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## Relation between $\text{PaO}_2/\text{F}_1\text{O}_2$ ratio and $\text{F}_1\text{O}_2$ : a mathematical description

Received: 23 July 2006  
Accepted: 24 July 2006  
Published online: 9 August 2006  
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### Introduction

The acute respiratory distress syndrome (ARDS) is characterized by severe hypoxemia, a cornerstone element in its definition. Numerous indices have been used to describe this hypoxemia, such as the arterial to alveolar  $\text{O}_2$  difference, the intrapulmonary shunt fraction, the oxygen index and the  $\text{PaO}_2/\text{F}_1\text{O}_2$  ratio. Of these different indices the  $\text{PaO}_2/\text{F}_1\text{O}_2$  ratio has been adopted for routine use because of its simplicity. This ratio is included in most ARDS definitions, such as the Lung Injury Score [1] and in the American–European Consensus Conference Definition [2]. Ferguson et al. recently proposed a new definition including static respiratory system compliance and  $\text{PaO}_2/\text{F}_1\text{O}_2$  measurement with PEEP set above 10  $\text{cmH}_2\text{O}$ , but  $\text{F}_1\text{O}_2$  was still not fixed [3]. Important for this discussion, the  $\text{PaO}_2/\text{F}_1\text{O}_2$  ratio is influenced not only by ventilator settings and PEEP but also by  $\text{F}_1\text{O}_2$ . First, changes in  $\text{F}_1\text{O}_2$  influence the intrapulmonary shunt fraction, which equals the true shunt plus ventilation–perfusion mismatching. At  $\text{F}_1\text{O}_2$  1.0, the effects of ventilation–perfusion mismatch are eliminated and true intrapulmonary shunt is measured. Thus, the estimated shunt fraction may decrease as  $\text{F}_1\text{O}_2$  increases if V/Q mismatch is a major component in inducing hypoxemia

(e.g., chronic obstructive lung disease and asthma). Second, at an  $\text{F}_1\text{O}_2$  of 1.0 absorption atelectasis may occur, increasing true shunt [4]. Thus, at high  $\text{F}_1\text{O}_2$  levels ( $> 0.6$ ) true shunt may progressively increase but be reversible by recruitment maneuvers. Third, because of the complex mathematical relationship between the oxy-hemoglobin dissociation curve, the arterio-venous  $\text{O}_2$  difference, the  $\text{PaCO}_2$  level and the hemoglobin level, the relation between  $\text{PaO}_2/\text{F}_1\text{O}_2$  ratio and  $\text{F}_1\text{O}_2$  is neither constant nor linear, even when shunt remains constant.

Gowda et al. [5] tried to determine the usefulness of indices of hypoxemia in ARDS patients. Using the 50-compartment model of ventilation–perfusion inhomogeneity plus true shunt and dead space, they varied the  $\text{F}_1\text{O}_2$  between 0.21 and 1.0. Five indices of  $\text{O}_2$  exchange efficiency were calculated ( $\text{PaO}_2/\text{F}_1\text{O}_2$ , venous admixture,  $\text{P(A-a)}\text{O}_2$ ,  $\text{PaO}_2/\text{alveolar PO}_2$ , and the respiratory index). They described a curvilinear shape of the curve for  $\text{PaO}_2/\text{F}_1\text{O}_2$  ratio as a function of  $\text{F}_1\text{O}_2$ , but  $\text{PaO}_2/\text{F}_1\text{O}_2$  ratio exhibited the most stability at  $\text{F}_1\text{O}_2$  values  $\geq 0.5$  and  $\text{PaO}_2$  values  $\leq 100$  mmHg, and the authors concluded that  $\text{PaO}_2/\text{F}_1\text{O}_2$  ratio was probably a useful estimation of the degree of gas exchange abnormality under usual clinical conditions. Whiteley et al. also described identical relation with other mathematical models [6, 7].

This nonlinear relation between  $\text{PaO}_2/\text{F}_1\text{O}_2$  and  $\text{F}_1\text{O}_2$ , however, underlines the limitations describing the intensity of hypoxemia using  $\text{PaO}_2/\text{F}_1\text{O}_2$ , and is thus of major importance for the clinician. The objective of this note is to describe the relation between  $\text{PaO}_2/\text{F}_1\text{O}_2$  and  $\text{F}_1\text{O}_2$  with a simple model, using the classic Berggren shunt equation and related calculation, and briefly illustrate the clinical consequences.

