On Orlicz capacities and a nonexistence result for certain elliptic PDEs

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Abstract. In this paper a new relation between a capacity of second order and an Orlicz capacity of first order is established. As a consequence, we construct a class of semilinear equations of second order, satisfying an assumption given in terms of first order capacities, which has no distributional solutions.

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1. Introduction

The foundations for the study of Orlicz capacity were established by Aïssaoui in [2] and Aïssaoui and Benkirane in [4, 5] in view of the several applications in Nonlinear Potential Theory, in Harmonic Analysis and in PDEs theory. Notwithstanding, the subject is still of interest in recent years, see e.g. [3] and [14] for a broad treatment of the topic.

In order to study the properties of the capacities, the comparison theorems turn out to be relevant (see Section 5.1 in [1]). Moreover, this kind of results are a key tool in questions related to the existence of solutions for some nonlinear elliptic equations involving measures ([8, 16]). The aim of this paper is to give a contribution in these directions. We remark that connections between Orlicz capacities and quasilinear or semilinear elliptic equations are of current interest, see e.g. [13, 15, 19].

It is well known that there is a natural ordering in the two-parametric family of (α, p) -capacities, $0 < \alpha p \leq N$, and that no two capacities in the family are equivalent, in the sense that if $C_{\beta, q}(E) \leq c C_{\alpha, p}(E)$ for arbitrary sets E , then there exist sets E such that $C_{\beta, q}(E) = 0$ but $C_{\alpha, p}(E) > 0$. This phenomenon has been analyzed in the Orlicz setting only partially ([8]). In Sect. 3 we provide a result in this sense for some first order capacities of Orlicz

type, when compared with a classical second order capacity. The interest in this question relates to the study of the nonexistence of solutions to the problem

$$\begin{cases} -\Delta u + |u|^{q-1}u = \lambda & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where Ω is a bounded open subset of $\mathbb{R}^N$, $N > 2$, $q > N/(N-2)$, and λ is a bounded Radon measure. Theorem 4.1 in [6] asserts that λ “absolutely continuous” with respect to the $(2, q')$ -capacity is a necessary and sufficient condition for the existence of a solution (for the non linear case see [7, 17]). Of course if λ is “absolutely continuous” with respect to some $(2, q' + \epsilon)$ -capacity one cannot deduce neither existence, nor nonexistence, because the “absolute continuity” with respect to the $(2, q')$ -capacity implies the one with respect to the $(2, q' + \epsilon)$ -capacity but the opposite does not hold. A similar argument is still true if one considers $(1, 2q' + \epsilon)$ -capacity instead of $(2, q' + \epsilon)$ -capacities.

In the setting of first order Orlicz $(1, A)$ -capacities, with $A(t)$ growing faster than $t^{2q'}$, these conclusions are not so immediate because sharpness results for the comparison between Orlicz capacities do not seem to be available in literature. In our Theorem 3.2 we prove a sharpness result, finding the existence of a set E such that $C_{2,q'}(E) = 0$ and a first order capacity $C_{1,A}(E) > 0$ for N -functions $A(t)$ growing “some logarithm” faster than $t^{2q'}$. These N -functions are exactly those ones for which $C_{1,A}(E) = 0 \Rightarrow C_{2,q'}(E) = 0$ holds (see Sect. 3 for details). As a consequence, we will obtain in Sect. 4 a nonexistence result. We stress that the novelty here is not in different conditions on the datum for the nonexistence of the solutions, but in the consistency of the statement, in the sense that we exhibit a concrete class of measures not “absolutely continuous” with respect to the $(2, q')$ -capacity.

