ERRATUM TO EQUIVARIANT CHOW CLASSES OF MATRIX VARIETIES

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Abstract. We correct two formulae in Section 5 of [BF17] in which, as published, the wrong lift of a class on a Grassmannian to the space of matrices was chosen.

We maintain the notation of [BF17]. In Section 5 of *loc. cit.* we consider a generic G orbit closure in $\mathbf{A}^{r \times n}$. Theorem 5.1 presents two formulas: The first formula is for the T -equivariant Chow class of a generic T -orbit closure in the Grassmannian, and the second formula is for the G -equivariant Chow class of a generic G orbit closure in $\mathbf{A}^{r \times n}$. The second formula of Theorem 5.1 is incorrect.

As noted by Lee–Patel–Spink–Tseng [LPST20], the proof of the second claim in Theorem 5.1 shows the weaker assertion that the formula in question is a *lift* of the class $[Y_{\pi(v)}]_T$ to $A_G^*(\mathbf{A}^{r \times n})$. However this lift may not, and generally does not, satisfy the degree bound of Theorem 3.5.

A formula for the G -equivariant Chow class of a G -orbit closure in $\mathbf{A}^{r \times n}$ is given by Lee–Patel–Spink–Tseng in [LPST20, Thm. 1]. Their result gives a rational function which simplifies to the Chow class of X_v for any matrix v . The Schur expansion, or any otherwise enlightening geometric expansion, of this rational function is unavailable at this time.

It follows that the formula for $[X_v]_{\mathrm{GL}_r(\mathbf{C})}$ in Section 5 is incorrect, as is the formula for the degree in Corollary 5.4. These two results can be corrected as follows.

Proposition 0.1. *Let $v \in \mathbf{A}^{r \times n}$ have a uniform rank r matroid. Then,*

$$[X_v]_{\mathrm{GL}_r(\mathbf{C})} = \sum_{\lambda \subset (n-r-1)^{(r-1)}} \sum_{\nu: \nu_1 \leq n-r} c_{\lambda\bar{\lambda}}^\nu s_\nu(u).$$

The degree of X_v is

$$\deg(X_v) = \sum_{\lambda \subset (n-r-1)^{(r-1)}} \sum_{\nu: \nu_1 \leq n-r} c_{\lambda\bar{\lambda}}^{\nu} s_{\nu}(1^r).$$

Proof. As noted above, the incorrect formula in Theorem 5.1 for $[X_v]_G$ does yield a lift of the class $[Y_{\pi(v)}] \in A_T^*(G(r, n))$. Upon setting all the t -variables to 0 this lift becomes

$$\sum_{\lambda \subset (n-r-1)^{(r-1)}} s_{\lambda}(u) s_{\bar{\lambda}}(u) = \sum_{\lambda \subset (n-r-1)^{(r-1)}} \sum_{\nu} c_{\lambda\bar{\lambda}}^{\nu} s_{\nu}(u),$$

which is now a lift of $[Y_{\pi(v)}] \in A^*(G(r, n))$ to $A_{\mathrm{GL}_r(\mathbf{C})}^*(\mathbf{A}^{r \times n})$. The kernel of the map

$$A_{\mathrm{GL}_r(\mathbf{C})}^*(\mathbf{A}^{r \times n}) \rightarrow A^*(G(r, n))$$

is freely generated as a $\mathbf{Z}$ -module by those Schur polynomial $s_{\lambda}(u)$ with $\lambda_1 > n-r$. The results follow by subtracting off those terms which lay in the kernel. $\square$

We remark that this simple correction does not work for the G -equivariant Chow ring. This is because the kernel of $A_G^*(\mathbf{A}^{r \times n}) \rightarrow A_T^*(G(r, n))$ is generated as a $\mathbf{Z}[t]$ -module by the double Schur polynomials.

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References

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