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ORIGINAL PAPER

Differential fertility, school enrollment, and development

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Abstract

This paper develops a model wherein parents choose the number of children, enroll some children in school at indivisible education costs, and receive supplemental earnings from uneducated children. The model accounts for the positive relationship between enrollment ratios and parental earnings and the N-shaped relationship between fertility and parental earnings in Brazil and Indonesia. When children's living costs are high (low) relative to education costs and children's earnings, fertility increases (decreases) with parental earnings due to a dominant income (substitution) effect. A decline in the ratio of child earnings to parental earnings or a rise in education subsidy rates can increase enrollment ratios and decrease fertility. Under progressive income taxes and favorable education subsidies for poor families, educated parents' fertility could be higher than that of illiterate parents' when incomes are low. However, the relationship will be reversed partially because of the rising education subsidy.

Keywords School enrollment · Fertility differential · Growth · Demographic transition

JEL Classification I24 · J13 · O10

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1 Introduction

During the period 1950 to 1980, many developing countries had high illiteracy rates, high fertility rates, and substantial differences in fertility and children's school enrollment ratios between educated and illiterate parents. Conventional views hold that, when illiterate parents have higher fertility than literate parents, this fertility differential reduces average education for the next generation and impedes the demographic transition and economic growth. However, Brazil achieved faster increases in school enrollment ratios and steeper declines in average fertility than Indonesia even though illiterate parents' fertility was 2.5 times as high as educated parents' fertility in Brazil but below educated parents' fertility in Indonesia in 1960–1980. In developed countries with a high human development index (HDI), fertility increases with parental earnings, as in Myrskylä et al. (2009) and Bar et al. (2018). This paper explores fertility rates and school enrollment ratios between educated and illiterate parents (mothers) in a single gender model as well as the role of education subsidies in promoting education and reducing fertility for development.

To motivate the study, this paper first presents some relevant facts in two large developing countries, Brazil and Indonesia, using multiple rounds of census data in 1960–2010 for fertility rates and school enrollment ratios among families (mothers). Previous empirical studies on fertility and school enrollment ratios are mostly based on cross sectional variation.[1] Using census data with a large sample size and a long sample period helps us observe the puzzling evolution of the fertility distribution and changes in education status across generations. The data suggest a positive relationship between school enrollment ratios and parental education and overall positive trends of school enrollment ratios in all parental groups. The data also show that more than one-third of households enrolled only some rather than all children in school in the 1970s.

The decline in parental illiteracy rates played an important role in reducing average fertility in Brazil because illiterate parents had higher fertility than educated parents. In Indonesia, however, average fertility increased over 1960 to 1980 despite falling illiteracy rates because illiterate parents had lower fertility than educated parents. After the 1980s, educated parents' fertility fell below illiterate parents' fertility in Indonesia and average fertility started to decline.

Voluminous studies have explored fertility and child education and their relationships with parental earnings. From the Malthus theory (e.g., Malthus 1872), fertility increases with parental earnings due to subsistence costs for children (e.g., Boyer 1989; de la Croix et al. 2019).[2] Conversely, from the quantity-quality theory (Becker 1960), fertility decreases with parental earnings/education due to the opportunity costs of time spent on child rearing (e.g., Kremer and Chen 2002; de la Croix and Doepke

[1] Vogl (2016) uses data from the Demographic and Health Survey (DHS). The advantage of DHS data is that it contains fertility information in 48 developing countries, whereas the disadvantage is its relatively small sample size and short sample period.

[2] Similar to the subsistence cost for children, Baudin et al. (2015, 2020) assume a minimum level of consumption below which parents cannot give birth to any children in a unified model of childlessness, fertility, and marriage for agents with different education levels.

2003, 2004; Fan and Zhang 2013; Yasui 2017; Bar et al. 2018; Raute 2019).[3] Using US data, Bar et al. (2018) show a typical negative correlation between fertility and parental income in 1980 but an N-shaped relationship between fertility and parental income in 2010. They attribute the eventual positive relationship between fertility and parental income for rich households to the marketization of parental time costs at home.

In de la Croix and Doepke (2003), with unequal initial parental human capital, poor families have more children and invest little in education, and this fertility-differential effect accounts for most of the empirical relationship between inequality and growth. When inequality is high, public education yields higher growth by reducing fertility differentials (de la Croix and Doepke 2004). Using micro-data, Vogl (2016) finds that the relationship between fertility and parental socioeconomic status flipped from positive at the early stage of the demographic transition to negative. The reason for the flip in Vogl (2016) is a corner solution in which low-skill parents forgo investment in children as in a dynastic model of Becker et al. (1990). However, even illiterate parents enroll a fraction of their children in school in the Indonesia census data prior to the flip of fertility differential.

In these models, rearing children costs parents time and educating them costs parents money. In addition, Vogl (2016) assumes a living cost for a child and a subsistence constraint on consumption that creates a threshold level for human capital above which parents cease to be subsistence-constrained when making decisions on fertility. A common assumption in previous models is that a parent chooses equal or no education investment in children. This model allows parents constrained by children's education and living costs to choose the number of children and the fraction of children who receive education, with the other children working.[4] The earnings of working children weaken the incentive for education and strengthen the incentive for more children, particularly in poor families. This model also assumes a progressive labor-income tax to finance education subsidies that may be higher for children in poor households. The taxes reduce the opportunity cost of time rearing children and the subsidies reduce the private cost of school enrollment. Thus, progressive income taxes and favorable education subsidies for poor households may cause higher fertility for educated parents than for illiterate parents. A partial catch-up rise in education subsidies for educated parents enlarges the decline in their fertility and thus may induce the flip of the fertility differential when illiterate parents also enroll some children in school.

We show that the implications of this mechanism can account for the patterns of enrollment ratios and fertility in relation to parental education attainments in the data and the roles of progressive labor-income taxes and education subsidies in the demographic transition and development. Our first contribution is to account for the positive relationship between school enrollment ratios and parental education or earnings in the

[3] Many studies use this theory to explain the rise in education investment and demographic transition from the Malthusian regime (e.g., Becker et al. 1990; Galor and Weil 2000; Galor and Moav 2002; Greenwood and Seshadri 2002; Choi et al. 2020). Among these studies, Galor and Moav (2002) propose an overlapping generations model with heterogeneous tastes for the quantity and quality of children and fixed land inputs to develop a unified theory of the evolution of population, technology, and growth.

[4] The education system is typically structured as fixed bundles, such as primary, secondary, or tertiary education, rather than continuously divisible subjects assumed in related models.

census. When the opportunity costs of children's education (e.g., education spending and child earnings) are high relative to the forgone earnings of rearing children and living costs, parents have as many children as possible and no incentive for children's education. When income grows relative to the education costs, parents enroll a fraction of children in school. Thus, identical children at birth in a family become educated or illiterate workers. The enrollment ratio increases with the returns to education, forgone earnings of rearing children, and living costs but decreases with the education costs and child earnings.

Our second contribution is to account for the N-shaped relation between fertility and parental earnings in the data. When children's living costs are high relative to children's education costs, fertility increases with parental earnings due to a dominant income effect and thus fertility is higher for educated parents than for illiterate parents. When the living costs are low relative to the education costs, fertility decreases with parental earnings due to a dominant substitution effect and thus fertility is lower for educated parents than for illiterate parents. A restriction on child labor can reduce fertility and increase enrollment ratios.

Third, the illiteracy rate of an economy converges to a steady state—a poverty trap for illiterate parents when child earnings and the costs of education and living increase proportionately with parental earnings for stationary education subsidy rates. This motivates a rise in education subsidy rates in the attempt to reduce illiteracy for prosperity.[5] The decline in illiteracy rates may raise average fertility in the economy when illiterate parents have lower fertility than educated parents as in Indonesia until 1990. Conversely, the rise in education subsidy rates is particularly effective in reducing illiteracy rates and average fertility when illiterate parents have higher fertility than educated parents.

The results help explain the Brazil-Indonesia comparison with their, respectively, high and low education subsidies. In particular, rising education subsidies in favor of poor families financed by progressive labor-income taxes can initially generate higher enrollment ratios and fertility for rich families than for poor families at low average income. Then, a partial catch-up rise in education subsidy rates for other families can enlarge the decline in their fertility and thus yield the flip of the fertility differential as observed in Indonesia. When all children complete education, fertility may relate positively to parental earnings again as in developed countries with high HDI (e.g., Myrskylä et al. 2009; Bar et al. 2018; Vogl 2020).[6]

The rest of this paper proceeds as follows. Section 2 provides facts in the census data. Section 3 proposes the model. Section 4 presents equilibrium analyses. Section 5 provides quantitative results. Section 6 gives the conclusion.

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[5] Empirical evidence indicates positive effects of education subsidies on school enrollment ratios from micro-data in developing countries (e.g., Attanasio et al. 2010; Duflo et al. 2015; Ravallion and Wodon 2000; Wang 2018) and in developed countries (e.g., Mitch 1986), as well as from cross-country data (e.g., Casagrande and Zhang 1998).

[6] When the child subsistence cost is high in Becker et al. (1990), parents make no investment in children and fertility relates positively to parental earnings. When the subsistence cost is negligible, parents invest the same education time in children and fertility declines. This inverted-U pattern differs from the N-shaped pattern here.

2 Facts in census data from Brazil and Indonesia

We focus on census data from Brazil and Indonesia to motivate our paper for the following reasons. First, both countries have the most extended sample coverage in developing countries, which helps observe declining fertility rates and rising school enrollment ratios.[7] Second, Brazil has one of the highest ratios of illiterate parents' fertility to educated parents' fertility in the world but Indonesia has one of the lowest ratios as noted by Kremer and Chen (2002). Thus, a comparison between these two countries may shed light on the distribution and evolution of fertility and school enrollment ratios across parental groups and over time.

We extract data from the Integrated Public Use Microdata Series (IPUMS). We use *actual* fertility at the survey time and restrict our sample to focus on parents (women) aged 40 to 49 in the census year because the number of children born over parents' lifetime can only be accurately constructed for those at the end of their reproductive cycle.[8] As a result, we observe the fertility and child school enrollment of six birth cohorts in Brazil and seven in Indonesia.[9] Since one can only match parents with their children if they lived in the same household, we focus on the education outcomes of school-aged children between 7 and 15 to avoid over-representing children's sample by children who left home at older ages.[10] We measure child education by school enrollment status because many children in this age group have not completed their education yet.

Table 1 reports the descriptive statistics of the key variables. To check whether changes in school enrollment ratios differ between genders, we report school enrollment ratios for boys and girls separately. Both countries' demographic transition in the sample period had similar average fertility rates and comparable fertility trends. For example, on average, the sampled parents born in the 1920s had 5.5 children in Brazil and 5.2 children in Indonesia. The respective numbers declined to 2.4 and 3.0 for parents born in the 1960s.