2. Notation and preliminaries

We recall that for $0 < \alpha < N$ and $p > 1$, the (α, p) -capacity of a compact set $K \subset \Omega$ is defined as

$$C_{\alpha,p}(K) := \inf\{\|u\|_{W_0^{\alpha,p}(\Omega)}^p : u \in C_c^\infty(\Omega), u \geq \chi_K\}$$

where χ_K is the characteristic function of K . The (α, p) -capacity of an open set $U \subset \Omega$ is defined as

$$C_{\alpha,p}(U) := \sup\{C_{\alpha,p}(K), K \text{ compact}, K \subset U\}$$

and the (α, p) -capacity of any set $E \subset \Omega$ as

$$C_{\alpha,p}(E) := \inf\{C_{\alpha,p}(U), U \text{ open}, E \subset U\}.$$

Let $A : [0, +\infty[\rightarrow [0, +\infty[$ be a N -function i.e. a strictly increasing and convex function such that

$$\lim_{t \rightarrow 0} \frac{A(t)}{t} = 0 \qquad \lim_{t \rightarrow +\infty} \frac{A(t)}{t} = +\infty.$$

For simplicity, we shall consider only N -functions A which are differentiable.

The conjugate of the function A , denoted by $\tilde{A}$, is defined by

$$\tilde{A}(s) = \sup_{t \geq 0} \{st - A(t)\}, \quad s \geq 0. \quad (2.1)$$

It is well-known that also the function $\tilde{A}$ is a N -function.

Let Ω be an open bounded set of $\mathbb{R}^N$. The Orlicz class $L^A(\Omega)$ is defined by

$$L^A(\Omega) := \left\{ f \in L^1_{loc}(\Omega) : \int_{\Omega} A\left(\frac{|f|}{k}\right) dx < +\infty \text{ for some } k > 0 \right\}$$

and, equipped with the norm

$$\|f\|_A := \inf \left\{ k > 0 : \int_{\Omega} A\left(\frac{|f|}{k}\right) dx \leq 1 \right\},$$

becomes a Banach space. For exhaustive treatments of the Orlicz space theory we refer to [11] and [18].

For a compact subset K of Ω , the A -capacity of K with respect to Ω is defined by

$$C_{1,A}(K) := \inf \{A(\|\nabla u\|_A) : u \in C_c^\infty(\Omega), u \geq \chi_K\}$$

where χ_K is the characteristic function of K . The A -capacity of any open subset U of Ω is then defined as

$$C_{1,A}(U) := \sup \{C_{1,A}(K), K \text{ compact}, K \subset U\}$$

and finally the A -capacity of any set $E \subset \Omega$ is

$$C_{1,A}(E) := \inf \{C_{1,A}(U), U \text{ open}, E \subset U\}.$$

It is well known that if $A(t)$ grows faster than t^N , one cannot find a set E such that $C_{1,A}(E) = 0$. We will therefore assume that $A(t) \leq t^N$. On the other hand, since we will be interested in N -functions $A(t)$ growing faster than $t^{2q'}$ (where $q > 1$ is fixed), we will have the natural restriction $q > \frac{N}{N-2}$.

We shall use also the following definition given in [4]

$$C'_{1,A}(E) := \inf \{\|\nabla u\|_A : u \in C_c^\infty(\Omega), u \geq \chi_E\}.$$

Observe that $A(C'_{1,A}(E)) = C_{1,A}(E)$.

Following [1], we denote by $\mathcal{F}$ the Fourier transform of a function $f \in L^1(\mathbb{R}^N)$, that is $\mathcal{F}f(\xi) = \int_{\mathbb{R}^N} f(x)e^{-ix\xi} dx$, and by G_α the Bessel kernel

$$G_\alpha := \mathcal{F}^{-1}((1 + |\xi|^2)^{\alpha/2}) \quad \alpha > 0.$$

The function G_α is positive, radial (i.e. depends on $|x|$ only) and integrable on $\mathbb{R}^N$. Consider the space $C(\Omega)$ of the real valued continuous functions on Ω equipped with the topology of uniform convergence on compact subsets of Ω . If K is compact, $C(K)$ is usually normed with the supremum norm $\|\cdot\|_{L^\infty(K)}$. $C_c(\Omega)$ is the subset of $C(\Omega)$ consisting of functions with compact support contained in Ω . The dual of the space $C_c(\Omega)$ is denoted by $\mathcal{M}(\Omega)$, the bounded Radon measures on Ω . The subset of positive measures on Ω , is denoted by $\mathcal{M}^+(\Omega)$.

