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Annals of Combinatorics

Foreword

Analysis of Algorithms was created by Donald Knuth around 1962 and largely developed in his famous series of books *The Art of Computer Programming*. Typically, one asks for the average behaviour of certain parameters of algorithms and data structures, often related to running time or storage requirements. More recently, the investigation of the limiting distribution (under a precisely defined probability model) has become a focus in the theory. Methods from *analytic combinatorics* and *probability theory* are now advanced enough to tackle such questions. Knuth's original intention to put the quantitative analysis of algorithms on a serious mathematical basis has found many followers over the last 35 years. This is documented by a series of special issues on the subject in various journals.

This is the first time that *Annals of Combinatorics* hosts such a special issue. The editors are extremely thankful to the editor-in-chief Bill Chen for giving them this opportunity. While the subject has quite a combinatorial flavour anyway, we have asked especially for submissions with a strong emphasis on (analytic) combinatorics. The papers were refereed according to the usual standards, and the authors were asked to present their material in a gentle and reader-friendly manner, so that the usual readers of the journal will have the opportunity to become familiar with this attractive subject.

Contents.

The paper by Dong, Gao, and Panario deals with decomposable combinatorial objects coming from the exp-log class. To give an example, permutations decompose into cycles, whence their exponential generating function is $\exp\log\frac{1}{1-z}=\frac{1}{1-z}$. The emphasis is on restrictions, for instance, in the permutation case about which cycle lengths can occur and how often. The (asymptotic) enumeration is performed by attacking appropriate generating functions with methods from complex analysis.

Special cases of *increasing trees* existed in the literature for a long time, but the first systematic treatment was performed by Bergeron, Flajolet, and Salvy. Drmota now considers the *height*. For the general case, he can derive bounds. For special families of increasing trees (which are quite natural ones, as they also appeared in other analyses), much more precise results are possible. These cases include (increasing) *binary*, *recursive*, and *plane oriented* trees. The latter family is also known as *heap ordered trees*.

Simply generated families of trees (which contain, e.g., Catalan trees, Cayley trees and ordered trees as special instances) are important tree models with applications in computer science. After a suitable rescaling, simply generated trees converge to the continuum random tree, which has been introduced by Aldous, and this continuum tree has a contour process: The normalized Brownian excursion. Hence, a lot of functionals (as, e.g., the height, the width, the total path length and the Wiener index) of simply generated trees converge to a functional of the normalized Brownian excursion and so a detailed study of their properties will help to improve our understanding of simply generated trees. In the paper by Fill and Janson a general theorem for functionals X of the normalized Brownian excursion is given leading to precise asymptotic results for $\mathbb{P}\{X > x\}$ with $x \to \infty$, thus establishing the so-called *tail estimates*. This theorem is applied to many examples of interest.

The space requirements of a sorting algorithm introduced by af Hällström are analysed in detail in the paper by Janson. The algorithm under consideration maintains during its execution a list of sorted complete subintervals of the data set that are eventually merged. By using probabilistic techniques such as martingale theory, a precise asymptotic study of the space requirements of the algorithm is given assuming the random permutation model for the input sequence, leading to asymptotic expansions for the moments and limiting distribution results.

Knuth asked about the joint distribution of left respectively right path length in binary trees. This and related questions found the attention of several researchers, but Panholzer's combinatorial approach produces the richest results in this specific setup. The probability distribution is exactly and explicitly described. This is based on a generating function in 4 variables. Mixed moments of suitably scaled random variables can be pumped out using operators.

The *leader election* algorithm proceeds by consecutive rounds of coin flippings, where only the winners of a round continue. If there is no winner, the round must be repeated. Here, the instance of a biased coin is revisited. The analysis presented in the paper by Louchard and Prodinger is based on an approach developed by Louchard et al., which successfully used the fact that some underlying distributions are of the *Gumbel* type. Here, old results by Janson, Knessl, and Szpankowski are rederived without much effort; new results (variance) are obtained, and some analysis of Knessl's is simplified, using Mellin transforms.

Contention trees arise in the context of protocols for communication networks. Wagner explains that they are essentially tries, and uses the technical machinery that is known for the analysis of tries to describe parameters of contention trees. This paper is of the semi-expository type, but contains also new material.

A short paper by Sandra Sattolo, based on her master's thesis (!) has recently created some activity in the algorithms community. Her algorithm produces a random *cyclic* permutation, by a sequence of $n-1$ random interchanges. Wilson found out that an older algorithm by Fisher and Yates (to create a random permutation) is of a similar spirit. He sets up an algebraic framework for such algorithms, and analyses the parameters *number of moves* and *distance*, rederiving existing expressions and adding some new ones. Multivariable generating functions are used in a consistent way.

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