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Results in Mathematics

ERRATUM

Erratum to: On the Closure of Translation–Dilation Invariant Linear Spaces of Polynomials

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In our recently published paper "On the closure of translation–dilation invariant linear spaces of polynomials" (see original article), there was an error. Namely, the decomposition of Ω_V given after Lemma 8 in original article does not hold for $d \geqslant 3$ for some TDI-subspaces of $\mathbb{R}[x]$. For example, the TDI-subspace $V = \mathbf{span}\{x_1^a x_2^b x_3^c : a \leqslant 1, b \leqslant 2\}$ of $\mathbb{R}[x_1, x_2, x_3]$ does not possess such a decomposition. This fact was pointed to the authors by Prof. Georg Zimmermann after publication of the paper. Below we give a new proof of the main result of original article avoiding such a hypothesis. We use the notation introduced in original article.

Theorem 1. *If a sequence in a TDI-subspace in the polynomial ring* $\mathbb{R}[x]$ *converges pointwise to a polynomial, then this polynomial belongs to the subspace, too.*

Proof. We note that if V is a TDI-subspace of $\mathbb{R}[x]$ such that there exist natural numbers $N_1, \ldots, N_d$ satisfying

$$\Omega_V = \bigcup_{k=1}^{d} Z_i,$$

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where $Z_i = \{\alpha \in \mathbb{N}^d : \alpha_i \leqslant N_i\}$ for $i = 1, 2, \ldots, d$, then our statement is proved in original article, in Case 1 in the proof of Theorem 9 in original article.

Now let V be an arbitrary TDI-subspace of $\mathbb{R}[x]$ and let the sequence $(p_n)_{n\in\mathbb{N}}$ of elements of V converge pointwise to the polynomial p. Assume that

$$p(x) = \sum_{|\gamma|\leqslant \deg(p)} a_\gamma x^\gamma$$

with $a_\alpha \neq 0$ for some $\alpha = (\alpha_1, \ldots, \alpha_d)$ which is not in Ω_V. We define

$$\widetilde{\Omega} = \{\beta : \beta_k < \alpha_k \text{ for at least one } k\}, \text{ and } \widetilde{V} = \mathbf{span}\{x^\alpha : \alpha \in \widetilde{\Omega}\}.$$

Then $\widetilde{V}$ is a TDI-subspace of $\mathbb{R}[x]$ and $\Omega_V \subseteq \widetilde{\Omega}$. Indeed, assuming that β is in Ω_V with $\beta \notin \widetilde{\Omega}$ gives $\beta_k \geqslant \alpha_k$ for all k, so $\alpha \leqslant \beta$, hence α is in Ω_V, a contradiction.

Obviously, $\widetilde{\Omega} = \bigcup_{k=1}^d Z_k$, where $Z_k = \{\beta : \beta_k < \alpha_k\}$. Hence, by our remark above, we conclude that p is in $\widetilde{V}$ which, by our construction, is impossible. This proves the theorem. $\square$

We note that Lemma 8 of original article is not needed in this proof.

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