Let $\mu \in \mathcal{M}^+(\mathbb{R}^N)$. Since G_α is positive, the function

$$(G_\alpha \star \mu)(y) := \int_{\mathbb{R}^N} G_\alpha(x-y) d\mu(x) \quad y \in \mathbb{R}^N$$

is positive too and the following Wolff's inequality holds (see Theorem 4.5.2, p. 109 in [1]). Here and in the sequel the letters c, c_1, c_2 will denote constants, not necessarily the same at each occurrence.

Theorem 2.1. *Let $1 < p < \infty$, $0 < \alpha p \leq N$ and $\mu \in \mathcal{M}^+(\mathbb{R}^N)$. Then there exists a constant c such that*

$$\int_{\mathbb{R}^N} (G_\alpha \star \mu)^{p'} dx \leq c \int_{\mathbb{R}^N} W_{\alpha,p}^\mu(x) d\mu \quad (2.2)$$

where

$$W_{\alpha,p}^\mu(x) = \int_0^1 \left(\frac{\mu(B(x,t))}{t^{N-\alpha p}} \right)^{p'-1} \frac{dt}{t} \quad (2.3)$$

In the following we shall use next result which is a generalization of Theorem 2.1 in the framework of Orlicz spaces (see Theorem 4.3 in [8] and also [12]; set $A(t) = t^p$ to recover Theorem 2.1). Since we consider integrals over intervals of the type $(0, a)$ and the precise value of a is not important, we use the notation $\int_0^a$ instead of writing $\int_0^a$.

Theorem 2.2. *Let $0 < \alpha < N$ and $\mu \in \mathcal{M}^+(K)$. Then*

$$\tilde{A}(\|G_\alpha \star \mu\|_{\tilde{A}}) \leq c \Psi_{\tilde{A}} \left(\int_{\mathbb{R}^N} W_{\alpha,A}^\mu(x) d\mu \right) \quad (2.4)$$

where

$$\Psi_A(s) = \sup_{t>0} A \left(\frac{t}{A^{-1} \left(\frac{A(t)}{s} \right)} \right) \quad s > 0, \quad \Psi_A(0) = 0 \quad (2.5)$$

and

$$W_{\alpha,A}^\mu(x) = \int_0 t^\alpha \tilde{A}' \left(\frac{\mu(B(x,t))}{t^{N-\alpha}} \right) \frac{dt}{t} \quad (2.6)$$

for some constant c depending on $\tilde{A}, \alpha, N$ but independent of μ .

Let us conclude this section recalling the following useful construction.

Let $\mathcal{L} = \{l_n\}_{n=0}^{+\infty}$ be a decreasing sequence such that $l_0 = 1, 0 < 2l_{n+1} < l_n$, for $n > 0$. Let $E_0 \subset \mathbb{R}$ be a closed interval of length l_0 and let E_1 be the set obtained by removing an open interval of length $l_0 - 2l_1$ in the middle, so that E_1 consists of two closed intervals of length l_1 . Then remove an interval of length $l_1 - 2l_2$ in the middle of each of these intervals to obtain E_2 consisting of 2^2 intervals of length l_2 . Continuing this way, we obtain after n steps a set E_n consisting of 2^n intervals of length l_n . Denote the Cartesian product of N copies of E_n by $E_n^{(N)}$ and set $E_{\mathcal{L}} = \bigcap_{n=0}^{\infty} E_n^{(N)}$. The set $E_{\mathcal{L}}$ is called the Cantor set corresponding to $\mathcal{L}$.

