Erratum to: Singular Adams inequality for biharmonic operator on Heisenberg Group and its applications

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Erratum to: Nonlinear Differ. Equ. Appl. (2016) 23:58
DOI 10.1007/s00030-016-0412-z

In [1], we have established Adams-type inequality for biharmonic operator on Heisenberg group and proved the existence of solution to a biharmonic equation involving a singular potential and a nonlinearity satisfying critical and subcritical exponential growth condition.

We observed that there is a technical mistake in the homogeneous dimension of the Heisenberg group that is under consideration. For our results to be meaningful, we need to work with bounded domains in $\mathbb{H}^1$ instead of bounded domains in $\mathbb{H}^4$. The reason of this change is as follows:

Let $\Omega \subseteq \mathbb{H}^n$ be a bounded domain and $Q = 2n + 2$ be homogeneous dimension of $\mathbb{H}^n$. When $Q > 4$ ($n > 1$), we know that $D_0^{2,2}(\Omega) \hookrightarrow L^q(\Omega)$, $1 \leq q \leq \frac{2Q}{Q-4}$. In the critical case, $Q = 4$ ($n = 1$), $D_0^{2,2}(\Omega) \not\hookrightarrow L^\infty(\Omega)$. Then it is natural to ask, what is the best possible space for this embedding? To answer this question, we need an Adams-type inequality with $Q = 4$. Thus, we need to work with $\mathbb{H}^1$ instead of $\mathbb{H}^4$ in [1]. For the sake of clarity, we restate the main results of [1]. However, all the proofs remain unchanged.

Theorem 0.1. *Let Ω be a bounded domain in $\mathbb{H}^1$, $p = \frac{Q}{2}$, $p' = \frac{Q}{Q-2}$, where $Q = 4$ is homogeneous dimension of $\mathbb{H}^1$. Then there exists a constant $C_0(\Omega)$ such that for all $u \in C_0^\infty(\Omega)$ and $\|\Delta_{\mathbb{H}^1} u\|_p \leq 1$,*

$$\int_{\Omega} \exp \left(A_Q |u(\xi)|^{p'} \right) \leq C_0 < \infty, \quad (0.1)$$

where

$$\begin{aligned} A_Q &= \frac{Q}{c_0 \gamma_1^{p'}}, \\ c_0 &= \int_{\Sigma} d\mu, \quad \Sigma = \{\xi \in \mathbb{H}^1 : |\xi| = 1\} \end{aligned} \quad (0.2)$$

and

$$\gamma_n = \left(n(n+1) \int_{\mathbb{H}^n} |z|^2 (|z|^4 + t^2 + 1)^{-\frac{n+4}{2}} d\xi \right)^{-1}. \quad (0.3)$$

Furthermore, if we choose any number greater than A_Q , then inequality fails to hold.

Theorem 0.2. *Let Ω be a bounded domain in $\mathbb{H}^1$ and $0 \leq a < Q$, where $Q = 4$ is homogeneous dimension of $\mathbb{H}^1$. Then there exists a constant $C_0(\Omega)$ such that for all $u \in C_0^\infty(\Omega)$ and $\|\Delta_{\mathbb{H}^1} u\|_{Q/2} \leq 1$,*

$$\int_{\Omega} \frac{\exp\left(A_Q \left(1 - \frac{a}{Q}\right) |u(\xi)|^{Q/(Q-2)}\right)}{\rho(\xi)^a} \leq C_0 < \infty,$$

where $\rho(\xi) = \sqrt{(|z|^4 + t^2)^{\frac{1}{4}}}$, $A_Q = \frac{Q}{c_0 \gamma_1^{p'}}$ and γ_1 and c_0 are as defined by (0.2) and (0.3), respectively. Furthermore, if we choose any number greater than $A_Q \left(1 - \frac{a}{Q}\right)$, then inequality fails to hold.

Next, we established the existence results to the following problem:

$$\begin{cases} \Delta_{\mathbb{H}^1}^2 u = \frac{f(\xi, u)}{\rho(\xi)^a} & \text{in } \Omega, \\ u|_{\partial\Omega} = 0 = \frac{\partial u}{\partial \nu} \Big|_{\partial\Omega}, \end{cases} \quad (0.4)$$

where $0 \in \Omega$ is a bounded domain in one dimensional Heisenberg group $\mathbb{H}^1$, $0 \leq a < Q$, $Q = 4$ is the homogeneous dimension of $\mathbb{H}^1$ and $f : \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ satisfies either subcritical or critical exponential growth condition. It is interesting to observe that in case of $\Omega \subseteq \mathbb{H}^n$, $n > 1$, by the Sobolev embedding theorem, the nonlinearity cannot exceed the degree $\frac{2Q}{Q-4}$, while in the case of $n = 1$, the Adams' inequality allows the nonlinearities to have exponential growth. Therefore, Adams' inequality motivates us to discuss the above problem with exponential growth in $\Omega \subseteq \mathbb{H}^1$. We assume the following conditions on the nonlinearity f :

- (H1) $f : \bar{\Omega} \times \mathbb{R} \rightarrow \mathbb{R}$ is continuous, $f(\xi, u) \geq 0$ on $\Omega \times [0, \infty)$, $f(\xi, u) \leq 0$ when $u \leq 0$.
 (H2) There exists $R > 0$, $M > 0$ such that, $\forall u \geq R$, $\forall \xi \in \Omega$

$$0 < F(\xi, u) \leq M f(\xi, u),$$

where $F(\xi, u) = \int_0^u f(\xi, s) ds$.

(H3) There exist $R > 0$, $\theta > 2$ such that $\forall |u| \geq R$, $\forall \xi \in \Omega$,

$$\theta F(\xi, u) \leq uf(\xi, u).$$

(H4) $\limsup_{u \rightarrow 0^+} \frac{2F(\xi, u)}{|u|^2} < \Lambda$.

(H5) $\lim_{u \rightarrow \infty} uf(\xi, u) \exp(-\alpha_0 |u|^2) \geq \beta_1 > \frac{(4-a)A_Q}{4\alpha_0 R^{4-a}\mathcal{M}}$, where $\mathcal{M}$ and A_Q are defined in [1, Section 2].

Theorem 0.3. *Assume that f satisfies the subcritical growth condition and (H1)–(H5) hold, then Problem (0.4) has a weak solution for $0 < a < 4$.*

Theorem 0.4. *Assume that f satisfies the critical growth condition and (H1)–(H5) hold, then Problem (0.4) has a weak solution for $0 < a < 4$.*

We say (0.4) has a critical potential case when $a = 4$. In this case there is no singular Adams-type inequality. In critical potential case, we establish the following:

Theorem 0.5. *Assume that f satisfies the subcritical growth condition and (H1)–(H5) hold, then Problem (0.4) has a weak solution for $a = 4$.*

Theorem 0.6. *Assume that f satisfies the critical growth condition and (H1)–(H5) hold, then Problem (0.4) has a weak solution for $a = 4$.*

Acknowledgements

Authors thank the referee for the constructive comments.

Reference

- [1] Dwivedi, G., Tyagi, J.: Nonlinear Differ. Equ. Appl. **23**, 58 (2016). doi:[10.1007/s00030-016-0412-z](https://doi.org/10.1007/s00030-016-0412-z)

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