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ERRATUM

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Erratum to: Minimization of semicoercive functions: a generalization of Fichera's existence theorem for the Signorini problem

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Unfortunately, the author has missed to correct the below changes in the original publication. The correct versions are given below:

In the Introduction section,

Line starting with "Thus, it remains to control the decay of G along the directions……" is corrected to "Thus, it remains to control the decay of G along the directions of recession of the effective domain of F. As shown by Theorem 5.3, an appropriate assumption is that the slope of G along these directions be bounded by a positive constant and that there is a bounded set out of which G is constant along its own directions of recession."
Line starting with "The assumption on G made in Theorem 5.3……." is corrected to "The assumption on G made in Theorem 5.3 then reduces to the requirement that the projection $\mathrm{b_k}$ of b belongs to the normal cone to the recession cone of the effective domain of F."

The revised version of Theorem 5.3 should read as

Theorem 5.3 *A function F of the form* (5.1) *is semicoercive if a is semicoercive and*

(i) *G is subdifferentiable,*
(ii) $\mathrm{dom}\, G$ *is boundedly generated,* $\mathrm{dom}\, G = \mathbb{K}_o + \mathbb{C}_o$, *with $\mathbb{K}_o$ and $\mathbb{C}_o$ closed and convex,*
(iii) *there is a positive constant k such that*

$$\begin{aligned} G(v_o+\eta) - G(v_o) &\geq -k\|\eta^\perp\| \quad \forall \eta \in \mathrm{rc}(\mathrm{dom}\, G), \qquad \forall v_o \in \mathbb{K}_o, \\ G(v+\eta) &= G(v) \qquad \forall \eta \in \mathcal{N}(a) \cap \mathrm{rc}\, G, \quad \forall v \in \mathrm{dom}\, G \backslash \mathbb{K}_o. \end{aligned} \tag{5.7}$$

Proof For $v = v_o + \eta$ with $v_o \in \mathbb{K}_o$ and $\eta \in \mathbb{C}_o$,

$$F(v) = F(v_o+\eta) = \frac{1}{2}\, a(\eta^\perp, \eta^\perp) + a(v_o, \eta^\perp) + \frac{1}{2}\, a(v_o, v_o) + G(v_o+\eta). \tag{5.8}$$

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Since $\mathbb{C}_o = \mathrm{rc}(\mathrm{dom}\, G)$ by (2.11), from assumption (5.7)$_1$, we have

$$G(v_o + \eta) \geq G(v_o) - k\|\eta^\perp\| \qquad \forall \eta \in \mathbb{C}_o.$$

Because G is subdifferentiable and dom G is closed and convex, G attains a minimum in the bounded set $\mathbb{K}_o$ by Proposition 2.2 and Theorem 3.2. Moreover, if a is semicoercive, inequality (5.5) holds. Then, there are positive constants c, c_1, c_2 such that

$$F(v) \geq \frac{1}{2}\, c\, \|\eta^\perp\|^2 - c_1\, \|\eta^\perp\| - c_2 \qquad \forall\, \eta \in \mathbb{C}_o.$$

For every $\varepsilon > 0$, the algebraic inequality

$$c_1\|\eta^\perp\| \leq \frac{1}{2\varepsilon}c_1^2 + \frac{\varepsilon}{2}\|\eta^\perp\|^2$$

holds. Therefore,

$$F(v) \geq \frac{1}{2}(c-\varepsilon)\|\eta^\perp\|^2 - \frac{1}{2\varepsilon}c_1^2 - c_2 \qquad \forall\, \eta \in \mathbb{C}_o. \tag{5.9}$$

Then, $F(v) > \omega$ if $\varepsilon < c$ and

$$\|\eta^\perp\|^2 \geq \frac{2}{c-\varepsilon}\left(\omega + \frac{1}{2\varepsilon}c_1^2 + c_2\right).$$

This proves the semicoerciveness condition (4.2).

To prove condition (4.3), it is sufficient to observe that for all $v \in \mathrm{dom}\, F$ and $\eta \in \mathcal{N}(a)$,

$$F(v+\eta) = \frac{1}{2}a(v,v) + G(v+\eta) = F(v) + G(v+\eta) - G(v). \tag{5.10}$$

Then, $G(v+\eta)$ constant in $\mathcal{N}(a) \cap \mathrm{rc}\, G = \mathrm{rc}\, F$ implies $F(v+\eta)$ constant in rc F. The semicoerciveness condition (4.3) is then verified for $\mathbb{K}_\phi = \mathbb{K}_o$. $\square$

Note that, in condition (5.7)$_2$, $\mathbb{K}_o$ can be replaced by any bounded closed convex set $\mathbb{K}_\phi$ containing $\mathbb{K}_o$. Indeed, the Motzkin decomposition dom $G = \mathbb{K}_o + \mathbb{C}_o$ implies that dom $G = \mathbb{K}_\phi + \mathbb{C}_o$ is also a Motzkin decomposition.

Thus, for functions with a quadratic smooth part, we have the following version of Theorem 4.3.

In Sect. 6 (The semicoercive Signorini problem) after the Eq. 6.4, the text is corrected as,

The conditions (5.7) then reduce to

$$b \cdot \eta \leq k\|\eta^\perp\| \qquad \forall\, \eta \in \mathbb{C}_o, \qquad b \cdot \eta = 0 \qquad \forall\, \eta \in \mathcal{N}(a) \cap \mathbb{C}_o \cap \mathcal{H}_b. \tag{6.5}$$

They are now independent of the point v. In particular, since $b \cdot \eta \geq 0$ for all η in $\mathcal{H}_b$, the second condition is implied by the first. Moreover, considering the decomposition $\eta = \eta^\parallel + \eta^\perp$, we see that the first condition is satisfied if $b \cdot \eta^\parallel \leq 0$ for all $\eta \in \mathbb{C}_o$, and since $b \cdot \eta^\parallel$ is equal to $b^\parallel \cdot \eta$, we conclude that both conditions (5.7) are satisfied if

$$b^\parallel \cdot \eta \leq 0 \qquad \forall\, \eta \in \mathbb{C}_o. \tag{6.6}$$

In this case, F is semicoercive, and therefore, it has minimizers by Theorem 4.3.

Conversely, if v_ϕ is a minimizer, for $\eta \in \mathbb{C}_o$, we have

$$\begin{aligned} -b \cdot \eta &= G(v_\phi + \eta) - G(v_\phi) \\ &= F(v_\phi + \eta) - F(v_\phi) - \frac{1}{2}a(\eta^\perp, \eta^\perp) - a(v_\phi, \eta^\perp) \geq -c_3\|\eta^\perp\| + o(\|\eta^\perp\|), \end{aligned}$$

with c_3 a positive constant. This implies condition (5.7)$_1$ which, in turn, implies (5.7)$_2$. Therefore, as anticipated in the Introduction, *for the Signorini problem the sufficient condition* (6.6) *becomes necessary and sufficient for the existence of minimizers.*