3. The main result

We start pointing out the relation between capacities proved in [8] extending to the Orlicz setting the nonlinear potential techniques of [1]:

Theorem 3.1. *Let $s > 1$, $0 < \beta s < N$, $\beta \in \mathbb{N}$. If*

$$\int_0^1 (\tilde{A})'(t^{1-\beta s}) dt < \infty \quad (3.1)$$

then

$$C_{1,A}(E) = 0 \Rightarrow C_{\beta,s}(E) = 0$$

The sharpness of Theorem 3.1 in the case $\beta = 2$, $s = q'$ is proved in the following

Theorem 3.2. *Let $q > \frac{N}{N-2}$ and let $A = A(t)$ be a N -function such that*

$$\int_0^1 (\tilde{A})'(t^{1-2q'}) dt < \infty \quad (3.2)$$

Then there exists a set E such that $C_{2,q'}(E) = 0$ but $C_{1,A}(E) > 0$.

The proof of Theorem 3.2 lies on the construction of the Cantor set recalled at the end of Sect. 2. Note that it is a fundamental tool to prove the sharpness of the relation between different capacities also in the classical situation (see Theorem 5.5.1, p. 148 in [1]).

Proof. Set $h(t) = t^{N-2q'}$. Noting that $\frac{h(t)}{t^N}$ is strictly decreasing, we get

$$h(2t) \leq 2^N h(t). \quad (3.3)$$

Defining the sequence $\mathcal{L} = \{l_n\}_{n=0}^{+\infty}$ by

$$l_0 = 1 \quad h(l_n) = 2^{-nN} \quad n = 1, 2, \dots,$$

we easily verify that inequality (3.3) implies that $2l_{n+1} < l_n$ for every $n > 1$. It follows that we can consider the corresponding Cantor set $E_{\mathcal{L}}$. In the sequel we will show that $E = E_{\mathcal{L}}$ is the set we are looking for.

Let $\mu \in \mathcal{M}^+(E)$ be a positive measure such that

$$\mu(B(x, r)) \leq h(r) \quad \text{for small } r > 0 \quad (3.4)$$

(such μ exists by a standard construction, see e.g. the proof of Theorem 5.3.1 p. 143 in [1]).

Applying the Wolff's inequality (2.4) with $\alpha = 1$, we have

$$\tilde{A}(\|G_1 * \mu\|_{\tilde{A}}) \leq c \Psi_{\tilde{A}} \left(\int_{\mathbb{R}^N} \left(\int_0^{\frac{\mu(B(x,t))}{t^{N-1}}} \tilde{A}' \left(\frac{\mu(B(x,t))}{t^{N-1}} \right) dt \right) d\mu \right)$$

where the constant $c = c(A, N)$ does not depend on μ .

Therefore, by using (3.4), it follows that

$$\begin{aligned} \tilde{A}(\|G_1 * \mu\|_{\tilde{A}}) &\leq c \Psi_{\tilde{A}} \left(\int_{\mathbb{R}^N} \left(\int_0^{\frac{h(t)}{t^{N-1}}} \tilde{A}' \left(\frac{h(t)}{t^{N-1}} \right) dt \right) d\mu \right) \\ &= c \Psi_{\tilde{A}} \left(\int_{\mathbb{R}^N} H d\mu \right) = c \Psi_{\tilde{A}}(H\mu(E)), \end{aligned} \quad (3.5)$$

where, by (3.2),

$$0 < H = \int_0 \tilde{A}' \left(\frac{h(t)}{t^{N-1}} \right) dt = \int_0 \tilde{A}'(t^{1-2q'}) dt < \infty$$

and hence

$$\|G_1 * \mu\|_{\tilde{A}} \leq \tilde{A}^{-1}(c\Psi_{\tilde{A}}(H\mu(E))). \quad (3.6)$$

