values (symbols represent FE results)

between the prototype component and scaled-down model can be directly obtained through the scaling diagram in Fig. 11, as $C_{sp} = Sp/Sp' = 1.05$.

5.4 Experimental verification of scaling diagrams for CAF

In order to verify the effectiveness of the developed scaling diagrams in Figs. 7 and 10, scaling trials with CAF experiments were carried out and the results are presented in this section. Since dimensions of length and width of test plates are not related to scaling properties of CAF of singly curved products, as shown in Eqs. (40) and (41), test plates with length of 550 mm and width of 280 mm were selected for both prototype and scaled models. The prototype was defined to have a thickness of 4 mm and loading curvature radius of 700 mm. CAF with scaled models under proportional scaling condition ($C_h = C_R = 4/3$) and non-proportional condition ($C_h = 4/3$, $C_R = 1$) were performed for comparison.

CAF tests were carried out by the multi-point forming tool invented for flexible CAF [35], with which all tests for prototype and scaled models can be performed. Figure 12 shows the experimental setup and the loading process for CAF. The forming pins were first adjusted to generate designed singly curved tool shapes with radius of 700 mm (prototype) or 525 mm (scaled model). The setup tool was loaded by a compression press to make the test plate have full contact with forming pins and locked at the position as shown in Fig. 12b. The loaded tool setup was then moved into a furnace, heated to 160 °C and kept for 5 h. After that, the tool was released and cooled down at the ambient environment and then the formed plate was obtained to complete the entire CAF process.

The deflections of formed plates were measured by the coordinate measurement machine (CMM), and corresponding

Fig. 12 **a** The set-up of the multi-point tool for CAF test of AA6082-T6. **b** The fixed position of tools after loading

springback percentage (Sp) values of prototype and scaled models were calculated and are listed in Table 1. Under proportional scaling condition with $C_h = C_R = 1.33$, springback similarity (C_{sp}) was obtained as 1.011 from CAF experiments, which agrees well with the value of 1.000 from scaling diagram in Fig. 7. Under non-proportional scaling condition with $C_h = 1.33$ and $C_R = 1$, a good agreement of C_{sp} from CAF experiments (0.971) and scaling diagram of Fig. 10 (0.975) has been obtained. Hence, the scaling diagrams have been validated and these diagrams can be used for scaling design of CAF process for manufacturing singly curved structures.

The scaling criteria developed in this study have been successfully applied to scale advanced trials for most of general CAF cases: large singly and doubly curved components. Further investigations will be conducted due to the limitations of the current method:

1. Both the material of component and the material of tools used for CAF process need to keep the consistency in prototype and scaled models for the applications of the method developed in this study.
2. CAF is currently considered as an isothermal process, in general. Recent studies investigated the non-isothermal CAF process [36]; however, more fundamental investigations are required to be performed in order to quantify and model these non-isothermal effects, based on which scaling criteria for heating in CAF need to be updated.

3. Further investigations will be conducted on scaling of CAF with complicated loading conditions and complex-shaped components when the material potentially exceeds its yield after loading.

6 Conclusions

A scaling method utilising the theory of similarity for the creep age forming (CAF) process has been developed in this study to provide guidance for designing scaled-down experiments which can reflect the industrial-scale CAF manufacture of large/extra-large panel structures; the principles and theory can also be adapted for applications to other forming processes. The following conclusions can be drawn:

1. Similarity theory is utilised, for the first time, to scale down large/extra-large panels in CAF manufacture. The scaling criteria, correlating key physical similarity parameters to geometrical scaling ratios, have been developed based on similarity theory and governing equations for loading and stress-relaxation of aluminium alloys throughout all CAF stages (loading, stress-relaxation and springback).
2. Scaling down of singly curved and doubly curved large plates in CAF under either proportional or non-proportional

Table 1 Springback results of prototype and scaled models obtained from CAF experiments (Exp.) and corresponding scaling diagrams

	Prototype (Exp.)	Scaled models from Exp.		Scaled models from scaling diagrams	
		$C_h = C_R = 1.33$	$C_h = 1.33, C_R = 1$	$C_h = C_R = 1.33$	$C_h = 1.33, C_R = 1$
Sp	89.2%	88.3%	91.8%	—	—
C_{Sp}	—	1.011	0.971	1.000	0.975

conditions have been completed using the scaling method developed in this study. The analytical results from the method agree well with numerical results obtained by an established CAF FE modelling programme.

3. The scaling criteria enable the generation of scaling diagrams to describe the relationships between springback/flexural rigidity similarities and scaling ratios for the CAF process, and it has been validated by the conducted CAF experiments. The generated scaling diagrams can be directly used to design advanced experimental trials with scaled-down models for CAF of the prototypal large/extra-large singly curved and doubly curved panels, under the specific scaling constraints.

Acknowledgements Much appreciated is the strong support received from CRRC Qingdao Sifang Co., Ltd. The research was performed at the CRRC Sifang-Imperial Centre for Rail Transportation Manufacturing Technologies at Imperial College London.

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