### Berggren shunt equation (Equation 1)

The Berggren equation [8] is used to calculate the magnitude of intrapulmonary shunt ( $S$ ), “comparing” the theoretical  $\text{O}_2$  content of an “ideal” capillary with the actual arterial  $\text{O}_2$  content and taking into account what comes into the lung capillary, i.e., the mixed venous content.  $\text{Cc}'\text{O}_2$  is the capillary  $\text{O}_2$  content in the ideal capillary,  $\text{CaO}_2$  is the arterial  $\text{O}_2$  content, and  $\text{C}\bar{\text{v}}\text{O}_2$  is the mixed venous  $\text{O}_2$  content,

$$S = \frac{\dot{Q}_s}{\dot{Q}_t} = \frac{(\text{Cc}'\text{O}_2 - \text{CaO}_2)}{(\text{Cc}'\text{O}_2 - \text{C}\bar{\text{v}}\text{O}_2)}$$

This equation can be written incorporating the arterio-venous difference (AVD) as:

$$\text{Cc}'\text{O}_2 - \text{CaO}_2 = \left( \frac{S}{1 - S} \right) \times \text{AVD}.$$

Blood  $\text{O}_2$  contents are calculated from  $\text{PO}_2$  and hemoglobin concentrations as:

### Equation of oxygen content (Equation 2)

$$\text{CO}_2 = (\text{Hb} \times \text{SO}_2 \times 1.34) + (\text{PO}_2 \times 0.0031)$$

The formula takes into account the two forms of oxygen carried in the blood, both that dissolved in the plasma and that bound to hemoglobin. Dissolved  $\text{O}_2$  follows Henry's law – the amount of  $\text{O}_2$  dissolved is proportional to its partial pressure. For each mmHg of  $\text{PO}_2$  there is 0.003 ml  $\text{O}_2/\text{dl}$  dissolved in each 100 ml of blood.  $\text{O}_2$  binding to hemoglobin is a function of the hemoglobin-carrying capacity that can vary with hemoglobinopathies and with fetal hemoglobin. In normal adults, however, each gram of hemoglobin can carry 1.34 ml of  $\text{O}_2$ . Deriving blood  $\text{O}_2$  content allows calculation of both  $\text{Cc}'\text{O}_2$  and  $\text{CaO}_2$  and allows Eq. 1 to be rewritten as follows:

$$\begin{aligned} & [(\text{Hb} \times \text{Sc}'\text{O}_2 \times 1.34) + (\text{Pc}'\text{O}_2 \times 0.0031)] \\ & - [(\text{Hb} \times \text{SaO}_2 \times 1.34) + (\text{PaO}_2 \times 0.0031)] \\ & = \left( \frac{S}{1 - S} \right) \times \text{AVD} \end{aligned}$$

In the ideal capillary ( $c'$ ), the saturation is 1.0 and the  $\text{Pc}'\text{O}_2$  is derived from the alveolar gas equation:

$$\text{Pc}'\text{O}_2 = \text{PAO}_2 = (\text{P}_B - 47) \times \text{F}_1\text{O}_2 - \frac{\text{PaCO}_2}{R}.$$

This equation describes the alveolar partial pressure of  $\text{O}_2$  ( $\text{PAO}_2$ ) as a function, on the one hand, of barometric pressure ( $\text{P}_B$ ), from which is subtracted the water vapor pressure at full saturation of 47 mmHg, and  $\text{F}_1\text{O}_2$ , to get the inspired  $\text{O}_2$  fraction reaching the alveoli, and on the other hand of  $\text{PaCO}_2$  and the respiratory quotient ( $R$ ) indicating the alveolar partial pressure of  $\text{PCO}_2$ . Saturation,  $\text{Sc}'\text{O}_2$  and  $\text{SaO}_2$  are bound with  $\text{O}_2$  partial pressure ( $\text{PO}_2$ )  $\text{Pc}'\text{O}_2$  and  $\text{PaO}_2$ , by the oxy-hemoglobin dissociation curve, respectively. The oxy-hemoglobin dissociation curve describes the relationship of the percentage of hemoglobin saturation to the blood  $\text{PO}_2$ . This relationship is sigmoid in shape and relates to the nonlinear relation between hemoglobin saturation and its conformational changes with  $\text{PO}_2$ . A simple, accurate equation for human blood  $\text{O}_2$  dissociation computations was proposed by Severinghaus et al. [9]:

### Blood $\text{O}_2$ dissociation curve equation (Equation 4)

$$\text{SO}_2 = \left( \left( \left( \text{PO}_2^3 + 150\text{PO}_2 \right)^{-1} \times 23\,400 \right) + 1 \right)^{-1}$$