[7] The Brazilian education system consists of five years of elementary I, four years of elementary II, three years of upper secondary school, and three to six years of tertiary education. By the 1934 constitution, the first four years of primary education are mandatory and free. The 1967 constitution expanded compulsory schooling to 8 years. However, due to inadequate investment in teacher training and school infrastructure, primary school enrollment ratios were still lower than 70% even in the 1980s. By the 2009 amendment, school education is mandatory and free from the age of 4 to 17 (2-year pre-school education and 12-year primary and secondary education). The Indonesian system consists of 6-year primary education, 3-year junior high school, 3-year high school, and 3- or 4-year tertiary education. Children generally start their primary education at the age of six or seven. By the 1973 Decree, the 6-year primary education was compulsory, and by 1994, it was extended for children aged 13–15.

[8] The inclusion of younger parents would cause a downward bias because they may continue to give births at an older age.

[9] We exclude parents with missing fertility information and use the number of survival children rather than the number of live births as the measure for fertility. All of our conclusions are insensitive to which measure is used in the analysis.

[10] The proportions of 15-year-old living with their parents are 83% and 87% in the 1990 Indonesian Census and 1991 Brazilian Census, respectively. The corresponding numbers are 79% and 84% for the 16-year-old, and 68% and 75% for the 18-year-old. These restrictions lead to the use of an approximately 5% samples of the censuses in these two countries. We skip the 2005 Indonesia census due to missing information on fertility.

Table 1 Descriptive statistics

Census Year	Fertility	TFR	Mother's edu.	School enrollment		GDP per cap	Edu. exp.
				boys	girls		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Panel A: Brazil						
1960 (born in 1911–1920)	5.916	6.21	1.538	0.491	0.473	2469.1	–
	(4.517)		(2.261)	(0.500)	(0.499)		
1970 (born in 1921–1930)	5.481	5.02	2.115	0.662	0.657	3844.6	3.599
	(4.187)		(2.844)	(0.473)	(0.475)		
1980 (born in 1931–1940)	5.124	4.07	2.998	0.647	0.668	6943.1	3.599
	(3.801)		(3.481)	(0.478)	(0.471)		
1991 (born in 1942–1951)	4.105	2.72	4.553	0.738	0.781	6116.8	4.634
	(3.183)		(4.425)	(0.440)	(0.414)		
2000 (born in 1951–1960)	3.141	2.36	6.276	0.928	0.942	6834.3	3.881
	(2.444)		(4.619)	(0.259)	(0.233)		
2010 (born in 1961–1970)	2.405	1.8	8.084	0.958	0.967	8324.7	4.328
	(1.836)		(4.578)	(0.200)	(0.177)		

Table 1 continued

Census Year	Fertility	TFR	Mother's edu.	School enrollment		GDP per cap	Edu. exp.
				boys	girls		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Panel B: Indonesia						
1960 (born in 1911–1920)	–	5.67	–	–		665.15	–
1971 (born in 1922–1931)	5.227	5.41	0.909	0.605	0.547	873.7	0.767
	(3.277)		(2.023)	(0.489)	(0.498)		
1976 (born in 1927–1936)	5.268	4.93	1.296	0.688	0.624	1239.9	0.767
	(3.308)		(2.385)	(0.463)	(0.484)		
1980 (born in 1931–1940)	5.392	4.43	1.725	0.783	0.747	1496.5	0.767
	(3.254)		(2.816)	(0.412)	(0.435)		
1990 (born in 1941–1950)	4.793	3.12	3.638	0.813	0.805	2160.1	0.626
	(2.677)		(3.687)	(0.390)	(0.396)		
1995 (born in 1946–1955)	4.130	2.7	4.963	0.876	0.870	2882.6	0.61
	(2.337)		(3.774)	(0.330)	(0.336)		
2000 (born in 1951–1960)	3.790	2.45	5.722	–	–	2749.6	2.228
	(2.307)		(3.422)	–	–		
2010 (born in 1961–1970)	2.992	2.1	7.194	0.907	0.921	3965.8	4.328
	(1.875)		(4.100)	(0.291)	(0.270)		

Notes: The fertility rate and mother's schooling are calculated using a sample of women aged 40–49 at the census year. Enrollment is calculated using 7–15-year-old children in the census year. School enrollment information was not collected in the 2000 Indonesian population census. Numbers in parentheses are standard deviations. GDP per capita is valued at the 2005 constant prices, extracted from the Penn World Table version 7.1. TFR (total fertility rate) and Education Expenditure are measured as the percentage of GNI and are extracted from the World Development Indicator https://databank.worldbank.org/source/world-development-indicators, which only includes public expenditure

Parental education attainments and school enrollment ratios of both boys and girls increased considerably over time. Real GDP per capita in Brazil was more than twice as that in Indonesia and increased substantially in both countries. The 10-year average annual growth rates peaked first at 6% in 1970–1980 and again at 2% and 3.7%, respectively, in 2000–2010 in Brazil and Indonesia, following surging school enrollment ratios in the preceding decades. Public education spending was below 1% of national income (GNI) in Indonesia until 2000, but 4% in Brazil throughout the sample period. At the end of the sample period, public education spending as a share of GNI in Indonesia was comparable to that in Brazil.

To check whether parents enroll a fraction of children in schools, we focus on children aged 7–15 whose school enrollment is still mostly their parents' decision. Figure 1(a) plots the share of households who only enroll some of their children in schools, and Fig. 1(b) plots the share of households who enroll all their children in schools. The figures show that more than 30% of households only enroll some of their children to schools in the 1970s in both Indonesia and Brazil.

Figure 1(a) shows that this ratio increased in Indonesia from the 1970s to 1980s and then declined monotonically, suggesting a possibility that the initial increase in school enrollment is because more parents send at least one child in schools. Even at the end of the sample period, a considerable number of households still did not enroll all of their children in schools, particularly in Indonesia. Figure 1(b) shows that while the proportion of households who enrolled all of their children is lower in Brazil than in Indonesia before 1990, it is higher in Brazil afterwards. In 2010, about 93% of Brazilian households enrolled all of their children to schools, which is about 10 percentage points higher than that in Indonesia.

The figure clearly shows that allowing parents to enroll only some of their children could help us understand the demographic transition process. However, the enrollment data in the figure consist of all education levels. When focusing on secondary education that matters to parental choices between schooling and working for children, school

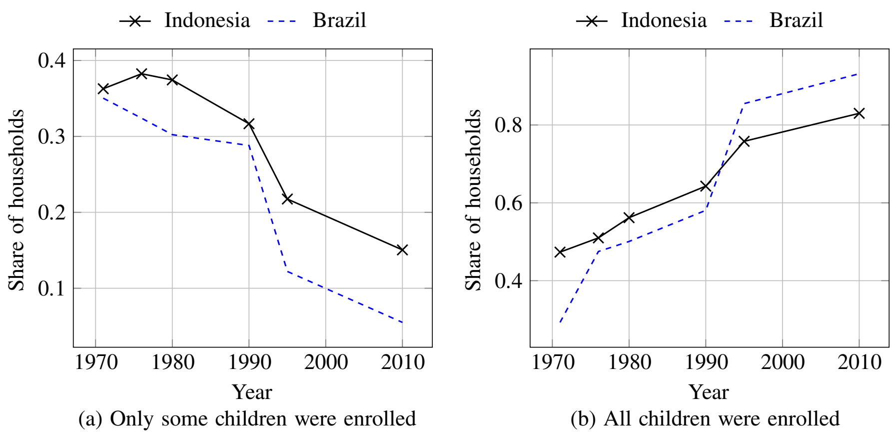


Fig. 1 Share of households by children's enrollment status. Data source: Various years of Brazil and Indonesia census

enrollment ratios in developing countries would be much lower than the ratios in Fig. 1 (World Bank data, various years). Consequently, the proportion of families that enroll only a fraction of children in secondary school would be much higher than the ratios in the figure.

In Table 2, we further divide our sample by the older sibling's enrollment status. Column (1) reports the unconditional school enrollment ratio of the youngest child, $P(S_y = 1)$, where $S_y = 1$ if the youngest child is enrolled in school and 0 otherwise. This ratio $P(S_y = 1)$ increased from 50.3% in 1960 to 97.14% in 2010 for Brazilian boys and from 50.2% to 97.7% for Brazilian girls. The increase in school enrollment ratio in Indonesia is in a similar order, from 66.6% in 1971 to 94.1% in 2010 for boys and from 65.6% to 84.8% for girls. Column (2) reports the conditional school enrollment ratio $P(S_y = 1|S_o = 1)$, where $S_o = 1$ if the oldest child had been enrolled in school and 0 otherwise. This ratio can only serve as an upper bound because younger children from poor families have more years of schooling and higher school enrollment ratios than their older siblings, as noted by Parish and Willis (1993). For comparison, we also report $P(S_y = 1|S_o = 0)$. The steady upward trend in $P(S_y = 1|S_o = 0)$ suggests an increasing probability of sending at least one child to school over time.

Figure 2 plots school enrollment ratios of children aged 7–15 by parents' education attainments in four groups: illiterate; incomplete primary education; completed primary education; and completed secondary education. It shows a positive relationship between school enrollment ratios and parental education attainments in both countries. School enrollment ratios were lower for children born to parents with some or completed primary education in Indonesia than in Brazil. School enrollment ratios of illiterate parent increased faster in Brazil than in Indonesia despite a much higher ratio of illiterate parents' fertility to educated parents' fertility in Brazil. This may arise from the much higher education subsidies in Brazil than in Indonesia from Table 1. Nevertheless, the data suggest that a higher ratio of illiterate parents' fertility to educated parents' fertility does not necessarily imply slower human capital accumulation during the demographic transition.

It should be noted that the upward trend revealed in Fig. 2 might be biased by changes in the age composition of sampled children. Because parents' age at childbirth might have increased with their education attainments over time, the average age of sampled children could differ systematically across census years. Given that many children dropped out of school after completing primary education, changes in the age profile of the sampled children could cause year to year variations in school enrollment ratios even if age-specific school enrollment ratios do not change. To control for the impact of changes in the age profile of sampled children on school enrollment ratios, we run the following logit regression

$$P(E_{it} = 1) = F(\gamma_0 + \gamma_1 M_i + \sum_{a=8}^{a=15} \gamma_a D_{ai} + \sum_{j=1}^{j=4} \sum_t Z_j C_t \delta_{jt}), \tag{1}$$

where $E_{it} = 1$ if child i is enrolled in school at time t; $M_i = 1$ if child i is a male; D_a is a series of age dummies; Z_j is a series of parents' education attainment dummies; and C_t is a series of census year dummies.

