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Results in Mathematics

Correction

Correction to: Pseudovarieties of Ordered Completely Regular Semigroups

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The original version of this article unfortunately contained a mistake. The typesetting of mathematical notation was incorrect. The corrected details are given below for your reading.

In "Preliminaries and Notation" section, third paragraph should read as:

A quasiorder $\leq$ on a semigroup S is *stable* if, for every $a, s, t \in S$, the following condition holds: if $s \leq t$ then $sa \leq ta$ and $as \leq at$. By an *ordered semigroup*, we mean a semigroup $(S, \cdot)$ which is equipped with a stable partial order. *Homomorphisms of ordered semigroups* are homomorphisms of semigroups which are isotone. Similar to the unordered case, *pseudovarieties of ordered semigroups* are classes of finite ordered semigroups closed under taking homomorphic images, (ordered) subsemigroups and finite products. For a pseudovariety V of finite ordered semigroups, we denote by V^d the pseudovariety of all dually ordered semigroups, i.e., $(S, \cdot, \leq) \in \mathsf{V}^d$ if and only if $(S, \cdot, \geq) \in \mathsf{V}$. We say that V is *self-dual* if $\mathsf{V} = \mathsf{V}^d$.[1] Such a pseudovariety of ordered semigroups is also characterized by the property that $(S, \cdot, \leq) \in \mathsf{V}$ implies $(S, \cdot, =) \in \mathsf{V}$: the identity mapping is a homomorphism from the ordered semigroup $(S, \cdot, =)$ onto an arbitrary ordered semigroup $(S, \cdot, \geq)$, while

[1] It is perhaps more common to consider in the purely algebraic context the *dual* of a semigroup $(S, \cdot)$ to be $(S, *)$ where $a * b = b \cdot a$. This is not the notion of duality concerning us here.

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the diagonal mapping embeds $(S, \cdot, =)$ into the product $(S, \cdot, \leq) \times (S, \cdot, \geq)$. If a pseudovariety of semigroups V is given, then we may consider the (self-dual) pseudovariety consisting of all ordered semigroups which are members of V equipped with every possible stable partial order. Conversely, if V is a self-dual pseudovariety of ordered semigroups, then we may consider the pseudovariety consisting of all semigroups $(S, \cdot)$ such that $(S, \cdot, =) \in \mathsf{V}$. These constructions give a one-to-one correspondence between self-dual pseudovarieties of ordered semigroups and pseudovarieties of semigroups. Note that usually the same symbol is used to denote both a pseudovariety of semigroups and the corresponding self-dual pseudovariety of ordered semigroups.

In "Preliminaries and Notation" section, (seventh sentence of the) ninth paragraph should read as:

By a *system of relations* $(\rho_n)_n$, we mean a family of relations indexed by the positive integers such that ρ_n is a relation on $\overline{\Omega}_n\mathsf{S}$ for every n. We say that the system $(\rho_n)_n$ is *fully invariant* if, for each continuous homomorphism $\varphi : \overline{\Omega}_n\mathsf{S} \to \overline{\Omega}_m\mathsf{S}$ and $u \; \rho_n \; v$, we have $\varphi(u) \; \rho_m \; \varphi(v)$. Additionally, if every relation ρ_n in this system is a closed stable quasiorder on $\overline{\Omega}_n\mathsf{S}$, then we call the system $\rho = (\rho_n)_n$ a *fully invariant system of closed stable quasiorders*. It is not clear whether every such system ρ determines a pseudovariety V such that $\rho = (\rho_{\mathsf{V},n})$ (see [5] for a discussion concerning a related conjecture). To explain what kind of property is potentially missing, first denote $\tilde{\rho} = (\widetilde{\rho_n})_n$, where $\widetilde{\rho_n}$ is the equivalence relation corresponding to ρ_n. Then, for each n, we consider $\overline{\Omega}_n\mathsf{S}/\widetilde{\rho_n}$, which is a compact ordered semigroup, where the partial order is $\leq_{\rho_n}$. If for each $n \geq 1$ the ordered semigroup $\overline{\Omega}_n\mathsf{S}/\widetilde{\rho_n}$ is residually finite, then $\rho = \rho_{\mathsf{V}}$, where V is given as the class of all finite ordered semigroups which are finite quotients of some ordered semigroup $(\overline{\Omega}_n\mathsf{S}/\widetilde{\rho_n}, \cdot, \leq_{\rho_n})$. If this property is true then we call ρ a *complete system of pseudoinequalities*. Note that, for the corresponding pseudovariety V of ordered semigroups, we may also write $\mathsf{V} = [\![\rho]\!]$ to mean $\mathsf{V} = [\![\rho_n : n \geq 1]\!]$. We talk about a *complete system of pseudoidentities* when V is a self-dual pseudovariety of ordered semigroups. Notice also that V is self-dual if and only if each ρ_n is symmetric.

In Theorem 4.8, second and third paragraph of proof should read as:

To show that $\gamma \circ \iota$ is the identity mapping, we need to prove that $(\mathsf{V} \cap \mathsf{NB}) \vee (\mathsf{V} \cap \mathsf{G}) = \mathsf{V}$ for every $\mathsf{V} \subseteq \mathsf{NOCR}$. This is true if $\mathsf{V} \models x^\omega \leq x^\omega y^\omega x^\omega$, by Lemma 4.7. Clearly, one may use the dual version of the lemma if $\mathsf{V} \models x^\omega y^\omega x^\omega \leq x^\omega$. So, we may assume that V does not satisfy any of these two pseudoinequalities. Thus, by Lemma 4.1, we know that $\mathsf{Sl} \subseteq \mathsf{V}$. From Theorem 3.8, we get that V is self-dual. This means that if we consider an arbitrary ordered semigroup $(S, \cdot, \leq) \in \mathsf{V}$, then the pseudovariety V contains also the ordered semigroup $(S, \cdot, =)$. Since $(S, \cdot, \leq)$ is a homomorphic image of $(S, \cdot, =)$, we may deal only with the case of unordered semigroups.

So, let $(S, \cdot, =) \in \mathsf{V}$ be arbitrary. Now, it is possible to modify the proof of Lemma 4.7 in such a way that $H_e^\top$ is replaced by H_e^0. Since the partial order on

S is equality, the mapping ψ_e is trivially isotone. Moreover, V contains SI and, therefore, $H_e^0 \in (\mathsf{V}\cap\mathsf{NB})\vee(\mathsf{V}\cap\mathsf{G})$. Thus, we conclude that $S \in (\mathsf{V}\cap\mathsf{NB})\vee(\mathsf{V}\cap\mathsf{G})$. We have proved the required equality $(\mathsf{V} \cap \mathsf{NB}) \vee (\mathsf{V} \cap \mathsf{G}) = \mathsf{V}$.

The original article has been corrected.

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