According to Theorem 11 in [4], we have

$$C'_{1,A}(E) = \sup\{\mu(E) \in \mathcal{M}^+(E) : \mu \text{ is concentrated on } E, \|G_1 * \mu\|_{\tilde{A}} \leq 1\}$$

and therefore

$$C_{1,A}(E) = A(C'_{1,A}(E)) \geq A \left(\frac{\mu(E)}{\tilde{A}^{-1}(c\Psi_{\tilde{A}}(H\mu(E)))} \right) > 0$$

In order to prove that $C_{2,q'}(E) = 0$, it will be enough to observe that

$$\int_0 \left(\frac{h(t)}{t^{N-2q'}} \right)^{q-1} \frac{dt}{t} = \int_0 \frac{dt}{t} = \infty.$$

and to apply Theorem 5.3.3 in [1]. $\square$

Example 3.3. Let $q > \frac{N}{N-2}$, $\gamma > 2q' - 1$. The N -function $A(t) = t^{2q'} \log^\gamma(e+t)$ satisfies the assumption of Theorem 3.2.

Example 3.4. Let $q > \frac{N}{N-2}$, $\gamma > 2q' - 1$. The N -function $A(t) = t^{2q'} \log^{2q'-1}(e+t) \log^\gamma \log(e+t)$ satisfies the assumption of Theorem 3.2.

Example 3.5. Let $q > \frac{N}{N-2}$, $\eta, \beta > 0$. The N -function $A(t) = t^{2q'+\eta} \log^\beta(e+t)$ satisfies the assumption of Theorem 3.2.

We show now that if the N -function $A = A(t)$ grows much faster than $t^{2q'}$, then the same conclusion of Theorem 3.2 holds with a shorter argument.

Proposition 3.6. Let $q > \frac{N}{N-2}$ and let $A = A(t)$ be a N -function such that

$$A(ct) \geq t^r \quad \text{for } t \text{ large,} \quad r > 2q'. \quad (3.7)$$

Then there exists a set E such that $C_{2,q'}(E) = 0$ but $C_{1,A}(E) > 0$.

Proof. In Theorem 5.5.1, p. 148 of [1] it is shown there exists a constant c such that the following comparison holds

$$C_{2,q'}(E)^{1/(N-2q')} \leq c C_{1,r}(E)^{1/(N-r)} \quad (3.8)$$

and that it is sharp, in the sense that there exists a set E_0 such that $C_{2,q'}(E_0) = 0$, $C_{1,r}(E_0) > 0$. Observe now that by (3.7) and the classical embedding theorem for Orlicz spaces, it is $\|\nabla u\|_r \leq c \|\nabla u\|_A$. Therefore, again by (3.7), we get

$$C_{1,r}(E_0) \leq c_1 C'_{1,A}(E_0)^r \leq c_2 A(C'_{1,A}(E_0)) = c_2 C_{1,A}(E_0)$$

and the conclusion easily follows. $\square$

In the critical situation $r = 2q'$, Lemma 2.24 in [8] proves that for $A(t) = t^{2q'} \log^{2q'-1+\varepsilon}(e+t)$, for some $\varepsilon > 0$, the following implication holds

$$C_{1,A}(E) = 0 \Rightarrow C_{2,q'}(E) = 0. \quad (3.9)$$

Note that (3.9) fails for $A(t) = t^{2q'}$ as can be deduced by inequality (Theorem 5.5.1, p. 148 of [1])

$$C_{1,2q'}(E) \leq c C_{2,q'}(E)$$

while it is still valid for $A(t) = t^{2q'+\eta} \log^\beta(e+t)$, $\eta, \beta > 0$. We observe that the class of N -functions A satisfying (3.2) is larger than the one considered in Lemma 2.24 in [8] (see Examples 3.3 and 3.4 above).