Table 2 Child school enrollment ratios by the older sibling's enrollment status

Census year	All $P(S_y = 1)$ (1)	Older sibling enrolled $P(S_y = 1 \mid S_o = 1)$ (2)	Observations (3)	Older sibling not enrolled $P(S_y = 1 \mid S_o = 0)$ (4)	Observations (5)
	Panel A: Brazilian boys				
1960	0.503	0.777	10,557	0.333	17,017
1970	0.664	0.838	27,594	0.422	19,839
1980	0.664	0.834	32,113	0.395	20,212
1991	0.726	0.844	48,759	0.457	21,392
2000	0.934	0.960	57,111	0.708	6,577
2010	0.971	0.985	41,370	0.700	2,194
	Panel B: Brazilian girls				
1960	0.502	0.786	10,454	0.321	16,337
1970	0.675	0.849	26,991	0.432	19,350
1980	0.680	0.851	30,794	0.407	19,246
1991	0.759	0.872	47,016	0.503	20,843
2000	0.947	0.969	55,608	0.753	6,455
2010	0.977	0.989	39,486	0.738	2,121
	Panel C: Indonesian boys				
1971	0.666	0.788	2,858	0.478	1,851
1976	0.787	0.867	1,589	0.667	1,055
1980	0.837	0.887	48,495	0.744	26,314
1990	0.907	0.949	5,183	0.816	2,345
1995	0.937	0.962	4,769	0.844	1,278
2010	0.941	0.961	131,917	0.813	20,130

Table 2 continued

Census year	All $P(S_y = 1)$ (1)	Older sibling enrolled $P(S_y = 1 \vert S_o = 1)$ (2)	Observations (3)	Older sibling not enrolled $P(S_y = 1 \vert S_o = 0)$ (4)	Observations (5)
	Panel D: Indonesian girls				
1971	0.656	0.798	2,798	0.420	1,687
1976	0.763	0.868	1,552	0.610	1,070
1980	0.832	0.888	46,902	0.729	25,216
1990	0.908	0.944	5,008	0.826	2,219
1995	0.939	0.966	4,541	0.834	1,164
2010	0.948	0.965	125,150	0.839	19,262

Notes: The sample consists of children ages 7–15 with at least one sibling in the same age range and mother aged 40–49 in the census year. Because the 2000 Indonesia census does not contain information on school enrollment, we cannot calculate the school enrollment ratio for that year. $P(S_y = 1)$ is the school enrollment ratio of the youngest child in a household. $P(S_y = 1|S_o = 1)$ is the school enrollment ratio of the youngest child in a household conditional on the oldest child being enrolled in school. $P(S_y = 1|S_o = 0)$ is the school enrollment ratio of the youngest child in a household whose oldest child is not enrolled in school

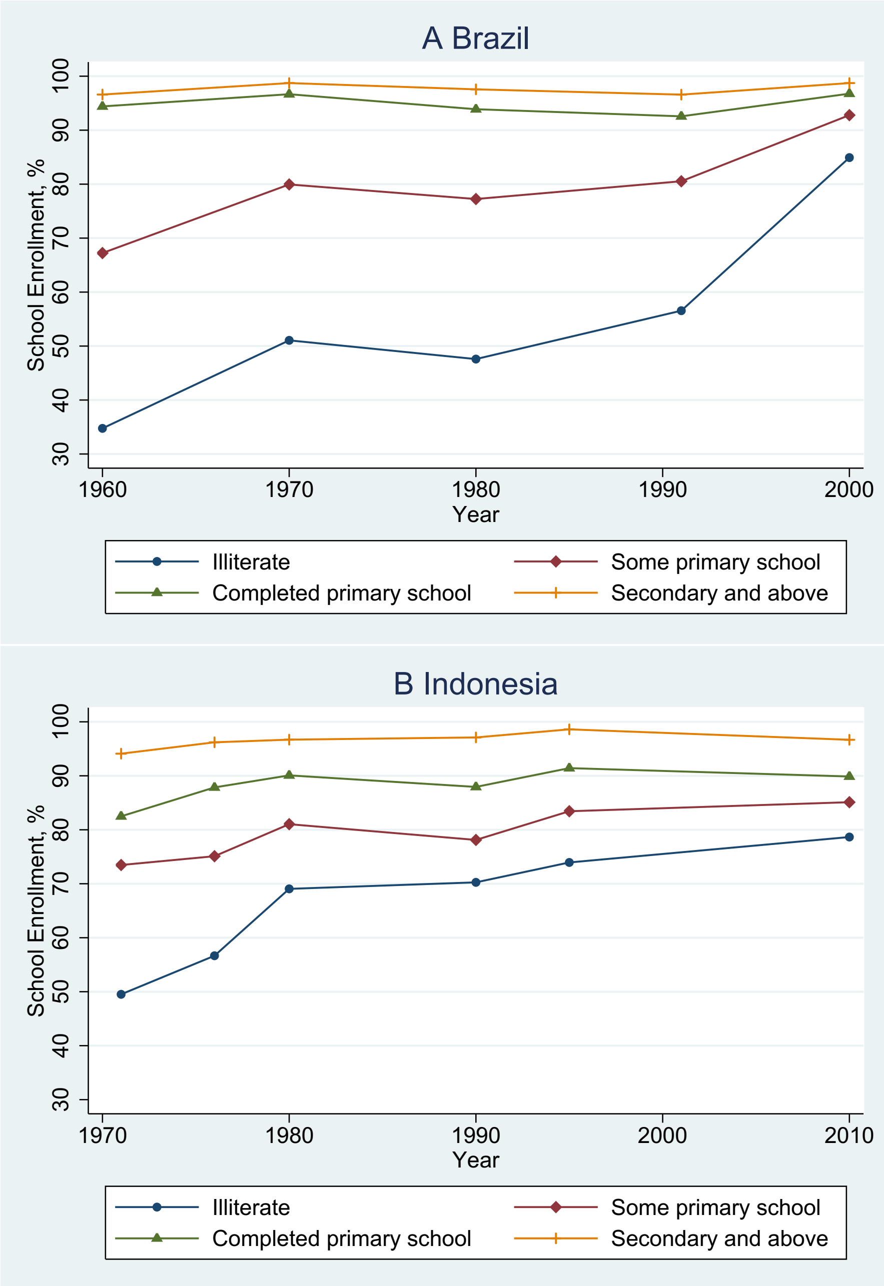


Fig. 2 Child school enrollment ratios by maternal education. Data source: Various years of Brazil and Indonesia census

Table 3 The impact of maternal education on child school enrollment

Census year	Coefficients				Marginal effect			
	E_1 (1)	E_2 (2)	E_3 (3)	E_4 (4)	E_1 (5)	E_2 (6)	E_3 (7)	E_4 (8)
	Panel A: Brazil							
1960 (mother born in 1911–1920)		1.408	3.603	4.131		0.186	0.477	0.547
		(0.015)	(0.086)	(0.178)		(0.002)	(0.011)	(0.023)
1970 (mother born in 1921–1930)	0.703	2.099	4.126	5.107	0.093	0.278	0.546	0.676
	(0.012)	(0.013)	(0.076)	(0.101)	(0.002)	(0.002)	(0.01)	(0.013)
1980 (mother born in 1931–1940)	0.586	1.991	3.564	4.512	0.078	0.263	0.471	0.597
	(0.012)	(0.012)	(0.046)	(0.062)	(0.002)	(0.002)	(0.006)	(0.008)
1991 (mother born in 1942–1951)	0.866	2.053	3.236	4.067	0.115	0.272	0.428	0.538
	(0.012)	(0.011)	(0.027)	(0.027)	(0.002)	(0.001)	(0.004)	(0.004)
2000 (mother born in 1951–1960)	2.452	3.286	4.159	5.119	0.324	0.435	0.55	0.677
	(0.015)	(0.013)	(0.027)	(0.031)	(0.002)	(0.002)	(0.004)	(0.004)
2010 (mother born in 1961–1970)	3.557	3.853	4.305	4.974	0.471	0.51	0.569	0.658
	(0.076)	(0.017)	(0.026)	(0.027)	(0.01)	(0.002)	(0.003)	(0.004)
Number of observations	2,209,392							
	Panel B: Indonesia							

Table 3 continued

Census year	Coefficients				Marginal effect			
	E_1 (1)	E_2 (2)	E_3 (3)	E_4 (4)	E_1 (5)	E_2 (6)	E_3 (7)	E_4 (8)
1971 (mother born in 1922–1931)	–	1.075	1.872	3.119	–	0.105	0.183	0.304
	–	(0.038)	(0.045)	(0.104)	–	(0.004)	(0.004)	(0.010)
1976 (mother born in 1927–1936)	0.216	1.194	2.162	3.698	0.021	0.116	0.211	0.361
	(0.028)	(0.041)	(0.071)	(0.173)	(0.003)	(0.004)	(0.007)	(0.017)
1980 (mother born in 1931–1940)	0.659	1.377	2.211	3.448	0.064	0.134	0.216	0.336
	(0.017)	(0.018)	(0.022)	(0.036)	(0.002)	(0.002)	(0.002)	(0.004)
1990 (mother born in 1941–1950)	0.827	1.285	2.061	3.622	0.081	0.125	0.201	0.353
	(0.025)	(0.026)	(0.033)	(0.074)	(0.002)	(0.003)	(0.003)	(0.007)
1995 (mother born in 1946–1955)	0.959	1.720	2.543	4.522	0.094	0.168	0.248	0.441
	(0.032)	(0.032)	(0.037)	(0.098)	(0.003)	(0.003)	(0.004)	(0.010)
2010 (mother born in 1961–1970)	1.282	1.772	2.246	3.465	0.125	0.173	0.219	0.338
	(0.018)	(0.018)	(0.017)	(0.018)	(0.002)	(0.002)	(0.002)	(0.002)
Number of observations	1,816,201							

Notes: The sample consists of children ages 7–15 whose mother was aged 40–49 in the census year. E_1 refers to illiterate women, E_2 to refers women with some primary school education, E_3 refers to women who completed primary school, and E_4 refers to women who graduated from secondary school. Children born to illiterate mothers in the 1960 Brazilian census (1971 Indonesian census) are the reference group. Numbers in the parentheses are standard errors, which are clustered at the household level. School enrollment information was not available in the 2000 Indonesia census. Columns (1)–(4) report the coefficients on the interaction between mother's education and census year dummy from the logit regression and columns (5)–(6) report the marginal effects. In addition to the interaction term, child gender and age are included as a vector of dummy variables in the regression

Table 3 reports estimation results. Columns (1)–(4) are the coefficients on the interaction between the parental education dummy and the census year dummy, $\hat{\delta}_{jt}$, whereas columns (5)–(8) are the marginal effects of these interaction terms. For the sake of brevity, the coefficients on the gender dummy and age dummies are not reported. We use children of illiterate parents in the 1960 Brazilian census (1971 Indonesian census) as the reference group. Consequently, the marginal effect of the interaction term captures the difference in school enrollment ratios between children in the reference group and children in a concerned census year whose parent belonged to the given education group, when evaluated at the sample mean. The vertical reading reveals the time trend of school enrollment ratios for a given level of parental education, while the horizontal reading shows the impact of parental education on school enrollment ratios at any given time.

Parental education has strong positive effects on school enrollment ratios. In Brazil, for instance, the school enrollment ratio of children from parents with secondary education was 54.7 percentage points higher than that of children from illiterate parents in 1960. The difference in school enrollment ratios of children across parental education attainments in Indonesia was smaller than that in Brazil. In 1980, for instance, the school enrollment ratio of children from parents with at least secondary education was 33.6 percentage points higher than that of children from illiterate parents in Indonesia and 59.7 percentage points in Brazil.

