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ERRATUM TO "STRATIFIED HYPERKÄHLER SPACES FROM SEMISIMPLE LIE ALGEBRAS"

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Abstract. This erratum concerns the Kähler potential F defined in the second displayed equation of [M, Sect. 4.2], which is incorrectly claimed to be proper.

This erratum concerns the Kähler potential F defined in the second displayed equation of Section 4.2, which is incorrectly claimed to be proper (we have, e.g., $F^{-1}(0) \cong K \times \mathfrak{k}$). Fortunately, we can replace F by another Kähler potential which is proper (inducing the same Kähler structure), and the proof is only a minor modification of the original incorrect one. The main results of the paper are unaffected. In particular, Proposition 4.6, which is the heart of Section 4.2, remains true, and everything outside Section 4.2 stays unchanged.

The exact mistake lies in the proof of Lemma 4.5. On the second line of the second paragraph, the minimum M is in fact 0, although it is claimed to be positive.

To circumvent this problem, we define F instead by

$$F(k, X) := \frac{1}{4}\int_0^1 \Big(2\|T_1^X(t)\|^2 + \|T_2^X(t)\|^2 + \|T_3^X(t)\|^2\Big)\,dt. \tag{1}$$

This is the average $F = \frac{1}{2}(F' + F'')$ of the two Kähler potentials F' and F'' for (ω_1, I_1) given by

$$F'(k, X) := \frac{1}{2}\int_0^1 \Big(\|T_1^X(t)\|^2 + \|T_2^X(t)\|^2\Big)\,dt,$$
$$F''(k, X) := \frac{1}{2}\int_0^1 \Big(\|T_1^X(t)\|^2 + \|T_3^X(t)\|^2\Big)\,dt.$$

The first one is the original Kähler potential used in the paper (modulo the $\frac{1}{2}$ factor which was forgotten) and the second one is also a Kähler potential for the same Kähler structure (ω_1, I_1) by a similar argument. Indeed, F'' is a moment

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map with respect to ω_2 for the U(1)-action fixing I_2 while rotating I_1 and I_3 [DS1, §3] so, by [HKLR, §3(E)], it is a Kähler potential for both (ω_1, I_1) and (ω_3, I_3).

To show that the new Kähler potential (1) is proper, we replace f_i in Lemma 4.5 by the map $f : W \to \mathbb{R}$ defined by $f(X) := \frac{1}{4}\int_0^1 \big(2\|T_1^X(t)\|^2 + \|T_2^X(t)\|^2 + \|T_3^X(t)\|^2\big)dt$. Then, $f(X) = 0$ if and only if $X = 0$, so the minimum M in the second paragraph of the proof is indeed positive. The rest of the proof is unaffected.

References

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