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ERRATA TO "STRUCTURE OF SOME $\mathbb{Z}$-GRADED LIE SUPERALGEBRAS OF VECTOR FIELDS"

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We shall give here the correct description of the Lie superalgebra $E(3,8)$ presented at the end of [Section 5.3, CK]. The superalgebra $E(3,8)$ has even part $E(3,8)_{\bar{0}} = W_3 + \Omega^0(3) \otimes sl(2) + d\Omega^1(3)$ where W_3 acts in the natural way on $\Omega^0(3) \otimes sl(2) + d\Omega^1(3)$ while, for $X, Y \in W_3$, $f, g \in \Omega^0(3)$, $A, B \in sl(2)$, $\omega_1, \omega_2 \in d\Omega^1(3)$, we have:

$$[X,Y] = XY - YX - \frac{1}{2} d(\operatorname{div} X) \wedge d(\operatorname{div} Y); \quad [f \otimes A, \omega_1] = 0;$$
$$[f \otimes A, g \otimes B] = fg \otimes [A,B] + \operatorname{tr}(AB) df \wedge dg; \quad [\omega_1, \omega_2] = 0.$$

The odd part of $E(3,8)$ is $E(3,8)_{\bar{1}} = \Omega^0(3)^{-\frac{1}{2}} \otimes \mathbb{C}^2 + \Omega^2(3)^{-\frac{1}{2}} \otimes \mathbb{C}^2$ where $E(3,8)_{\bar{0}}$ acts on $E(3,8)_{\bar{1}}$ as follows: for $X \in W_3$, $f \in \Omega^0(3)^{-\frac{1}{2}}$, $g \in \Omega^0(3)$, $v \in \mathbb{C}^2$, $A \in sl(2)$, $\omega \in d\Omega^1(3)$, $\sigma \in \Omega^2(3)^{-\frac{1}{2}}$,

$$[X, f \otimes v] = (X.f + \frac{1}{2} d(\operatorname{div} X) \wedge df) \otimes v;$$
$$[g \otimes A, f \otimes v] = (gf + dg \wedge df) \otimes Av; \quad [g \otimes A, \sigma \otimes v] = g\sigma \otimes Av;$$
$$[\omega, f \otimes v] = f\omega \otimes v; \quad [\omega, \sigma \otimes v] = 0.$$

Here, W_3 acts on $\Omega^2(3)$ by Lie derivative.

Let us now describe the bracket between two odd elements. We identify W_3 with $\Omega^2(3)^{-1}$ and $\Omega^3(3)^{-1}$ with $\Omega^0(3)$. In addition, we identify $\Omega^2(3)^{-\frac{1}{2}}$ with $W_3^{\frac{1}{2}}$ and we denote by X_ω the vector field corresponding to the 2-form ω under this identification.

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Let us note that if $\omega_1, \omega_2 \in \Omega^2(3)^{-\frac{1}{2}}$, then the Lie derivative $X_{\omega_1}(\omega_2)$ of ω_2 along X_{ω_1} is a nontwisted 2-form. Let $\omega_1, \omega_2 \in \Omega^2(3)^{-\frac{1}{2}}$, $f_1, f_2 \in \Omega^0(3)^{-\frac{1}{2}}$, $v_1, v_2 \in \mathbb{C}^2$. Then we have:

$$[f_1 \otimes v_1, f_2 \otimes v_2] = df_1 \wedge df_2 \otimes v_1 \wedge v_2;$$

$$[\omega_1 \otimes v_1, \omega_2 \otimes v_2] = (X_{\omega_1}(\omega_2) - (\text{div} X_{\omega_2})\,\omega_1) v_1 \wedge v_2;$$

$$[f_1 \otimes v_1, \omega_1 \otimes v_2] = f_1\omega_1 \otimes v_1 \wedge v_2 - \frac{1}{2}(f_1 d\omega_1 - \omega_1 df_1) \otimes v_1 \cdot v_2 + \frac{1}{2} df_1 \wedge d(\text{div} X_{\omega_1}) \otimes v_1 \wedge v_2.$$

By $v_1 \cdot v_2$ we denote an element in the symmetric square of $\mathbb{C}^2$, i.e., an element in $sl(2)$, and by $v_1 \wedge v_2$ an element in the skew-symmetric square of $\mathbb{C}^2$, i.e., a complex number. In particular, if E, F, H is the standard basis of $sl(2)$ and e_1, e_2 is the standard basis of $\mathbb{C}^2$ then $E = e_1^2/2$, $F = -e_2^2/2$, $H = -e_1 \cdot e_2$, and $e_1 \wedge e_2 = 1$.

We point out that under these identifications between $S^2\mathbb{C}^2$ and sl_2 and $\Lambda^2\mathbb{C}^2$ and $\mathbb{C}$, the commutator between two odd elements in the Lie superalgebra $E(3,6)$ becomes (see [Section 5.3, CK]):

$$[\omega_1 \otimes u_1, \omega_2 \otimes u_2] = (\omega_1 \wedge \omega_2) \otimes (u_1 \wedge u_2) + \frac{1}{2}(d\omega_1 \wedge \omega_2 + \omega_1 \wedge d\omega_2) \otimes u_1 \cdot u_2$$

for $\omega_1, \omega_2 \in \Omega^1(3)^{-\frac{1}{2}}$, $u_1, u_2 \in \mathbb{C}^2$.

Here are other corrections to [CK]:

p. 220: (I18) should be $(\Pi(\Lambda(2)), S(0,2) + \Lambda(2))$;
p. 220: (I19) should be $(\Pi(\Lambda(2)), S(0,2) + \mathbb{C}1 + \mathbb{C}\xi_1 + \mathbb{C}\xi_2)$;
p. 224, line 18 $\uparrow$: change the sign of the third summand to $+$;
p. 224, line 14 $\uparrow$: change the sign after E to $+$;
p. 230, line 3 $\uparrow$: should be $W(m,n)_k$;
p. 237, line 18 $\uparrow$: replace $S(n,1) + \mathfrak{a}$ by $S(n,1)$, $S(n,1) + \mathbb{C}E$;
p. 244, line 15 $\uparrow$: replace $\widehat{SHO'}(2,2)$ by $SKO'(2,3;1)$;
p. 244, line 14 $\uparrow$: replace $\widehat{SHO}(2,2)$ by $SKO(2,3;1)$;
p. 266, line 7 $\uparrow$: replace $dt^{-\frac{1}{2}}$ by $dt^{\frac{1}{2}}$ in the second summand;
p. 266, line 1 $\uparrow$: $d(fdt^{-\frac{1}{2}})gdt^{-\frac{1}{2}} - f(dt)^{-\frac{1}{2}}d(gdt^{-\frac{1}{2}})$ is interpreted as $(gf' - g'f)$;
p. 269: in the list of the irreducible gradations of $E(1,6)$, the gradations

$$(1|0,1,1,1,0,0) \quad \text{and} \quad (2|2,1,1,0,1,1)$$

were inadvertently omitted (see [S]);
p. 271: remove lines 15 $\downarrow$ and 16 $\downarrow$.

References

[CK] S.–J. Cheng, V. Kac, *Structure of some $\mathbb{Z}$-graded Lie superalgebras of vector fields*, Transform. Groups **4** (1999), 219–272.

[S] I. Shchepochkina, *The five exceptional simple Lie superalgebras of vector fields and their fourteen regradings*, Represent. Theory **3** (1999), 373–415.