Table 4 Population shares by maternal education attainment

Census year	E_1 (1)	E_2 (2)	E_3 (3)	E_4 (4)
	Panel A: Brazil			
1960 (born in 1911–1920)	0.566	0.387	0.035	0.012
1970 (born in 1921–1930)	0.472	0.445	0.041	0.042
1980 (born in 1931–1940)	0.368	0.489	0.064	0.079
1991 (born in 1942–1951)	0.240	0.487	0.092	0.180
2000 (born in 1951–1960)	0.114	0.434	0.155	0.298
2010 (born in 1961–1970)	0.009	0.330	0.215	0.447
	Panel B: Indonesia			
1971 (born in 1922–1931)	0.795	0.118	0.072	0.014
1976 (born in 1927–1936)	0.691	0.217	0.065	0.026
1980 (born in 1931–1940)	0.626	0.241	0.089	0.044
1990 (born in 1941–1950)	0.360	0.290	0.229	0.121
1995 (born in 1946–1955)	0.198	0.304	0.313	0.185
2000 (born in 1951–1960)	–	0.337	0.454	0.209
2010 (born in 1961–1970)	0.102	0.098	0.441	0.360

Notes: The sample consists of women ages 40–49 in the census year. E_1 refers to illiterate women, E_2 refers to women with some primary school education, E_3 refers to women who completed primary school, and E_4 refers to women who graduated from secondary school. Illiterate women and those who did not graduate from primary school were grouped together in the 2000 Indonesia census

The estimation results also show that school enrollment ratios increased over time regardless of parental education even after controlling for children's age. For children in the 1960 versus 2010 Brazilian censuses, school enrollment ratios increased by 47.1 percentage points for illiterate parents and by 11 percentage points for middle school graduates. As a result, the difference in child school enrollment ratios between illiterate parents and parents with secondary education narrowed from 54.7 percentage points in 1960 to 18.7 percentage points in 2010. Thus, the population share of less educated parents decreased, as shown in Table 4. The least educated parents' population share decreased from 47.2% for the 1920s cohort to 0.9% for the 1960s cohort in Brazil and from 79.5% to 10.2% in Indonesia.

Table 5 presents fertility rates by parental education with downward trends. The ratio of illiterate parents' fertility to that of parents who completed secondary education was 2.5 from the cohort born in 1911–1920 to the cohort born in 1951–1960 in Brazil but 0.9 in Indonesia. Such opposite fertility differentials may explain why average parental schooling years increased slightly slower in Brazil than in Indonesia. Yet, school enrollment ratios grew faster and fertility declined more sharply in Brazil than in Indonesia in contrast to the conventional view that economies with higher fertility from illiterate parents than from educated parents tend to have lower education for the next generation and higher average fertility. These challenging facts motivate our theoretical investigation.

Table 5 Fertility rates by maternal education

Census year	μ (1)	E_1 (2)	E_2 (3)	E_3 (4)	E_4 (5)
	Panel A: Brazil				
1960 (born in 1911–1920)	2.742	6.663	5.205	2.845	2.430
1970 (born in 1921–1930)	2.555	6.428	4.991	2.929	2.515
1980 (born in 1931–1940)	2.572	6.355	4.889	3.104	2.471
1991 (born in 1942–1951)	2.664	5.852	4.177	2.903	2.197
2000 (born in 1951–1960)	2.540	5.012	3.618	2.678	1.973
2010 (born in 1961–1970)	1.967	3.516	3.050	2.410	1.787
	Panel B: Indonesia				
1971 (born in 1922–1931)	0.901	5.019	5.995	6.188	5.568
1976 (born in 1927–1936)	0.960	5.038	5.784	6.001	5.249
1980 (born in 1931–1940)	0.952	5.101	5.939	5.967	5.358
1990 (born in 1941–1950)	1.062	4.596	5.088	4.977	4.329
1995 (born in 1946–1955)	1.134	4.062	4.393	4.242	3.583
2000 (born in 1951–1960)	–	–	4.038	3.830	3.303
2010 (born in 1961–1970)	1.198	3.181	3.434	3.125	2.655

Notes: The sample consists of women ages 40–49 in the census year. μ is the measure of fertility differential, defined the as the ratio of illiterate women's fertility to that of secondary school graduates. E_1 refers to illiterate women, E_2 refers to women with some primary school education, E_3 refers to women who completed primary school, and E_4 refers to women who graduated from secondary school. Illiterate women and those who did not graduate from primary school were grouped together in the 2000 Indonesia census

3 The model

Consider a quality-quantity model with infinite discrete time and two-period lived agents, children and parents. There are two types of parents in period t: illiterate (or less educated) parents with human capital h_t^L and literate parents with human capital $h_t^H > h_t^L$. A child who is not enrolled in school has a lower level of human capital $h_t^C < h_t^L$ than an illiterate parent does. Children are identical at birth within a family but may grow up as different workers with or without education.

The production of a single final good uses effective labor $l_t^k h_t^k$ for $k = H, L$:

$$y_t^k = l_t^k h_t^k. \tag{2}$$

The wage rates of parents are equal to their own human capital $W_t^k = h_t^k$ and thus $W_t^H > W_t^L$. Similarly, the wage rates for children are $W_t^C = h_t^C$.

The cost and level of education are externally structured to parents. The education cost, such as a tuition fee, is proportional to the human capital of literate adults, $e_t = \xi_t h_t^H$ where $\xi_t \in (0, 1)$ as the bulk of the cost is the implicit cost of teachers. The adulthood human capital of children from schooling depends positively on education investment received in period t

$$h_{t+1}^H = A(\psi + B\xi_t)^\phi h_t^H; \tag{3}$$

and the adulthood human capital of children without schooling is

$$h_{t+1}^L = A\psi^\phi h_t^H, \tag{4}$$

where $A > 0$ is the efficiency parameter; $B > 0$ is the relative advantage of schooling; $\psi > 0$ captures the spillovers from the human capital of literate parents to all children;[11] and $0 < \phi < 1$ measures the degree of returns to human capital accumulation. When compulsory education at the primary level is implemented, it can be represented by ψ when children at this school age are too young to work; and accordingly parents choose secondary school enrollment for children.

From (3) and (4), the return to education $\omega_{t+1} \equiv h_{t+1}^H / h_{t+1}^L$ is determined by

$$\omega_{t+1} = \frac{h_{t+1}^H}{h_{t+1}^L} = \left(\frac{\psi + B\xi_t}{\psi}\right)^\phi \equiv \omega(\xi_t) > 1. \tag{5}$$

Since $0 < \phi < 1$, ω_{t+1} increases with ξ_t, at a diminishing rate: $\omega'(\xi_t) > 0$ and $\omega''(\xi_t) < 0$. It is straightforward to extend this case to more education levels.

[11] Borjas (1995) finds evidence that the socioeconomic performance of workers depends on the average skills of the parents' generation in their ethnic groups and neighborhoods as assumed in de la Croix and Doepke (2003). Lei (2022) finds evidence for the effects of exposure to peers from disrupted families. Moretti (2004) also finds evidence for the spillovers of workers' education on productivity in production firms. Short-term job training and learning-by-doing are usual venues that allow workers without formal education to obtain certain skills for production from workers with education and skills as assumed in (4).

A parent allocates one unit of time endowment to labor or childcare. As in Becker et al. (1990), rearing a child needs $v \in (0, 1)$ units of time. A type-k parent chooses the number of children, $n_t^k \in [0, 1/v]$, and a fraction of them for education, $\lambda_t^k \in [0, 1]$, taking the cost and structure of education as given. Thus, vn_t^k is the units of time for child rearing and the remaining time $1 - vn_t^k = l_t^k$ is labor. The average adulthood human capital of children in the family is equal to $\bar{h}_{t+1}^k \equiv \lambda_t^k h_{t+1}^H + (1 - \lambda_t^k) h_{t+1}^L$.

Parents derive utility from their own consumption c_t^k, the number of children n_t^k, and the average human capital of children $\bar{h}_{t+1}^k$ as follows:

$$U_t^k = \frac{\left(c_t^k\right)^{1-\sigma} - 1}{1-\sigma} + \beta \frac{\left[\left(n_t^k\right)^{\alpha}\left(\bar{h}_{t+1}^k\right)^{\eta}\right]^{1-\gamma} - 1}{1-\gamma}, \tag{6}$$

where $1 > \beta > 0$ indicates the taste for utility from children; $1 > \alpha > 0$ or $1 > \eta > 0$ indicates the relative role of the number or average human capital of children; $1/\sigma > 0$ or $1/\gamma > 0$ indicates the elasticity of intergenerational substitution.

A type-k parent pays an income tax at rate $\tau_t^k \in [0, 1)$ and receives an education subsidy at rate $s_t^k \in [0, 1)$. As seen in Fig. 2, a common problem in developing countries is low school enrollment in poor families which motivates various public programs to mitigate poverty and low school enrollment in poor families. For instance, Morley and Coady (2003) study targeted education subsidies in the form of cash or food conditional on keeping children in school in many developing countries including Brazil. Given progressive income taxes and targeted education subsidies in developing countries, we expect that $\tau_t^H \geq \tau_t^L$ and $s_t^H \leq s_t^L$. From Table 1, the ratios of public education spending to GDP increased in both countries in 1970–2010 with declines only during times of rapid GDP growth, suggesting a positive trend in education subsidy rates for improvements in enrollment ratios.

Child earnings are supplemental income in parental budget constraints as in Doepke and Zilibotti (2005). The living cost per child exceeds a subsistence level $d_t \geq \tilde{d}$. Both the education cost and child living cost are externally given to families. The budget constraint of a type-k parent is given by

$$c_t^k = (1 - \tau_t^k) W_t^k (1 - vn_t^k) + W_t^C (1 - \lambda_t^k) n_t^k - d_t n_t^k - (1 - s_t^k) e_t \lambda_t^k n_t^k. \tag{7}$$

Since the budget-feasible set is non-convex due to the product of choice variables $\lambda_t^k n_t^k$, we need the following restriction as in Ehrlich and Lui (1991):

Assumption 1 $1 > \alpha > \eta > 0$.

This restriction means that the number of children is more important than the quality of children in parental preference. A parent maximizes utility in (6) by choosing the number of children $n_t^k \in [0, 1/v]$ and school enrollment ratio $\lambda_t^k \in [0, 1]$ subject to (7). Given education costs, if parents cannot afford education for all children, then enrolling a fraction of them in school may be more favorable than enrolling none. This partial enrollment ratio is consistent with the evidence in Parish and Willis (1993) and in Brazil and Indonesia. From the partial school enrollment ratio, inequality arises not only among children across families as in the literature (e.g., Córdoba et al. 2016) but also among identical siblings.

Overall, the assumption of enrolling a fraction of children in school is based on the low enrollment ratios and high illiteracy rates in 1960–1970 in the data. In contrast, the conventional assumption of equal investment in all children implies either full or zero illiteracy rates. In this model high illiteracy means persistent inequality and poverty for illiterate parents due to low school enrollment ratios, thus motivating relatively high education subsidy rates for low income families in developing countries as noted in Morley and Coady (2003).

