Correction to: Unicity on Entire Function Concerning Its Differential-Difference Operators

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Abstract. In this paper, we give a correction to Theorem 1 in a previous paper [Results Math. 76, Article number: 147 (2021)].

Mathematics Subject Classification. 30D35, 39A70.

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1. Introduction and main results

Recently, the present author originally considered the uniqueness of entire function sharing finite values with its differential-difference operators. Unfortunately, there was an error in the proof of [1, Theorem 1] (that is the equality (3.57) is wrong) and so there exists another case in the conclusion of [1, Theorem 1]. Here we correct it as follows.

Theorem 1. *Let $f(z)$ be a transcendental entire function of finite order, let $\eta \neq 0$ be a finite complex number, $n \geq 1, k \geq 0$ two integers and let a, b be two distinct finite complex values. If $f(z)$ and $(\Delta_\eta^n f(z))^{(k)}$ share a CM and share b IM, then either $f(z) \equiv (\Delta_\eta^n f(z))^{(k)}$ or $a = 2b$,*

$$f(z) \equiv be^{2(cz+d)} - 2be^{cz+d} + 2b,$$

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and

$$(\Delta_{\eta}^n f(z))^{(k)} \equiv b e^{cz},$$

where $c = (-2)^{-\frac{n+1}{k}}$ for $k \geq 1$ and d are two finite constants.

When $k = 0$, the Corollary 1 in [1] is true.

2. The Proof of Theorem 1

We only consider the case $ab \neq 0$. From the proof of Theorem 1 as in [1], we know that

$$a = 2b, \quad (2.1)$$

$$f(z) = b e^{2(cz+d)} - 2b e^{cz+d} + 2b, \quad (2.2)$$

and

$$(\Delta_{\eta}^n f)^{(k)}(z) = b e^{cz+d}, \quad (2.3)$$

where $c \neq 0$ and d are two finite constants. Then by (2.2) we have

$$(\Delta_{\eta}^n f)^{(k)}(z) = b(2c)^k (e^{2c\eta} - 1)^n e^{2(cz+d)} - 2b c^k (e^{c\eta} - 1)^n e^{cz+d}. \quad (2.4)$$

By (2.3) and (2.4), we claim that $e^{2c\eta} - 1 = 0$. Otherwise, we set $B = e^{2c\eta} - 1$ is a finite constant. Then by (2.4) and Lemma 2.8 in [1], we can obtain

$$T\left(r, (\Delta_{\eta}^n f)^{(k)}(z)\right) = 2T\left(r, e^{cz+d}\right) + S\left(r, e^{cz+d}\right). \quad (2.5)$$

But we can know from (2.3) that

$$T\left(r, (\Delta_{\eta}^n f)^{(k)}(z)\right) = T\left(r, e^{cz+d}\right) + S\left(r, e^{cz+d}\right), \quad (2.6)$$

it follows from (2.5) that

$$T\left(r, (\Delta_{\eta}^n f)^{(k)}(z)\right) = S\left(r, e^{cz+d}\right). \quad (2.7)$$

Since $f(z)$ and $(\Delta_{\eta}^n f)^{(k)}(z)$ share a CM, and b IM, and $f(z)$ is an entire function, then by the Second Nevanlinna's Fundamental Theorem and (2.7), we have

$$\begin{aligned} T(r, f(z)) &\leq \overline{N}(r, f(z)) + \overline{N}\left(r, \frac{1}{f(z) - a}\right) + \overline{N}\left(r, \frac{1}{f(z) - b}\right) + S(r, f) \\ &= \overline{N}\left(r, \frac{1}{(\Delta_{\eta}^n f)^{(k)}(z) - a}\right) + \overline{N}\left(r, \frac{1}{(\Delta_{\eta}^n f)^{(k)}(z) - b}\right) + S(r, f) \\ &\leq 2T\left(r, (\Delta_{\eta}^n f)^{(k)}(z)\right) + S\left(r, e^{cz+d}\right) = S\left(r, e^{cz+d}\right), \end{aligned} \quad (2.8)$$

which implies $T(r, f(z)) = S(r, e^{cz+d})$, a contradiction. Therefore, $e^{2c\eta} - 1 = 0$. We also claim that $e^{c\eta} = -1$. Otherwise, by (2.4) we obtain $(\Delta_\eta^{nl} f)^{(k)}(z) \equiv 0$, but it is impossible. Therefore, $e^{c\eta} = -1$ and we can get

$$(\Delta_\eta^n f)^{(k)}(z) = (-2)^{n+1} c^k b e^{cz+d}. \quad (2.9)$$

So we combine (2.3) and (2.5), we can get $c = (-2)^{-\frac{n+1}{k}}$ for $k \geq 1$. When $k = 0$, we can see from (2.3) and (2.5) that $(-2)^{n+1} = 1$, a contradiction.

This completes the proof of Theorem 1.

Reference

- [1] Huang, X.H.: Unicity on entire function concerning its differential-difference operators. *Results Math.* 76, Article number: 147 (2021)

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