Correction to: New Non-Existence Proofs for Ovoids of Hermitian Polar Spaces and Hyperbolic Quadrics

J. Bamberg, J. De Beule and F. Ihringer

Abstract. We correct some lemmas in Section 3 and Section 4 in 'New non-existence proofs for ovoids of Hermitian polar spaces and hyperbolic quadrics', Ann. Comb., 21(1):25–42, 2017.

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1. Corrections for Section 3

Proposition 3.4, Corollary 3.5, and Lemma 3.8 require that the group G needs to act generously transitively. This does not suffice. All aforementioned results are correct if we demand that G has rank 3. For instance, the following is a correct version of Proposition 3.4.

Proposition 1. *Let G be a rank 3 group on Γ . Let $\chi, \psi \in \mathbb{C}^n$ with $\chi, \psi \notin \langle j \rangle$. Then the following statements are equivalent.*

- (a) *One of the vectors χ and ψ is a non-trivial weighted m -ovoid, and one of the vectors is a non-trivial weighted i -tight set.*
- (b) *For all $g \in G$, we have $\psi^g \chi^\top = mi$.*

To prove this, we need the following result by Delsarte [3, Th. 6.8].

Theorem 1. *Let G be a group which acts generously transitively on Γ . Let $\chi, \psi \in \mathbb{C}^n$. Then the following statements are equivalent.*

- (a) *For all non-principal minimal idempotents E of the association scheme arising from G , we have $\chi E = 0$ or $\psi E = 0$.*

(b) Over all $g \in G$, the scalar product $\psi^g \chi^\top$ is constant.

Proof. (Proposition 1) As in [1], write $\mathbb{C}^n = \langle j \rangle \perp V_+ \perp V_-$ and let E_+ and E_- be the non-principal idempotents corresponding to V_+ and V_- . As G is rank 3, there are only two non-principal minimal idempotents in the view of Theorem 1, that is E_+ and E_- . So a nontrivial vector of $\langle j \rangle \perp V_-$ is precisely a weighted m -ovoid, and a nontrivial vector of $\langle j \rangle \perp V_+$ is precisely a weighted i -tight set. Suppose (a) holds, and without loss of generality, suppose χ is a (nontrivial) weighted m -ovoid and ψ is a (nontrivial) weighted i -tight set. Now every image of ψ under the action of G is also a weighted i -tight set, and so by [1, Lemma 3.3], (b) holds. Conversely, if (b) holds, then over all $g \in G$, the scalar product $\psi^g \chi^\top$ is constant. So by Theorem 1, we have $\chi E_+ = 0$ or $\psi E_+ = 0$, and $\chi E_- = 0$ or $\psi E_- = 0$, where $E_\pm$ is the projection to $V_\pm$. By definition, (a) holds. $\square$

To see why we need G to be rank 3, consider the following. Suppose that G is rank 4. Then we have three non-principal minimal idempotents. Denote the corresponding eigenspaces by V_1, V_2 and V_3 . Suppose that $V_+ = V_1$ and $V_- = V_2 \perp V_3$. Let χ , respectively, ψ be a non-trivial weighted m -ovoid, respectively, m' -ovoid. Suppose that $\chi \in \langle j \rangle \perp V_2$ and $\psi \in \langle j \rangle \perp V_3$. Then $\chi^g \in \langle j \rangle \perp V_2$ for all $g \in G$, so $\psi^g \chi^\top = mm'$ for all $g \in G$, but χ is not a weighted tight set.

Correction for Sect. 4 Lemma 4.3 is wrong, but not needed. A correct version of our proof of Theorem 4.5 due to Hendrik Van Maldeghem can be found in [2, §2.6].

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J. Bamberg
Centre for the Mathematics of Symmetry and Computation,
Department of Mathematics and Statistics
The University of Western Australia
35 Stirling Highway
Crawley, WA, 6009
Australia
e-mail: John.Bamberg@uwa.edu.au

J. De Beule
Department of Mathematics and Data Science
Vrije Universiteit Brussel
Pleinlaan 2
1050 Brussels
Belgium
e-mail: Jan.De.Beule@vub.be

F. Ihringer
Vakgroep Wiskunde: Analyse, Logica en Discrete Wiskunde
Universiteit Gent
Krijgslaan 281
9000 Gent,
Belgium
e-mail: Ferdinand.Ihringer@ugent.be