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ERRATUM

Erratum to: Symplectic embeddings of polydisks

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The original proof of Lemma 6.1 implicitly assumed that all upper level curves in the limiting building F had punctures asymptotic to geodesics in the classes $(a, 0)$. (Throughout we are adopting the same notation as in the main paper.) The purpose of this erratum is to justify this assumption; the justification will rely on our quantitative hypotheses. The statement of the lemma remains unchanged.

Suppose that F_0, the top level of our building, has a single curve f which is not a cover of curves in the foliation $\mathcal{F}$. Then since F has genus 0, each puncture of f must be matched to unions of limits of the foliation curves. Hence the limiting geodesics must indeed lie in the classes $(a, 0)$. It therefore suffices for us to establish the following.

Lemma 0.1 *F_0 contains a single curve that is not a cover of a curve in the foliation $\mathcal{F}$.*

Proof Observe first that since the space of closed foliation curves in $\mathcal{F}$ has two connected components, and the building F is a limit of curves having intersection number

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1 with the fiber (foliation) class, by positivity of intersection, there can be at most two curves in $X \setminus L$ that are not in our original foliation. Arguing by contradiction we assume there are two curves and denote them by f_1 and f_2. Note that f_1 and f_2 must be somewhere injective since they have intersection number 1 with certain closed foliation curves. Let $d_i = f_i \bullet \mathbb{C}P^1(\infty)$ and $e_i = f_i \bullet E$.

Let $C_0 \cup C_1$ be a broken configuration in the foliation $\mathcal{F}$, as in Proposition 5.1 and Lemma 5.2. If either curve f_i has a puncture asymptotic to a Reeb orbit covering a geodesic in class $(a, 0)$ for $a > 0$ [$a < 0$], then $f_i \star C_0 \geq 1$ [$f_i \star C_1 \geq 1$] and thus $f_i \bullet C \geq 1$ for every closed curve C in our foliation. This is a contradiction since the total intersection number of F with each C is 1.

Thus, f_1 and f_2 must converge at each of their punctures to an orbit representing a class (k, l) with $l \neq 0$. As F has genus 0 and these punctures cannot be matched with curves in the foliation, it follows then that f_1 and f_2 must both be planes. Denote the homology classes of their asymptotic limits by (k_1, l) and $(k_2, -l)$. Without loss of generality, we assume $l \geq 1$.

Now we compare the Fredholm indices of the various components.

The top level consists of f_1 and f_2, together with some number of planes that cover curves of type C_0 and C_1 (defined as in Proposition 5.1). Let K_0 and K_1 be the numbers of planes that cover curves of type C_0 and C_1 respectively and let a_i, $i = 1, \dots, K_0$ and b_i, $i = 1, \dots, K_1$ be their covering multiplicities, and r_i, $i = 1, \dots, K_0$ and s_i, $i = 1, \dots, K_1$ be the respective numbers of ends. By Proposition 3.1 such covers have Fredholm indices

$$\text{index} = r_i - 2 + 2a_i; \quad \text{index} = s_i - 2 + 2b_i$$

respectively.

The bottom levels consist of a sub-building in T^*L, passing through the $2d$ constraint points. The total Fredholm index is $2S - 2K$ where S is the number of positive ends and K is the number of components. Since the sub-building must satisfy $2d$ point constraints, we may assume $2S - 2K \geq 4d$. (This holds even if some components are multiply covered.)

Finally, the total Fredholm index of the whole building (omitting point constraints) is $4d$. We compute this as the sum of the indices of the components of the building, subtracting constraints from matching asymptotics:

$$\begin{aligned} 4d &= (2S - 2K) + \text{index}(f_1) - 1 + \text{index}(f_2) - 1 \\ &\quad + \sum_{i=1}^{K_0}((r_i - 2 + 2a_i) - r_i) + \sum_{i=1}^{K_1}((s_i - 2 + 2b_i) - s_i). \end{aligned}$$

Then since $2S - 2K \geq 4d$ and the sums are clearly nonnegative, we see that the equality is possible only if $(\text{index}(f_1) - 1) + (\text{index}(f_2) - 1) \leq 0$. However, the f_i are somewhere injective planes, and by Proposition 3.1 they have nonnegative odd index, so $\text{index}(f_1) = \text{index}(f_2) = 1$.

Now using Proposition 3.1 again for the index formula we observe

$$\begin{aligned} 0 < \text{area}(f_2) &= Rd_2 - e_2 + k_2 - 2l \\ &= (R-3)d_2 - l + 3d_2 - e_2 + k_2 - l \\ &= (R-3)d_2 - l + \frac{1}{2}(\text{index}(f_2) + 1) \\ &= (R-3)d_2 - l + 1. \end{aligned}$$

Note that $d_2 \geq 0$ and $R - 3 < 0$, so $0 < \text{area}(f_2) \leq 1 - l$. As we assume $l \geq 1$ this gives our contradiction. □