Structural and Multidisciplinary Optimization (2023) 66:184
https://doi.org/10.1007/s00158-023-03599-5

CORRECTION

# Correction to: An interface-enriched generalized finite element method for level set-based topology optimization

S. J. van den Boom[1] · J. Zhang[1] · F. van Keulen[1] · A. M. Aragón[1]

Published online: 2 August 2023


**Correction to: Structural and Multidisciplinary Optimization (2021) 63:1–20 https://doi.org/10.1007/s00158-020-02682-5**

The published article contains a mistake in the notation of the strain–displacement matrix $\mathbf{B}$ in Voigt notation concerning the isoparametric mapping and in the notation of the corresponding derivatives. As the mistakes relate only to the notation, and the implementation in the code was done correctly, they bear no consequences for the remainder of the manuscript and the results presented therein.

The $\mathbf{B}$ matrix defined inline after Eq. (11) on page 5 (repeated on pages 8 and 17) should be written as

$$\mathbf{B} = \begin{bmatrix} \boldsymbol{\Delta} N_1 & \boldsymbol{\Delta} N_2 & \dots & \boldsymbol{\Delta} N_n & \boldsymbol{\Delta}\psi_1 & \dots & \boldsymbol{\Delta}\psi_m \end{bmatrix} \quad (1)$$

for an enriched element with $n$ original shape functions and $m$ enrichment functions. The differential operator $\boldsymbol{\Delta}$ in Eq. 12 should then have been defined in Voigt notation and in global coordinates as

$$\boldsymbol{\Delta} \equiv \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}^{\top} \text{ and}$$
$$\boldsymbol{\Delta} \equiv \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix}^{\top} \quad (2)$$

for elastostatics in 2-D and 3-D, respectively, and

$$\boldsymbol{\Delta} \equiv \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix}^{\top} \text{ and } \boldsymbol{\Delta} \equiv \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix}^{\top} \quad (3)$$

for heat conductivity in 2-D and 3-D, respectively. The derivatives in global coordinates are computed from the derivatives in local coordinates as

$$\nabla_{\mathbf{x}} N_i = \mathbf{J}^{-1} \nabla_{\boldsymbol{\xi}} N_i, \quad \nabla_{\mathbf{x}} \psi_i = \mathbf{J}_e^{-1} \nabla_{\boldsymbol{\xi}} \psi_i \quad (4)$$

for standard and enriched shape functions, respectively, where $\mathbf{J}$ is the Jacobian of the intersected original element and $\mathbf{J}_e$ is the Jacobian of the integration element.

Furthermore, the derivative of $\mathbf{B}$ with respect to the enriched node location $\mathbf{x}_n$ in Equation 28 should be written as

$$\frac{\partial \mathbf{B}}{\partial \mathbf{x}_n} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \frac{\partial \boldsymbol{\Delta}\psi_1}{\partial \mathbf{x}_n} & \dots & \frac{\partial \boldsymbol{\Delta}\psi_m}{\partial \mathbf{x}_n} \end{bmatrix}. \quad (5)$$

Note that the terms in $\partial \mathbf{B}/\partial \mathbf{x}_n$ corresponding to the standard shape functions are zero *(this equation replaces Eq. 29 of the manuscript)*:

$$\frac{\partial \nabla_{\mathbf{x}} N_i}{\partial \mathbf{x}_n} = \cancelto{0}{\frac{\partial \mathbf{J}^{-1}}{\partial \mathbf{x}_n}} \nabla_{\boldsymbol{\xi}} N_i + \mathbf{J}^{-1} \cancelto{0}{\frac{\partial \nabla_{\boldsymbol{\xi}} N_i}{\partial \mathbf{x}_n}} = \mathbf{0}, \quad (6)$$

as the Jacobian of the parent element is not influenced by the enriched node location, and the derivatives of the standard shape functions with respect to local coordinates of the parent element are constant.

The components in $\partial \mathbf{B}/\partial \mathbf{x}_n$ corresponding to the enrichment functions are computed using *(this equation replaces Eq. 30 in the manuscript)*

$$\frac{\partial \nabla_{\mathbf{x}} \psi_i}{\partial \mathbf{x}_n} = \frac{\partial \mathbf{J}_e^{-1}}{\partial \mathbf{x}_n} \nabla_{\boldsymbol{\xi}} \psi_i + \mathbf{J}_e^{-1} \cancelto{0}{\frac{\partial \nabla_{\boldsymbol{\xi}} \psi_i}{\partial \mathbf{x}_n}}, \quad (7)$$

The original article can be found online at https://doi.org/10.1007/s00158-020-02682-5.

✉ A. M. Aragón
a.m.aragon@tudelft.nl

[1] Department of Precision, Microsystems Engineering (PME), Faculty of Mechanical, Maritime, Materials Engineering (3ME), Delft University of Technology (TU Delft), Mekelweg 2, 2628 CD Delft, Netherlands

where the second term is zero because the derivatives of the enrichment function with respect to local coordinates are constant and independent of the enriched node location in global coordinates.