The first-order condition with respect to the number of children is

$$\begin{aligned}\frac{\partial U_t^k}{\partial n_t^k} =& -(c_t^k)^{-\sigma}[(1-\tau_t^k)W_t^k v + d_t + (1-s_t^k)e_t\lambda_t^k - W_t^C(1-\lambda_t^k)] \\ & +\alpha\beta(n_t^k)^{\alpha(1-\gamma)-1}(\bar{h}_{t+1}^k)^{\eta(1-\gamma)} = 0 \ \text{ for } n_t^k \in (0, 1/v); \\ & \frac{\partial U_t^k}{\partial n_t^k} < 0 \text{ for } n_t^k = 0; \ \frac{\partial U_t^k}{\partial n_t^k} > 0 \text{ for } n_t^k = \frac{1}{v}. \end{aligned} \tag{8}$$

The first term in the net marginal benefit of having a child is the forgone marginal utility through the time cost $(1-\tau_t^k)W_t^k v$, living cost d_t, and education cost $(1-s_t^k)e_t\lambda_t^k$, net of child earnings $W_t^C(1-\lambda_t^k)$. The second term is the marginal utility gained from having an additional child. If child earnings are no less than the time and living costs, $W_t^C \geq (1-\tau_t^k)W_t^k v + d_t$, then the net marginal benefit of having a child $\partial U_t^k/\partial n_t^k$ will be positive at $\lambda_t^k = 0$. This implies that parents may choose maximum fertility $n_t^k = 1/v$ and no school enrollment for children. Since $W_t^H > W_t^L$, it is possible that illiterate parents choose maximum fertility while literate parents choose lower fertility and enroll a fraction of children in school.

The first-order condition with respect to the school enrollment ratio is

$$\begin{aligned}\frac{\partial U_t^k}{\partial \lambda_t^k} =& -(c_t^k)^{-\sigma} n_t^k[(1-s_t^k)e_t + W_t^C] \\ & +\beta\eta(h_{t+1}^H - h_{t+1}^L)(n_t^k)^{\alpha(1-\gamma)}(\bar{h}_{t+1}^k)^{\eta(1-\gamma)-1} = 0 \ \text{ for } \lambda_t^k \in (0, 1); \\ & \frac{\partial U_t^k}{\partial \lambda_t^k} < 0 \ \text{ for } \lambda_t^k = 0; \ \ \frac{\partial U_t^k}{\partial \lambda_t^k} > 0 \ \text{ for } \lambda_t^k = 1. \end{aligned} \tag{9}$$

The first term in the net marginal benefit of enrolling a child in school is the forgone marginal utility of direct education cost $(1-s_t^k)e_t n_t^k$ as well as forgone child earnings $n_t^k W_t^C$. The second term is the marginal utility from increasing average human capital through education $h_{t+1}^H - h_{t+1}^L > 0$.

Let N_t^k be the number of type k ($k = H, L$) parents in period t. The government runs a balanced budget:

$$N_t^H(1-vn_t^H)W_t^H\tau_t^H + N_t^L(1-vn_t^L)W_t^L\tau_t^L = N_t^H\lambda_t^H e_t s_t^H + N_t^L\lambda_t^L e_t s_t^L + G_t, \tag{10}$$

where G_t is government spending exclusive of education subsidies.

4 Equilibrium analysis

In equilibrium, $e_t = \xi_t h_t^H$, $W_t^H = h_t^H$, $W_t^L = h_t^L$, and $W_t^C = h_t^C$ where earnings are equal to human capital. Substituting a given state $(h_t^H, h_t^L, h_t^C, d_t, \xi_t)$ and $\omega(\xi_t) = h_{t+1}^H / h_{t+1}^L$ into (8) and (9), the unique interior solution to these two equations, denoted Λ_t^k, is as follows:

$$\Lambda_t^k \equiv \frac{\eta(\omega(\xi_t)-1)[(1-\tau_t^k)h_t^k v+d_t-h_t^C]-\alpha[(1-s_t^k)\xi_t h_t^H+h_t^C]}{(\alpha-\eta)(\omega(\xi_t)-1)[(1-s_t^k)\xi_t h_t^H+h_t^C]}.$$

Since $\omega(\xi_t > 1$, the necessary condition for $\Lambda_t^k > 0$ is $(1-\tau_t^k)h_t^k v+d_t > h_t^C$. Combining it with $0 \leq \lambda_t^k \leq 1$, the equilibrium school enrollment ratio for type-k parents is

$$\lambda_t^k = \begin{cases} \min\left\{1, \max\left\{0, \Lambda_t^k\right\}\right\} & \text{if } (1-\tau_t^k)h_t^k v+d_t > h_t^C, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

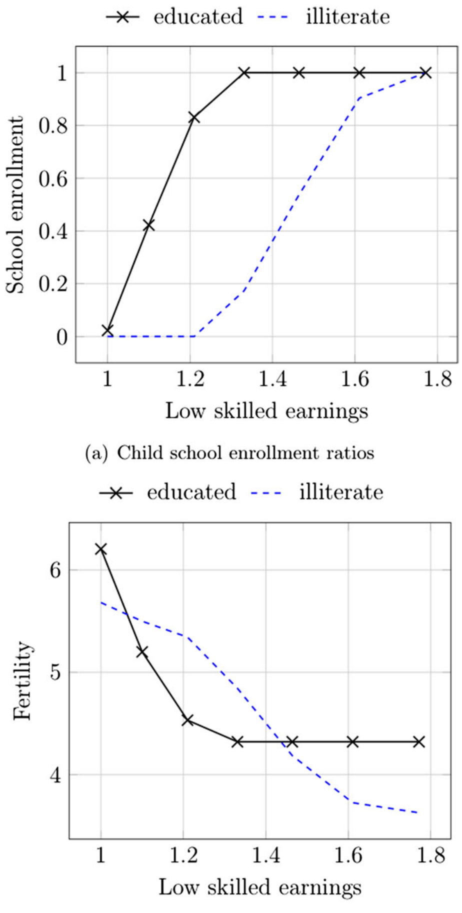
In (11), parents enroll a fraction of children in school only if forgone parental earnings for child rearing plus the living cost per child are greater than child earnings, $(1-\tau_t^k)h_t^k v + d_t > h_t^C$. Otherwise, parents would not enroll any child in school and would choose the maximum number of children $n_t^k = 1/v$ in (8). Notably, the rate of return to education $\omega(\xi_t) - 1 = (h_{t+1}^H - h_{t+1}^L)/h_{t+1}^L = (W_{t+1}^H - W_{t+1}^L)/W_{t+1}^L$ must be high enough, relative to the opportunity costs of education $(1-s_t^k)\xi_t h_t^H + h_t^C$, to motivate parents to enroll a fraction of children in school.

Figure 3(a) illustrates the relationships between school enrollment ratios and parental earnings under the conditions for school enrollment by numerically solving (11). In doing so, we set the living cost as 7.7% of illiterate parents' earnings in every period, the education cost as 3% of educated parents' earnings in every period, and child earnings as 4% of illiterate parents' earnings in the initial period. At a first glance, we also set an equal subsidy 56% on education spending for both types of parents financed by a 20% tax on educated parents' earnings and a 3% tax on illiterate parents' earnings in all periods.[12] We will discuss unequal education subsidies in favor of children of illiterate parents and the parameter values in the calibration for the simulation in Section 5. For illustration, we increase parental earnings by 10% in each period. In Fig. 3, the enrollment ratio for children of educated parents starts low in period 0 and grows to 100% in period 4. By contrast, the enrollment ratio for children of illiterate parents remains zero until period 4, and reaches 100% in period 8. Formally, we link school enrollment ratios to parental earnings and other factors below.

Proposition 1 *If $\eta(\omega(\xi_t) - 1)[(1-\tau_t^k)h_t^k v + d_t] > \alpha(1-s_t^k)\xi_t h_t^H + [\alpha + \eta(\omega(\xi_t) - 1)]h_t^C$, then school enrollment ratios are positive, increasing with returns to education $\omega(\xi_t)$, forgone earnings $h_t^k v$ for rearing a child, and child living costs d_t, but decreasing*

[12] The individual income tax rates are 0, 7.5%, 15%, 22.5%, and 27.5% at different income brackets in Brazil (Federal Tax Revenue Authority) and 5%–30% or zero below a low income level in Indonesia (Arnold 2012). Illiterate parents are most likely subject to the low tax rates, while the shares of parents completing secondary education, who are likely subject to the top tax rates, were very low in 1960–1980 in Table 4

Fig. 3 School enrollment ratios and fertility vs parental earnings. Parameters: $\beta = 3$, $\gamma = 1$, $s^H = s^L = 0.56$, $\tau^H = 0.2$, $\tau^L = 0.03$, $H_0^L = 1$, $H^H/H^L = 1.6803$, $d = 0.077H^L$, $H^C = 0.04H_0^L$. H^L grows by 10% per period. Other parameter values are in Table 6



(a) Child school enrollment ratios

(b) Fertility

with education costs $(1 - s_t^k)\xi_t H_t^H$, *income taxes* τ_t^k, *and child earnings* h_t^C. *If the growth rates of parental earnings, child earnings, and the costs of living and education are equal, then school enrollment ratios are constant over time for stationary rates of taxes and subsidies.*

Proof See Appendix A. □

These results differ from those in the related literature with equal or no education for children. Subsidizing education increases school enrollment ratios by decreasing direct education costs, thereby raising upward intergenerational mobility for all families. If tax and subsidy rates are equal, then school enrollment ratios increase with parental education or earnings because $v(h_t^H - h_t^L) > 0$. Thus, literate parents have higher school enrollment ratios than illiterate parents for $\lambda_t^H \in (0, 1)$, due to the difference in their forgone earnings of rearing a child. The positive relationship between school enrollment ratios and maternal education is observed in Fig. 2.[13]

The positive impact of rising child living costs on school enrollment ratios differs from those in the literature such as Becker et al. (1990). The intuition will become clear when we examine the impacts of child living costs on fertility. When child living costs increase proportionately with income, economies with higher income may not have higher school enrollment ratios unless they have higher education subsidies or lower child earnings than others. Unlike education subsidies, child earnings decrease enrollment ratios by increasing the opportunity cost of schooling. When child earnings increase proportionately with parental earnings, their opposite effects on enrollment ratios cancel out. When child earnings remain constant as in Fig. 3, the increase in parental earnings drives up enrollment ratios.