4. An application to PDEs

Consider the problem

$$\begin{cases} -\Delta u + |u|^{q-1}u = \lambda & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \quad (4.1)$$

where Ω is a bounded open subset of $\mathbb{R}^N$, $N > 2$, $q > 1$ and λ is a bounded Radon measure on Ω . A celebrated result due to Baras and Pierre (Theorem 4.1 in [6], see also [9, 10]) states that a distributional solution exists if and only if λ belongs to $L^1(\Omega) + W^{-1,q}(\Omega)$ or, equivalently, if and only if λ is “absolutely continuous” with respect to the $(2, q')$ -capacity; when it exists, the solution is unique. Thanks to the comparison results between the $(2, q')$ -capacity and the $(1, r)$ -capacity (see e.g. [1], Theorem 5.5.1), the existence of the solution is obtained also when λ is “absolutely continuous” with respect to the $(1, r)$ -capacity, $r \leq 2q'$. Due to the sharpness of such comparison results, the condition $r > 2q'$ does not guarantee the existence of the solution.

Some difficulties arise when trying to compare the $(2, q')$ -capacity with the first order Orlicz capacities. Of course, if we assume that λ is “absolutely continuous” with respect to the $(1, A)$ -capacity and $A(t) \preceq t^{2q'}$ (i.e. $A(ct) \leq t^{2q'}$ for t large), then a unique solution exists, because

$$C_{1,A} \leq c_1 C_{1,2q'} \leq c_2 C_{2,q'}.$$

It is natural to examine the case $A(t) \succ t^{2q'}$, $q > N/(N-2)$.

The condition λ “absolutely continuous” with respect to the $(1, A)$ -capacity does not give any information on the existence of the solution of problem (4.1). However, assuming in addition that λ is concentrated on a set E of null $(2, q')$ -capacity, i.e. $\lambda(B) = \lambda(B \cap E)$ for every Borelian subset B of Ω , by the result of Baras–Pierre one can conclude the nonexistence of the solution.

At this point a question arises, namely, the existence of measures λ which satisfy such assumptions. Our Theorem 3.2 gives a positive answer, because it suffices to consider any measure concentrated on the Cantor set $E_{\mathcal{L}}$ constructed therein.

Our conclusion about the nonexistence agrees with the result in [8] (and therefore also with the one in [16]), where more general nonlinear equations are studied and solutions (which are also distributional) are actually defined through a process of approximation. In fact, the assumption λ concentrated on a set of null $(1, t^{2q'} \log^{2q'-1+\epsilon}(e+t))$ -capacity (for some $\epsilon > 0$) of Proposition 2.19 in [8] is stronger than λ concentrated on a set of null $(2, q')$ -capacity.

In order to complete the discussion about the assumptions on the right hand side of the equation in problem (4.1) in terms of Orlicz capacities, we remark that even in the case of measures λ concentrated on a set of null $(1, A)$ -capacity we may obtain some conclusions. Namely, if $A(t) \preceq t^{2q'}$ the condition λ concentrated on a set of null $(1, A)$ -capacity does not give any information on the existence of the solution of problem (4.1). However, assuming in addition that λ is “absolutely continuous” with respect to the $(2, q')$ -capacity, by the result of Baras–Pierre one can conclude the existence of the solution. Finally, if A satisfies the assumption of Theorem 3.2, then again by the result of Baras–Pierre one can conclude the nonexistence of the solution: in fact,

$$C_{1,A}(E) = 0 \Rightarrow C_{2,q'}(E) = 0$$

and therefore λ is not “absolutely continuous” with respect to the $(2, q')$ -capacity, hence the nonexistence of solutions can be deduced.

Previous considerations allow us to state the following

Theorem 4.1. *Let Ω be a bounded open subset of $\mathbb{R}^N$, $N > 2$, $q > \frac{N}{N-2}$. If $A = A(t)$ is a N -function such that*

$$\int_0 \tilde{A}'(t^{1-2q'}) dt < \infty,$$

λ is a nontrivial bounded Radon measure on Ω concentrated on a set of null $(2, q')$ -capacity and such that

$$\forall E \subset \Omega, \quad C_{1,A}(E) = 0 \Rightarrow |\lambda|(E) = 0,$$

then problem (4.1) does not have distributional solutions.

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