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CORRECTION TO: MOTIVIC DECOMPOSITION OF PROJECTIVE PSEUDO-HOMOGENEOUS VARIETIES

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Abstract. We point out an error in the proof of a lemma in [Sri17] and correct it by proving a stronger version of the lemma using a theorem of Pierre Deligne.

It was pointed out by Pierre Deligne that the proof of Lemma 6.2 in [Sri17] is incorrect. Although the statement of the lemma is correct, here we state a stronger version of the lemma and prove it using a result of P. Deligne [Del18]. This gives an easy proof of Corollary 6.3 in [Sri17] which we also comment on. The rest of the paper is unaffected.

First we will describe the error in the proof of Lemma 6.2. Recall that $\widetilde{X}$ is a projective pseudo-homogeneous variety for G and X is the corresponding projective homogeneous variety. The base field of these varieties, denoted by k, is assumed to be perfect. The lemma claims that for any field extension F of k, X has an F-point if and only if $\widetilde{X}$ has an F-point. The "only if" direction is clear because of the existence of canonical G-equivariant k-morphism $X \to \widetilde{X}$ obtained via the universal property of quotients(see below). The proof of the "if " direction cites Exercise 13.2.5(4) in [Spr09] and wrongly concludes that the Tits index of G over F and F' are the same where F' is the perfect closure of F. The conclusion is wrong because the exercise implies that $T_{s,F'} = T_{s,F}$ where for an inseparable extension E of F and for an F-torus T, $T_{s,E}$ denotes the unique maximal E-split subtorus of T. This does not necessarily mean that the Tits index of G over F and F' are the same, as the field extension F'/F could possibly give rise to a different F'-split torus of larger rank in G that is not necessarily contained in the torus T that we started with.

We now fix this error by replacing the lemma with a stronger version and give a proof.

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Lemma 1 (Stronger version of Lemma 6.2 in [Sri17]). *There exist k-morphisms* $f : X \to \widetilde{X}$ *and* $g : \widetilde{X} \to X$.

Proof. With notations as in the paper, recall that $X_K \simeq G/P$ and $\widetilde{X}_K \simeq G/\widetilde{P}$ where K is the algebraic closure of k (which is assumed to be perfect). Since $P \hookrightarrow \widetilde{P}$, by the universal property of quotients, there exists a unique G-equivariant morphism $\mathcal{F} : G/P \to G/\widetilde{P}$ over K. We now use the following descending argument to get a k-morphism f. The uniqueness of $\mathcal{F}$ together with the fact that the G-action on X and $\widetilde{X}$ is defined over k, implies that $\mathcal{F}$ is $\mathcal{G}_k$ invariant where $\mathcal{G}_k$ is the absolute Galois group of k. Hence the K-morphism $\mathcal{F}$ descends to a k-morphism $f : X \to \widetilde{X}$. To get a k-morphism from $\widetilde{X}$ to X we proceed as follows. Let $G^{(n)} : G \times_{\mathrm{Frob}^n} k$ denote the n-th order Frobenius twist of G, i.e., $G^{(n)}$ as a k-scheme has the same underlying topological space as G but has a k-structure twisted by the ring homomorphism $a \mapsto \sqrt[p^n]{a}$. Then $G^{(n)}$ is an algebraic group of the same type as G and the n-th order Frobenius induces a surjective k-morphism of algebraic groups

$$\mathrm{Frob}^n : G \to G^{(n)}$$

with kernel G_n yielding a k-isomorphism of algebraic groups

$$G/G_n \simeq G^{(n)}.$$

Now by [Lau93, §2], there is an embedding

$$\widetilde{P} \hookrightarrow G_m P$$

for some large enough m. Recall that by Deligne ([Del18]), there exists $n \geq m$ such that $G^{(n)}$ is k-isomorphic to G. Call this isomorphism ϕ. Then X is projective homogeneous for G where G acts via the k-morphism

$$G \to G/G_n \simeq G^{(n)} \xrightarrow{\phi} G.$$

Over K this implies that $X_K \simeq G/G_nP$. Since $n \geq m$, by the universal property of quotients we get a unique G-equivariant map

$$G/\widetilde{P} \to G/G_nP.$$

Using the descending argument as above we get k-morphism

$$g : \widetilde{X} \to X.$$

The proof of [Sri17, Cor. 6.3] can now be easily derived as follows.

Corollary 2 ([Sri17, Cor. 6.3]). *Let X and $\widetilde{X}$ be as above. Then in* $\mathrm{Chow}(k, \Lambda)$, $U_X \simeq U_{\widetilde{X}}$.

Proof. By [Kar13, Cor. 2.15], it suffices to show multiplicity one correspondences $\alpha : \mathcal{M}(X) \to \mathcal{M}(\widetilde{X})$ and $\beta : \mathcal{M}(\widetilde{X}) \to \mathcal{M}(X)$. Take α and β respectively to be the correspondence induced from k-morphisms f and g constructed in Lemma 6.2.

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