Equations (7), (8) and (11) yield an explicit solution for fertility in a special case with logarithmic utility $\sigma = \gamma = 1$:

$$n_t^k = \begin{cases} \frac{1}{v} & \text{if } (1-\tau_t^k)h_t^k v + d_t \le h_t^C; \\ & \text{else if } \sigma = \gamma = 1: \\ \frac{\alpha\beta(1-\tau_t^k)h_t^k}{(1+\alpha\beta)[(1-\tau_t^k)h_t^k v + d_t - h_t^C]}, & \forall\ \lambda_t^k = 0; \\ \frac{(\alpha-\eta)\beta(1-\tau_t^k)h_t^k(\omega(\xi_t)-1)}{(1+\alpha\beta)\{[(1-\tau_t^k)h_t^k v + d_t - h_t^C](\omega(\xi_t)-1) - (1-s_t^k)\xi_t h_t^H - h_t^C\}}, & \forall \lambda_t^k \in (0,1); \\ \frac{\alpha\beta(1-\tau_t^k)h_t^k}{(1+\alpha\beta)[(1-\tau_t^k)h_t^k v + d_t + (1-s_t^k)\xi_t h_t^H]}, & \forall\ \lambda_t^k = 1. \end{cases} \tag{12}$$

Figure 3(b) depicts the relationships between fertility rates and parental earnings in (12) when the living cost grows proportionately with parental earnings. At low income levels, educated parents initially have higher fertility than illiterate parents. As parental earnings and the living cost grow, fertility rates decline. The decline in fertility is relatively slow at zero school enrollment for children of illiterate parents. Thus, at higher levels of parental earnings, the fertility rate of educated parents falls below the fertility rate of illiterate parents. When the enrollment ratio reaches 100% for children of educated parents, the fertility rate of educated parents becomes constant, while the fertility rate of illiterate parents continues to fall as they increase the enrollment ratio for their children. Eventually, when the enrollment ratio reaches 100% for all types of children, the fertility rate is higher again for educated parented than for illiterate parents. Formally, we link fertility to parental earnings and other factors as follows:

Proposition 2 *Under* $(1-\tau_t^k)h_t^k v + d_t > h_t^C$ *with* $\sigma = \gamma = 1$*, fertility increases with parental earnings if* $d_t > h_t^C$ *at* $\lambda_t^k = 0$*, or* $d_t > [(1-s_t^k)e_t + h_t^C\omega(\xi_t)]/(\omega(\xi_t)-1)$ *at* $\lambda \in (0,1)$*, or* $\lambda = 1$*; however, fertility decreases with parental earnings if* $d_t < [(1-s_t^k)e_t + h_t^C\omega(\xi_t)]/(\omega(\xi_t)-1)$ *at* $\lambda \in (0,1)$*. Fertility also increases with child earnings at* $\lambda \in [0,1)$*, or with education costs at* $\lambda \in (0,1)$*, but decreases with child living costs at* $\lambda \in [0,1]$*, returns to education at* $\lambda \in (0,1)$*, and education costs at*

[13] There is overwhelming evidence on this positive relationship; see Haveman and Wolfe (1995) for a review on this issue based on the US data. Using data in Australia, Arnup et al. (2022) also find evidence that economically disadvantaged children are more likely than other children to experience worse cognitive outcomes.

$\lambda = 1$. *If the growth rates of parental earnings, child earnings, and the costs of living and education are equal, then fertility is constant for stationary rates of taxes and subsidies.*

Proof See Appendix B. □

The conditions signing the effects of parental earnings on fertility are complementary to those in the literature. In the extreme case when parental education is so low that school enrollment ratios are zero ($\lambda^k = 0$), fertility increases with parental earnings if child living costs are greater than child earnings as the (Malthusian) income effect dominates the (Beckerian) substitution effect. A rise in child living costs strengthens the income effect, whereas a rise in child earnings weakens it. When these two factors offset $d_t = h_t^C$, logarithmic utility would imply that the income and substitution effects of parental earnings should cancel out.

When parental earnings are high enough such that enrollment ratios are positive and below 100%, the impact of parental earnings on fertility is positive if child living costs are high relative to education costs and child earnings, $d_t > [(1 - s_t^k)e_t + h_t^C \omega(\xi_t)]/(\omega(\xi_t) - 1)$, and negative otherwise. Thus, this model can generate a hump-shaped relationship between fertility and parental skills or education with positive investment in child education in contrast to the corner solution in Becker et al. (1990) and Vogl (2016) at low human capital or skill.[14] A rise in education costs tips the quality-quantity tradeoff in favor of the latter, whereas a rise in returns to education tips the tradeoff against the latter. Subsidizing education reduces the education cost for $\lambda^k \in (0, 1)$, thus reducing fertility.

When parental education is so high that school enrollment ratios are 100% ($\lambda^k = 1$), children do not work and thus fertility increases with parental earnings as opposed to the Beckerian theory since now the income effect dominates as in Bar et al. (2018). This N-shaped relationship between fertility and parental education or earnings extends the hump-shaped relation in previous models. When all children are enrolled in school, a rise in education costs reduces fertility and thus subsidizing education may raise fertility by reducing education costs.

In all cases, fertility decreases with child living costs, explaining why school enrollment ratios increase with child living costs. If the growth rates of parental earnings, child earnings, and the costs of living and education are equal, then their effects on fertility completely cancel out with logarithmic utility and thus fertility remains constant. In Fig. 3, due to the constant child earnings, fertility rates decline when the living and the direct education costs grow proportionately with parental earnings. This is because the constant child earnings strength the negative substitution effect of rising parental earnings on fertility.

When the rates of taxes and subsidies are equal for all parents, fertility is higher for literate parents than for illiterate parents under the conditions for a positive relationship between fertility and parental earnings. If these conditions are reversed, fertility is lower for parents with higher education attainments. The mixed results for fertility

[14] Vogl (2016) also finds a positive relationship between fertility and parental human capital with positive investment in children's education when the goods cost for children and subsistence consumption are sufficiently high relative to the endowed human capital.

differentials are consistent with the mixed observations of fertility differentials in Brazil and Indonesia.

The predictions for partial school enrollment ratios are comparable to those observed in Brazil and Indonesia in Tables 2 and 3. In these two countries, the majority of parents were illiterate or had some incomplete primary education in census years in 1960–1990, as shown in Table 4. From Table 5, fertility rates decreased with parental education in Brazil, but increased with parental education in Indonesia except for those who completed secondary education in census years in 1970–1990. The contrasting patterns can be explained by Proposition 2 with $\lambda^k \in (0, 1)$ if parental earnings were much higher in Brazil than in Indonesia relative to subsistence costs for children in these years.

This argument may be relevant as real GDP per capita adjusted by purchasing power parity in Brazil was more than three times as that in Indonesia in these years in Table 1. The low income in Indonesia means that subsistence costs were high relative to income, especially in the 1960s. When income per capita in Indonesia reached the 1960s level of Brazil after the 1990s, the majority of sampled parents in Indonesia completed primary education and fertility decreased with parental education. As a result, the fertility differential in Indonesia flipped in Table 5 when reaching the 1960 income per capita in Brazil (2,469 dollars at 2005 price).

Propositions 1 and 2 help explain the demographic transition, the change in school enrollment ratios, as well as the evidence on the flip of fertility differentials in Vogl (2016) across nations. In the early development stage when parental earnings are low relative to child living costs, illiterate parents have fewer children and lower school enrollment ratios for children than literate parents. Rising parental earnings during early development may induce higher fertility but little change in school enrollment ratios, unless governments subsidize education. As income grows, parental earnings and education costs are high relative to child living costs, and thus fertility is higher for illiterate parents than for literate parents. Now, a rise in parental earnings decreases fertility and increases school enrollment ratios.

Eventually, when most children complete all levels of education, fertility increases with parental earnings again in contrast to a downward trend to low fertility in models with equal investment in siblings. According to Myrskylä et al. (2009), developed countries with the human development index (HDI) value exceeding 0.9 have observed increases in fertility along with increases in HDI in the previous decade. These countries have very high tertiary education enrollment ratios, e.g., over 80% in Australia and over 90% in the US.

We explore the equilibrium transition for the model economy as follows. The number of type-L parents in generation $t + 1$ is determined by

$$N_{t+1}^L = N_t^L n_t^L (1 - \lambda_t^L) + N_t^H n_t^H (1 - \lambda_t^H). \tag{13}$$

Similarly, the number of type-H parents in generation $t + 1$ is determined by

$$N_{t+1}^H = N_t^L n_t^L \lambda_t^L + N_t^H n_t^H \lambda_t^H. \tag{14}$$

Let $\rho_t \equiv N_t^L/N_t^H$ be the ratio of illiterate to literate parents and $\mu_t \equiv n_t^L/n_t^H$ be the ratio of illiterate parents' fertility to literate parents' fertility. Here, ρ_t magnifies illiteracy rates to the range $(0, \infty)$ monotonically. Dividing (13) and (14) by the number of children of literate parents $N_t^H n_t^H$ gives

$$\frac{N_{t+1}^L}{N_t^H n_t^H} = \rho_t \mu_t (1 - \lambda_t^L) + (1 - \lambda_t^H), \tag{15}$$

$$\frac{N_{t+1}^H}{N_t^H n_t^H} = \rho_t \mu_t \lambda_t^L + \lambda_t^H. \tag{16}$$

Thus, the transition equation of the ratio of illiterate to literate parents is

$$\rho_{t+1} = \frac{\rho_t \mu_t (1 - \lambda_t^L) + (1 - \lambda_t^H)}{\rho_t \mu_t \lambda_t^L + \lambda_t^H}. \tag{17}$$

The evolution of this ratio has the following properties:

Proposition 3 *The ratio of illiterate to literate parents in the next period ρ_{t+1} increases with its current ratio ρ_t and the ratio of illiterate parents' fertility to literate parents' fertility μ_t but decreases with school enrollment ratios λ_t^k.*

Proof See Appendix C. □

The positive relationship between the future and current ratios of illiterate to literate parents suggests the difficulty in improving the education status of the population or reducing illiteracy rates for development. High ratios of illiterate parents' fertility to literate parents' fertility also impede the reduction in illiteracy rates. Education subsidies can improve parental education status across generations by increasing school enrollment ratios and reducing fertility.

From (13) and (14), the average fertility is determined by

$$\frac{N_{t+1}}{N_t} = \frac{n_t^H (1 + \rho_t \mu_t)}{1 + \rho_t}. \tag{18}$$

The average fertility increases with the fertility rates of all parents or the ratio of illiterate parents' fertility to literate parents' fertility; however, its relationship with the illiteracy rate depends on fertility differentials as follows:

Proposition 4 *If illiterate parents have higher fertility than literate parents $\mu_t > 1$, then average fertility increases with the ratio of illiterate to literate parents ρ_t; otherwise, average fertility decreases with ρ_t.*

Proof See Appendix D. □

This result has novel implications. When illiterate parents have higher fertility than literate parents, falling illiteracy rates will decrease average fertility. Conversely, when literate parents have higher fertility than illiterate parents, falling illiteracy rates will increase average fertility. Thus, education subsidies are particularly effective to reduce illiteracy rates and average fertility in countries with higher fertility from illiterate parents than from literate parents. As noted in Tables 1 and 5, Brazil with relatively high fertility from illiterate parents and a high ratio of public education spending to GDP attained a larger rise in school enrollment ratios and a sharper decline in average fertility than Indonesia did.

The growth rate of average output in the economy from period t to $t+1$ is

$$g_{t+1} \equiv \frac{N^H_{t+1}(1 - vn^H_{t+1})h^H_{t+1} + N^L_{t+1}(1 - vn^L_{t+1})h^L_{t+1}}{N^H_t(1 - vn^H_t)h^H_t + N^L_t(1 - vn^L_t)h^L_t}. \quad (19)$$

Given a state $(N^L_t, N^H_t, h^L_t, h^H_t, h^C_t)$, the growth rate of average output (g_{t+1}) increases if fertility rates fall over time (lower n^k_{t+1}) or if school enrollment ratios rise (higher N^H_{t+1}). The growth rate also increases with current fertility (n^k_t) since high n^k_t means low labor (hence low income) given the time cost of child rearing.