This equation can be introduced in Eq. 1:

$$\begin{aligned} & \left[ \left( \text{Hb} \times \left( \left( \left( \left( (\text{P}_B - 47) \times \text{F}_1\text{O}_2 - \frac{\text{PaCO}_2}{R} \right)^3 \right. \right. \right. \right. \right. \right. \right. \\ & + 150 \left( (\text{P}_B - 47) \times \text{F}_1\text{O}_2 - \frac{\text{PaCO}_2}{R} \right) \right)^{-1} \\ & \times 23\,400 \left. \right) + 1 \right)^{-1} \times 1.34 \left. \right) + \left( (\text{P}_B - 47) \right. \\ & \left. \times \text{F}_1\text{O}_2 - \frac{\text{PaCO}_2}{R} \right) \times 0.0031 \left. \right] \\ & - \left[ \left( \text{Hb} \times \left( \left( \left( \left( \text{PaO}_2^3 + 150\text{PaO}_2 \right)^{-1} \times 23\,400 \right) \right. \right. \right. \right. \right. \right. \right. \\ & + 1 \left. \right)^{-1} \times 1.34 \left. \right) + (\text{PaO}_2 \times 0.0031) \left. \right] \\ & = \left( \frac{S}{1 - S} \right) \times \text{AVD} \end{aligned}$$

Equation 1 modified gives a relation between  $\text{F}_1\text{O}_2$  and  $\text{PaO}_2$  with six fixed parameters:  $\text{Hb}$ ,  $\text{PaCO}_2$ , the respiratory quotient  $R$ , the barometric pressure ( $\text{P}_B$ ),  $S$  and  $\text{AVD}$ . The resolution of this equation was performed here with Mathcad® software, (Mathsoft Engineering & Education, Cambridge, MA, USA).

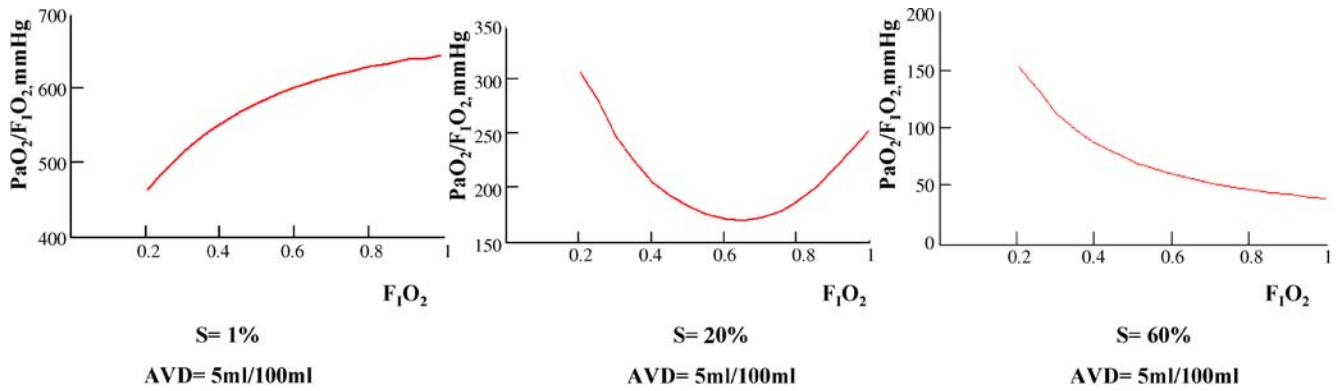


Fig. 1 Relation between  $\text{PaO}_2/\text{FIO}_2$  and  $\text{FIO}_2$  for a constant arterio-venous difference (AVD) and different shunt levels ( $S$ )

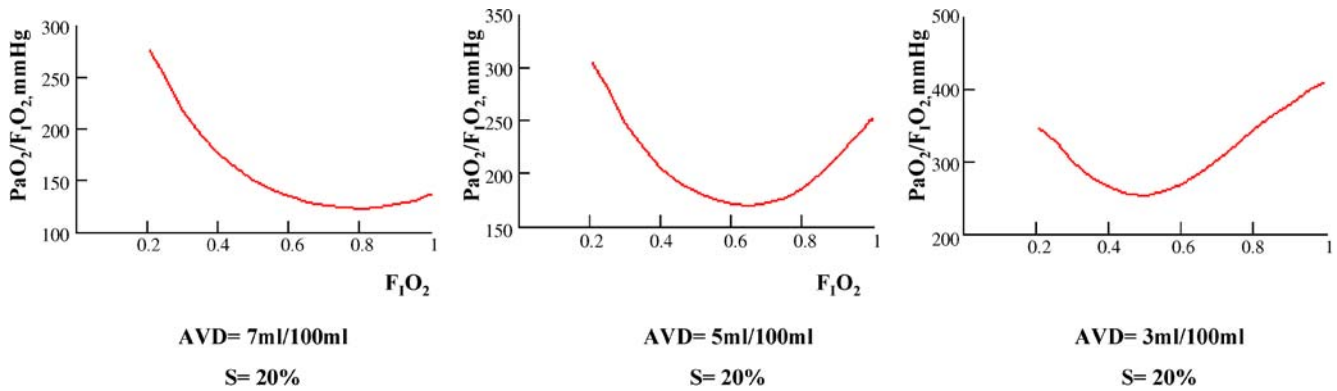


Fig. 2 Relation between  $\text{PaO}_2/\text{FIO}_2$  and  $\text{FIO}_2$  for a constant shunt ( $S$ ) level and different values of arterio-venous differences (AVD)