From (17) and (19), the long-run equilibrium is as follows:

Proposition 5 *For stationary* $(\xi, \tau^L, \tau^H, s^L, s^H, d_t/h^k_t, h^C_t/h^k_t)$ *and* $\sigma = \gamma = 1$, *the economy converges to a unique steady-state growth path with*

$$\rho^* = \frac{\mu(1 - \lambda^L) - \lambda^H + \sqrt{[\mu(1 - \lambda^L) - \lambda^H]^2 + 4\mu\lambda^L(1 - \lambda^H)}}{2\mu\lambda^L} \geq 0, \quad (20)$$

$$g^* = A(\psi + B\xi)^\phi \left[\frac{\omega(\rho^* n^L\lambda^L + n^H\lambda^H) + \rho^* n^L(1 - \lambda^L) + n^H(1 - \lambda^H)}{\omega(1 - vn^H) + \rho^*(1 - vn^L)}\right]. \quad (21)$$

Proof See Appendix E. □

The convergent and stable illiteracy rate creates a poverty trap, calling for rises in education subsidy rates. A permanent rise in education subsidy rates raises school enrollment ratios of all children, leading to a lower illiteracy rate and a higher growth rate of average output in the long run. However, the steady state applies to a special case with stationary education subsidy rates, stationary ratios of the living cost, education cost and child earnings to parental earnings, and logarithmic utility.

In Fig. 3, the rise in school enrollment ratios and decline in fertility rates in the process of rising parental earnings arise from constant child earnings under the assumptions of stationary education subsidy rates and stationary ratios of the living and education costs to parental earnings. Although the illustrated numerical results match the observed trends of enrollment ratios and fertility, child earnings and education subsidy rates may rise over time, education subsidy rates may favor children of poor parents, and the utility function may be more general than logarithmic. The illustrated numerical results also have limitations: The fertility rates at high enrollment ratios are still higher than the observed levels in the data; the gap in enrollment ratios for children across families is larger than observed in the early stage; the first flip of fertility differentials occurs when poor parents forgo investment in education for children as in the literature. As shown numerically in Greenwood et al. (2005), if $\gamma > \sigma = 1$ (relatively inelastic fertility), then fertility may decline when income increases. We consider this scenario and more realistic education subsidy rates and child earnings for quantitative implications next.

5 Quantitative implications

We now simulate the equilibrium path with $1/\gamma < 1$ for quantitative implications to account for the patterns of fertility rates and school enrollment ratios in the data of Brazil and Indonesia. Table 6 presents the parameter values for simulations in Cases I and II for Brazil and Indonesia, respectively. One period of the model is 20 years. In de la Croix and Doepke (2003) and Vogl (2016) with logarithmic utility from consumption and fertility, the fraction of parental time for rearing a child is set at 7.5%. Vogl (2016) also sets the living cost per child at 7.7% of median full income. Given that 80% of parents are illiterate in 1971 in Indonesia in Table 4 (57% in 1960 in Brazil), we use the time cost in their studies and the living cost per child as 7.7% of an illiterate parent's earnings in both cases.

As noted in Table 1, the ratios of public education spending to GNI were as high as 4.3% in both countries. Thus, we assume the parameter values for the education technology ($\xi = 0.03$, $A = 7.5$, $B = 2.6$, $\phi = 0.9$, and $\psi = 0.1$) such that the return to education spending is $\omega(\xi) = 1.6803$ and that education spending per child is $\xi H^H/H^L$ =5.05% of an illiterate parent's earnings. From Assumption 1, we set the relative roles of the number and average quality of children in the utility function at $\alpha = 0.6$ and $\eta = 0.4$ to permit plausible values for the enrollment ratios in equation (11) in both countries with different tax rates and education subsidies.

From the progressive income tax rates in Indonesia and Brazil in footnote 12, the respective income tax rates for educated and illiterate parents are 20% and 3% in Case II as used for Fig. 3. Since the top tax rate is lower but the bottom rate is higher in Brazil, we set the respective tax rates at 18% and 5% in Case I. As per capita GDP in Indonesia was between a quarter and one-third of that in Brazil in the 1960s in Table 1, the initial levels of human capital or earnings in Cases I and II of Table 6 reflect the gap in initial income levels in these two countries.

The remaining parameter values to account for the enrollment ratios in Fig. 2 are child earnings and education subsidies. From Fig. 2 and Table 1, the enrollment ratios of children in families with illiterate parents were lower in 1960 in Brazil despite a higher ratio of public education spending to GDP than in Indonesia, suggesting that child earnings were higher in Brazil. Thus, we set child earnings as 3.8% of an illiterate parent's earnings in Case I and 2% in Case II. Since child earnings and the costs of living and education are all proportional to parental earnings in Table 6, school enrollment ratios should remain constant over time by Proposition 1 unless education subsidy rates rise as observed in the data.

As in Fig. 3, equal subsidy rates for all families yield a much larger gap in enrollment ratios across families than the observed gap. Although the census data has no information about education subsidy rates for each type of family, education subsidy rates are higher for poor families in developing countries as noted by Morley and

Table 6 Parameterization

Parameter	Definition	Value
α	relative taste for the number of children	0.6
β^{I}	taste for children's quantity and quality in Case I	26
β^{II}	taste for children's number and quality in Case II	10.5
η	relative taste for average human capital	0.4
γ	1/elasticity of substitution for fertility	2.8
σ	1/elasticity of substitution for consumption	1
υ	time for rearing a child	0.075
ξ	portion of high human capital for child education	0.03
A	productivity of human capital	7.5
B	advantage of enrollment for education	2.6
ϕ	degree of returns to the access to education	0.9
ψ	spillover from high human capital	0.1
$\tau^{H\mathrm{I}}$	income tax on high earnings in Case I	0.18
$\tau^{H\mathrm{II}}$	income tax on high earnings in Case II	0.2
$\tau^{L\mathrm{I}}$	income tax on low earnings in Case I	0.05
$\tau^{L\mathrm{II}}$	income tax on low earnings in Case II	0.03
$s_0^{L\mathrm{I}}$	high education subsidy in Case I at $t=0$ (up by 1.6% at $t>0$)	0.82
$s_0^{H\mathrm{I}}$	low education subsidy in Case I at $t=0$ (up by 1.5% at $t>0$)	0.675
$s_0^{L\mathrm{II}}$	high subsidy in II at $t=0$ (up by 2% at $t<2$, 1.8% at $t\geq 2$)	0.35
$s_0^{H\mathrm{II}}$	low subsidy in II at $t=0$ (up by 2.3% at $t<2$, 1.5% at $t\geq 2$)	0.16

Table 6 continued

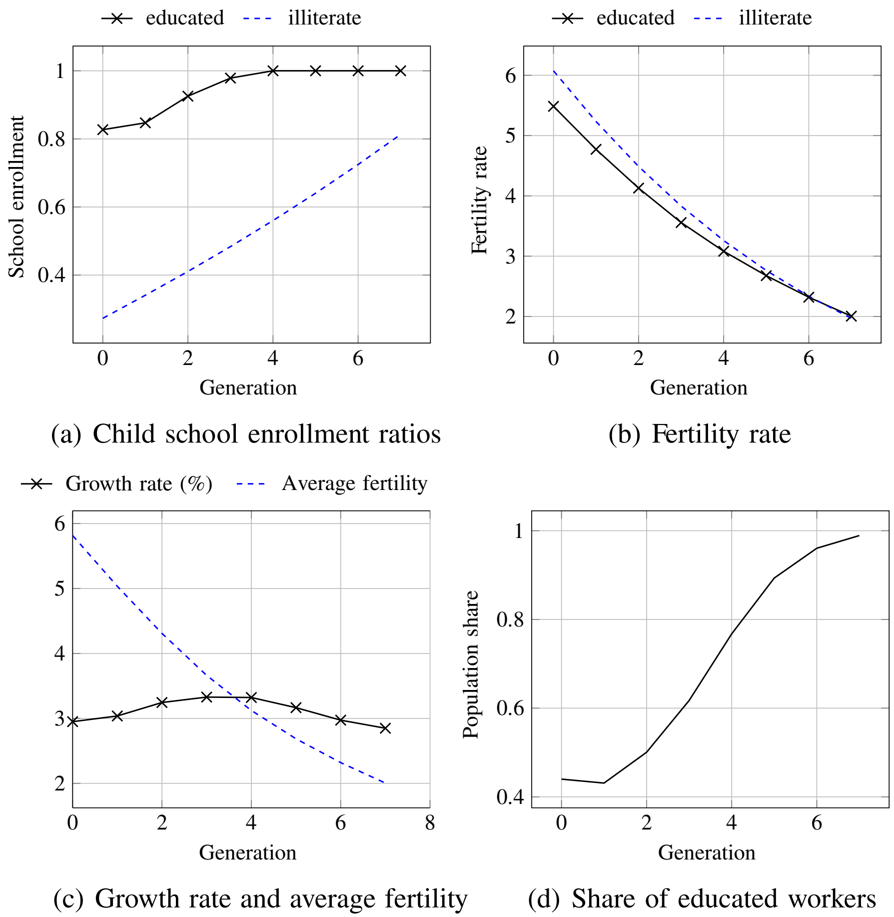
Parameter	Definition	Value
$H_0^{H\mathrm{I}}$	initial high human capital in Case I	13.962
$H_0^{L\mathrm{I}}$	initial low human capital in Case I	8.3
$H_0^{H\mathrm{II}}$	initial high human capital in Case II	3.173
$H_0^{L\mathrm{II}}$	initial low human capital in Case II	1.8884
d_t	child living cost	$0.077 H_t^L$
$H_t^{C\mathrm{I}}$	child human capital	$0.038 H_t^L$
$H_t^{C\mathrm{II}}$	child human capital	$0.02 H_t^L$
$(N_0^H/N_0)^{\mathrm{I}}$	ratio of literate to total population at $t = 0$ in Case I	44%
$(N_0^H/N_0)^{\mathrm{II}}$	ratio of literate to total population at $t = 0$ in Case II	20.5%

Coady (2003). From Table 1, public spending on education subsidies in Indonesia started at much lower levels (0.767% of GDP) than in Brazil (3.599%) in 1970–1980 but caught up (to 4.328%) in 2010. Both Brazil and Indonesia have been hit by several political or economic shocks relating to education policy or funding. Brazil passed a new federal constitution in 1988 that expanded the rights and resources in the field of education (Weller et al. 2020). In Indonesia, the government doubled the expenditures on regional development in 1973–1980, particularly for school construction or funding, following the oil boom (Duflo 2001). Yet, the lower education subsidies in Indonesia in 1970–1980 supported a higher enrollment ratio for children of illiterate parents in Fig. 2, who accounted for nearly 80% of the parent population in Table 4, than that in Brazil even though Brazil also had higher GDP per capita and lower illiteracy in 1960. The comparisons suggest a possibility that education subsidies were highly in favor of poor families in Indonesia initially and became more equal over time.

Specifically, we assume that education subsidies begin at respective rates 67.5% and 82% for children of educated parents and children of illiterate parents and grow respectively at 1.6% and 1.5% per period in Case I to account for the rises in enrollment ratios in Brazil in Fig. 2. In Case II, the respective subsidy rates begin at low rates 16% and 35%, and grow respectively at 2.3% and 2% in the first two periods (1.5% and 1.8% afterwards) to account for the rises in enrollment ratios in Indonesia.

From Table 1, the fertility rate in 1960 was much higher in Brazil than in Indonesia despite a much higher income level in Brazil, suggesting stronger taste for utility from children in Brazil. Fertility rates also declined dramatically to the replacement level in both countries in 2010, suggesting an inelastic preference for the number of children relative to consumption. The decline in fertility in data is much larger than the illustrated level in Fig. 3 that arises from the constancy of child earnings and education subsidy rates. To match the patterns of fertility rates in different families in Table 5, we set the taste for utility from children's quantity and quality at $\beta = 26$ in Case I and 10.5 in Case II, the elasticity of substitution with respect to consumption at $1/\sigma = 1$ (logarithmic utility), and the elasticity of substitution with respect to fertility at $1/\gamma = 1/2.8$. In Greenwood et al. (2005), the elasticity of substitution with respect to consumption is $1/\sigma = 1$ and the elasticity of substitution with respect to fertility is $1/\gamma = 1/1.2$ that reduces fertility over time when income increases. The reason for the lower elasticity of substitution for fertility than in their model is rising child earnings in this model that induce parents to have high fertility. If child earnings increase at a lower rate than parental earnings, then fertility can fall as in Fig. 3 and thus the elasticity of substitution for fertility can be higher than 1/2.8 to match the observed decline in fertility. However, the census data has no information about child earnings.

Figures 4 and 5 present simulation results in Cases I and II in 8 periods based on parameter values in Table 6. Each figure has four panels. Figures 4(a) and 5(a) show that educated parents always have higher school enrollment ratios for children than illiterate parents. Both enrollment ratios increase over time until reaching 100% due to rising education subsidy rates when child earnings and the costs of living and education grow proportionately with parental earnings. From the higher education subsidy rates for children of illiterate parents, enrollment ratios are positive for all children in all



(a) Child school enrollment ratios

(b) Fertility rate

(c) Growth rate and average fertility

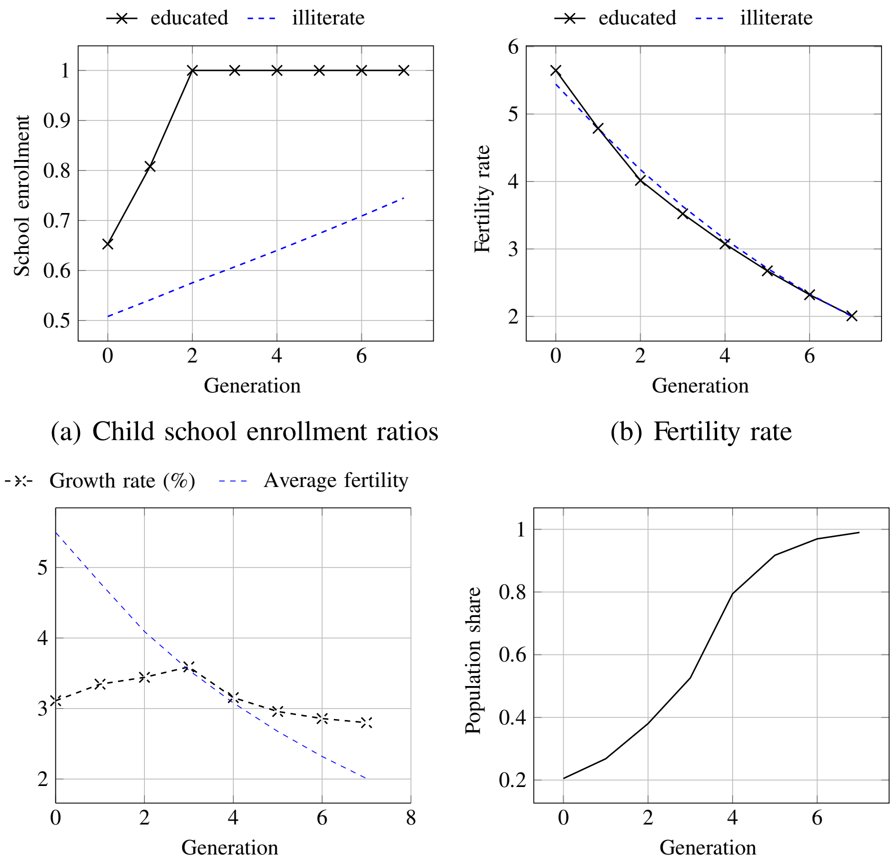
(d) Share of educated workers

Fig. 4 Fertility, school enrollment and economic growth I

periods as in the data and the gap in the enrollment ratios across families is much smaller than in Fig. 3 and closer to the observed level.

In Figs. 4(b) and 5(b), the simulated fertility rates have a downward trend due to rising parental income, rising subsidy rates, and inelastic preference for fertility $1/\gamma < 1$. In Fig. 4, the fertility rate is lower for educated than for illiterate parents in all periods as in Brazil. In Fig. 5, the fertility rate of educated parents starts higher but decreases more rapidly than that of illiterate parents as in Indonesia. Consequently, the fertility differential flips to a lower fertility rate of educated parents than that of illiterate parents in the first three periods as observed in Indonesian. The difference between the two figures are due to higher earnings and education subsidies in Fig. 4 than in Fig. 5.

The flip of the fertility differential in Fig. 5 arises from the difference in education subsidy rates for children in different families at low income levels under progressive taxes. On the one hand, the higher subsidy rate for children of illiterate parents induces



(a) Child school enrollment ratios (b) Fertility rate

(c) Growth rate and average fertility (d) Share of educated workers

Fig. 5 Fertility, school enrollment and economic growth II

the parents to have higher enrollment ratios and lower fertility than they would have otherwise. On the other hand, the partial catch-up rise in education subsidy rates for children of educated parents in the first two periods yields a sharper decline in the fertility of educated parents than that of illiterate parents. The high tax rate for educated parents also induces educated parents for high fertility, particularly in the initial period or early development when income is low. The flip in the fertility differential occurs when the enrollment ratios are positive for children in all types of families in contrast to Fig. 3 where illiterate parents forgo investment in children's education at the flipping point as in the aforementioned literature.

Figures 4(c) and 5(c) plot the growth rate of average output and average fertility. Average fertility declines from above five to the replacement level of two as observed in Table 1. The growth rate displays large swings due to the uneven paces of changes in school enrollment ratios and fertility rates over time and across different groups of families. By contrast, changes in growth rates are relatively small in models with equal and divisible education investment for all children within families.

In Figs. 4(d) and 5(d), the share of educated parents in parental population or the literacy rate starts from a low level, and initially declines in Case I but rises slowly in Case II due to the opposing effects of rising school enrollment ratios but rapidly falling fertility of educated parents. The opposite fertility differentials in Cases I and II are harmful for the average education attainment in Case I but conducive in Case II, while the larger increases in education subsidy rates in Case II are also conducive to the average education attainment. When the school enrollment ratios increase and fertility rates decline further for all families, the share of educated parents rises rapidly above 80%. Overall, the simulated results capture the patterns of the movements observed in Brazil and Indonesia in Table 1 and Fig. 2. In particular, the rise in education subsidy promotes development by accelerating human capital accumulation and the demographic transition.

When accounting for the stylized facts, both the endogenous dynamics and the exogenous rises in education subsidies play important roles. Education is a driving force for growth in parental earnings that, in turn, decreases fertility in this endogenous growth model with the choices of school enrollment and fertility. However, when child earnings, the cost of living, and the cost of education increase proportionately with parental earnings, enrollment ratios would remain constant for stationary education subsidy rates by Proposition 1. Thus, restricting child labor or raising the education subsidy rate is important to increase enrollment ratios and decrease illiteracy rates as in Figs. 3 and 4.

At the early development stage with low average earnings but high fertility and illiteracy rates, differentiated education subsidies in favor of children of illiterate parents are useful to induce them to enroll at least some children in school. Otherwise, illiterate parents may have little incentive for children's education because child earnings are particularly important for illiterate parents with low earnings. Sufficiently favorable education subsidies for children of illiterate families can also yield lower fertility for illiterate parents than for educated parents under progressive labor-income tax rates that reduce the opportunity cost of time for rearing children. In this case, a partial catch-up rise in the education subsidy rate for children of educated parents can enlarge the decrease in their fertility for the observed flip of the fertility differential in early development with positive school enrollment ratios in all families as in Fig. 5.

6 Conclusion

This paper explored differential fertility rates and school enrollment ratios among parents with different education attainments. In this model parents constrained by children's education and living costs choose the number of children, a fraction of children for education, and the remaining children for supplemental income. This distinct mechanism yields rich implications to account for the observed patterns.

When the opportunity costs of enrolling children in school are high relative to the forgone earnings of rearing children and living costs, parents have as many children as possible and no incentive for children's education. When income grows relative to the subsidized education costs, parents enroll a fraction of children in school. The enrollment ratio increases with the returns to education, forgone earnings of rearing children, and living costs but decreases with the subsidized education costs and child earnings. The positive relationship between enrollment ratios and parental education or earnings matches the pattern in the census data of Brazil and Indonesia.

The model also accounts for the N-shaped relation between fertility and parental earnings in the data. When children's living costs are high relative to children's education costs, fertility increases with parental earnings due to a dominant income effect and thus fertility is higher for parents with higher education attainments as in the data of Indonesia before 1990. When the living costs are low relative to the education costs, fertility decreases with parental earnings due to a dominant substitution effect and thus fertility is lower for parents with higher education attainments as observed in Indonesia after 1990 and in other countries.

When child earnings and the costs of education and living increase with parental earnings, school enrollment ratios remain constant and the illiteracy rate of the economy is convergent and stable for stationary education subsidy rates, creating a hurdle for development. An increase in education subsidy rates raises school enrollment ratios and reduces fertility, thus reducing illiteracy rates. However, the decline in illiteracy rates may raise the average fertility in the economy when illiterate parents have lower fertility than educated parents, especially under progressive income taxes and favorable education subsidies for poor families, as observed in Indonesia in 1960–1990. A partial catch-up rise in education subsidies for children of literate parents can enlarge the decrease in their fertility to flip the fertility differential. Conversely, when literate parents have lower fertility than illiterate parents, the decline in illiteracy rates reduces average fertility and an increase in education subsidy rates is particularly effective to reduce average fertility and illiteracy rates for development as in Brazil. When most children complete all education levels, fertility may relate positively with parental earnings again as observed in developed countries with high tertiary enrollment ratios.

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Declarations

Conflict of interest The authors declare no competing interests.

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