## Resolution of the equation

The equation results in a nonlinear relation between  $\text{FIO}_2$  and  $\text{PaO}_2/\text{FIO}_2$  ratio. As previously mentioned, numerous factors, notably nonpulmonary factors, influence this curve: intrapulmonary shunt, AVD,  $\text{PaCO}_2$ , respiratory quotient and hemoglobin. The relationship between  $\text{PaO}_2/\text{FIO}_2$  and  $\text{FIO}_2$  is illustrated in two situations. Figure 1 shows this relationship for different shunt fractions and a fixed AVD. For instance, in patients with 20% shunt (a frequent value observed in ARDS), the  $\text{PaO}_2/\text{FIO}_2$  ratio varies considerably with changes in  $\text{FIO}_2$ . At both extremes of  $\text{FIO}_2$ , the  $\text{PaO}_2/\text{FIO}_2$  is substantially greater than at intermediate  $\text{FIO}_2$ . In contrast, at extremely high shunt ( $\cong 60\%$ )  $\text{PaO}_2/\text{FIO}_2$  ratio is greater at low  $\text{FIO}_2$  and decreases at intermediate  $\text{FIO}_2$ , but does not exhibit any further increase as inspired  $\text{FIO}_2$  continue to increase, for instance above 0.7. Figure 2 shows the same relation but with various AVDs at a fixed shunt fraction. The larger is AVD, the lower is the  $\text{PaO}_2/\text{FIO}_2$  ratio for a given  $\text{FIO}_2$ . AVD can vary substantially with cardiac output or with oxygen consumption.

These computations therefore illustrate substantial variation in the  $\text{PaO}_2/\text{FIO}_2$  index as  $\text{FIO}_2$  is modified

under conditions of constant metabolism and ventilation-perfusion abnormality.

## Consequences

This discussion and mathematical development is based on a mono-compartmental lung model and does not take into account dynamic phenomena, particularly when high  $\text{FIO}_2$  results in denitrogenation atelectasis. Despite this limitation, large nonlinear variation and important morphologic differences of  $\text{PaO}_2/\text{FIO}_2$  ratio curves vary markedly with intrapulmonary shunt fraction and AVD variation. Thus, not taking into account the variable relation between  $\text{FIO}_2$  and the  $\text{PaO}_2/\text{FIO}_2$  ratio could introduce serious errors in the diagnosis or monitoring of patients with hypoxemia on mechanical ventilation.

Recently, the accuracy of the American-European consensus ARDS definition was found to be only moderate when compared with the autopsy findings of diffuse alveolar damage in a series of 382 patients [10]. The problem discussed here with  $\text{FIO}_2$  may to some extent participate in these discrepancies. A study by Ferguson et al. [11] illustrated the clinical relevance of this dis-

cussion. They sampled arterial blood gases immediately after initiation of mechanical ventilation and 30 min after resetting the ventilator in 41 patients who had early ARDS based on the most standard definition [2]. The changes in ventilator settings chiefly consisted of increasing  $F_{I}O_2$  to 1.0. In 17 patients (41%), the hypoxemia criterion for ARDS persisted after this change ( $PaO_2/F_{I}O_2 < 200$  mmHg), while in the other 24 patients (58.5%) the  $PaO_2/F_{I}O_2$  had become greater than 200 mmHg after changing the  $F_{I}O_2$ , essentially “curing” them of their ARDS in a few minutes. Of note, outcome varied greatly between the “persistent” and “transient” ARDS groups. There was a large difference in mortality, and duration of ventilation, favoring the “transient” ARDS group. Thus, varying  $F_{I}O_2$  will alter the  $PaO_2/F_{I}O_2$  ratio in patients with true and relative intrapulmonary shunt of  $\geq 20\%$ . In clinical practice, when dealing with patients with such shunt levels, one should know that the increasing  $PO_2/F_{I}O_2$  with  $F_{I}O_2$  occurs only after  $F_{I}O_2$  increase to  $> 0.6$  (depending on the AVD value). Thus, the use of the  $PO_2/F_{I}O_2$  ratio as a dynamic variable should be used with caution if  $F_{I}O_2$ , as well as other ventilatory settings, varies greatly.

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